On parametric generalized equations

Radek Cibulka

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Suppose that a Lipschitz function $p : [a, b] \to \mathbb{R}^n$ is given. By a parametric generalized equation we mean a problem to find a Lipschitz continuous function $z : [a, b] \to \mathbb{R}^n$ such that

$$p(t) \in f(z(t)) + F(z(t)) \quad \text{whenever} \quad t \in [a, b],$$

where a single-valued function $f : \mathbb{R}^n \to \mathbb{R}^n$ is continuously differentiable and a set-valued mapping $F : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ has closed graph. Hence $z(\cdot)$ is a selection for the solution mapping

$$S : [a, b] \ni t \longmapsto S(t) := \{z \in \mathbb{R}^n : \ p(t) \in f(z) + F(z)\},$$

that is, $z(t) \in S(t)$ whenever $t \in [a, b]$.

First, we present sufficient conditions for the existence of such a selection. Second, we investigate a modification of the Euler-Newton continuation method for tracking solution trajectories developed by Dontchev, Krastanov, Rockafellar, and Veliov. This allows us to deal with a non-differentiable input signal $p(\cdot)$. Finally, implementing this method (in Matlab) we provide a simulation of the behavior of some basic non-regular electrical circuits, that is, the circuits where various types of diodes are present.