

# On Second-Order Optimality Conditions for Discontinuous Optimal Controls

Nikolai P. Osmolovskii

Moscow State University of Civil Engineering;  
University of Technology and Humanities in Radom, Poland;  
Systems Research Institute, Polish Academy of Sciences  
Email: `osmolovski@uph.edu.pl`

We observe some results contained in monographs [1] and [2]. First we formulate no-gap necessary and sufficient second order conditions in optimal control problems with ordinary differential equations considered on a non-fixed time interval, subject to end-points and mixed state-control constraints. The conditions admit discontinuities of the first kind of the reference control and take them into account. Next we formulate such conditions for optimal control problems with a vector control variable having two components: a continuous unconstrained control appearing nonlinearly in the control system and a bang-bang control appearing linearly and belonging to a convex polyhedron. Such type of control problem arises in many applications. Particular emphasis is given to bang-bang control problems. Bang-bang controls induce an optimization problem with respect to the switching times of the control. It turned out that the classical second-order sufficient condition for the Induced Optimization Problem (IOP), together with the so-called strict bang-bang property, ensures second-order sufficient conditions (SSC) for the bang-bang control problem. We mention a number of numerical examples in different areas of application which illustrate the verification of SSC for both regular controls and bang-bang controls.

## References

- [1] Milyutin, A.A., Osmolovskii, N. P., Calculus of variations and optimal control. Translations of mathematical monographs, vol. **180**, American Mathematical Society, 1998.
- [2] Osmolovskii, N.P. and Maurer, H., Second-Order Necessary and Sufficient Optimality Conditions in Calculus of Variations and Optimal Bang-Bang Control: Applications to Regular and Bang-Bang Control. Society for Industrial and Applied Mathematics, Philadelphia, USA, 2012.