Metric regularity and convergence of iterative schemes

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Given Banach spaces X and Y, a single-valued (possibly non-smooth) mapping $f : X \to Y$ and a multivalued mapping $F : X \rightrightarrows Y$, we investigate the properties of the solution mapping corresponding to a generalized equation:

$$\text{Find} \quad x \in X \quad \text{such that} \quad 0 \in f(x) + F(x). \tag{1}$$

This model has been used to describe in a unified way various problems such as equations, inequalities, variational inequalities, and in particular, optimality conditions. In the first part, we present a result concerning the stability of metric regularity under set-valued perturbations as well as an infinite-dimensional generalization of the Izmailov's theorem [6] which is an extension both of the Clarke's theorem [4] and finite-dimensional version of the Robinson's theorem [8]. In the latter part, we study the convergence properties of the following iterative process for solving (1): *Choose a sequence of set-valued mappings* $A_k : X \times X \rightrightarrows Y$ *approximating the function* f *and a starting point* $x_0 \in X$, *and generate a sequence* (x_k) *in* X *iteratively by taking* x_{k+1} *to be a solution to the auxiliary generalized equation*

$$0 \in A_k(x_{k+1}, x_k) + F(x_{k+1}) \quad \textit{for each} \qquad k \in \{0, 1, 2, \dots\}. \tag{2}$$

The results from the first part are applied in the study of (super-)linear convergence of (2). Especially, several particular cases are discussed in detail. The presentation is based on the forthcoming papers with Samir Adly and Huynh Van Ngai; as well as on the one with Asen Dontchev and Michel H. Geoffroy.

References

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