Viennese Vintage Workshop, 27-28, October 2003

Age-Structured Models in Population Dynamics and Economics

Abstracts

Optimal prevention programs in an age-structured drug initiation model

Christian Almeder and *Gernot Tragler*

We start with a brief review of dynamic models of illicit drug use consisting of ODE's (both descriptive and normative). Even though these models do a good job in approximating empirical data such as the numbers of 'light' and 'heavy' cocaine users in the U.S., they neglect an important fact, which is the strong dependence of drug initiation on age. This leads to the core topic of this talk, which is the presentation of a model for drug initiation that extends traditional dynamic models by considering explicitly the age distribution of the users. On the basis of a 2-groups model in which the population is split into a user and a non-user group the advantage of a continuous age distribution is shown by considering more details and by yielding new results. Neglecting death rates reduces the model to a single state (1-group) descriptive model which can still simulate some of the complex behavior of drug epidemics such as repeated cycles. Furthermore, prevention programs – especially school-based programs – can be targeted to certain age classes. So in order to discover how best to allocate resources to prevention programs over different age classes we formulate and solve optimal control models.

Preserving Transfer Benefit for Present and Future Generations

Noël Bonneuil and Romina Boarini

How should state interventions be calibrated in time and magnitude to guarantee a positive account for each generation as well as a minimal welfare and some equity across age classes? The social contract turns into the dynamic maintenance of a positive account for every present and future generation. It requires either to accelerate or to slow down both pension and education spending. The ensuing dynamic trade-off between generational benefit and equity delineates an optimal interest rate in human capital. Moreover, the favorable effect of population growth on the capacity to pay for pensions is thwarted when fairness between age classes together with the preservation of a positive net benefit for future generations is also considered.

Modeling vintage structures with differential-difference equations: specification, stability assessment and optimal control

Raouf Boucekkine

We present an overview of the differential-difference (DDE hereafter) approach to vintage capital growth models, first suggested by Benhabib and Rustichini (1991). We start with a vintage capital model of the Solow type, ie. with a constant saving rate, in order to simply illustrate the main specifications within this framework, and the typical stability and numerical solution tools. We then turn to Ramsey vintage capital models with and without a stationary environment, i.e. with and without forcing functions. The dynamic properties of such models, together with the associated analytical tools, are briefly summarized. Finally, we deal with two vintage capital models displaying endogenous growth. The first one describes the dynamics of human capital accumulation under a realistic survival law and a vintage human capital structure. The second is the one hoss shay model with constant return to capital. When handling this model, we survey some optimal control material required to treat the presence of lagged controls.

Endemic states of an immunising infection with positive case fatality in a growing population

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The classical epidemiological model of Daniel Bernoulli* for a potentially fatal infection inconstant population is generalised for a growing population. We consider age-dependent birth- and death rates with a positive Malthusian parameter in the absence of the infection. Critical bounds for the age-dependent case-fatality rates are derived which are compatible with an endemic state. The force of infection is also allowed to depend on age. General formulas for the basic reproduction number in terms of the model parameters and in terms of the equilibrium prevalence of the susceptibles are derived. It is assumed that the duration of the infectious period is not affected by the mortality which is induced by the infection. We concentrate in particular on infections whose infectious period is of negligible duration compared to the life expectancy of an individual.

* Dietz, K. and Heesterbeek, J.A.P., Daniel Bernoulli's epidemiological model revisited, *Math Biosciences* **180**, 2002, 1-21.

Population cycles in the age dependent McKendrick model

Rui Dilão

Easterlin cycles are oscillations in population growth. They have been predicted by Easterlin in the sixties with a very simple two-generation model. In this approach, population cycles have a period of the order of the age of two generations, which, for human populations is of the order of 50 years. In the context of a McKendrick model with age-dependent birth and death rates, here we show the existence of demography cycles. For a population with only one reproductive age class, independently of the stability of the weak solutions and after a transient time, the temporal evolution of the number of individuals of the population is always modulated by a time periodic function. The periodicity of the cycles (Easterlin cycles) is equal to the age of the reproductive age class, and the population retains the memory from the initial data through the amplitude of oscillations of the Easterlin cycles. For a population with a continuous distribution of reproductive age classes, we prove the existence of damped cycles. The periodicity of the damped cycles is associated with the age of the first reproductive age class. Damping increases as the dispersion of the fertility function around the age class with maximal fertility increases. In general, the period of the demographic cycles is associated with the time that a species takes to reach the reproductive maturity.

Boundary Control Problems in Economic models: a dynamic programming approach

Silvia Faggian

We consider a class optimal control problems which can be modeled by an initial-boundary value problem for a linear PDE where the control is acting on the boundary. In particular, we focus on investment with vintage capital, on water management and population dynamics. We use semigroup theory to rephrase the problem as optimal control for a ODE in infinite dimension and apply Dynamic programming to show that the value function satisfies the associated Hamilton-Jacobi-Bellman equation in some generalised sense. We investigate both finite and infinite horizon problems, and properties of the trajectories, discussing in detail the application to the examples.

Optimal age specific labor force: effects of education, productivity and wages

Alexia Fuernkranz-Prskawetz, P. Kort and V. Veliov

We study the optimal hiring and firing policy of a single firm that is subject to hiring and firing costs. More specifically, we allow for age dependency of the labor force and age dependency of the adjustment costs, wages and productivity. In particular we investigate the effect of the age specific wage-productivity gap on the optimal age schedule of the work force.

Optimal control of stochastic PDE's with age structure and applications

Fausto Gozzi

We first introduce a class of optimal control of stochatic PDE's which include some examples arising in economics and population dynamics.
The dynamic programming method in these cases give rise to infinite dimensional Hamilton-Jacobi-Bellman (HJB) equations that may be very difficult to study.
We discuss the state of the art and we present some new results
obtained with Ben Goldys.

Some results on the modeling of Tuberculosis

Mimmo Iannelli

We consider a general epidemic model that includes some peculiar aspects of a disease such as Tuberculosis. In fact long periods of latency and the emergence of antibiotic resistance due to incomplete treatment are very important features of TB dynamics. We formulate a two-strain TB model with an arbitrarily distributed delay in the latent stage of individuals infected with the sensitive strain, and look at the effects of variable periods of latency on the disease dynamics. In this model we discuss invasion and coexistence of the two strains versus a significant parameter such as the rate at which infected people develops drug resistant TB, due to incomplete treatment. Furthermore we consider the single (drug-sensitive) strain case to discuss control policies through screening programs.

Endemic Threshold Results in an Age-Structured Population Model for HIV/AIDS Epidemic

Hisashi Inaba

In this talk we consider an age-duration-structured population model for the HIV infection in a homogeneous community. First we investigate the invasion problem of the HIV/AIDS epidemic by using positive operator theory. Next we examine conditions under which existence and uniqueness of endemic steady state can be shown. In particular, we will show that a backward bifurcation can occur at the threshold, that is. multiple endemic steady states can exist for our structured model for HIV infection. Finaly we discuss stability results for endemic steady states.

Environmental Policy, the Porter Hypothesis and the Composition of Capital: Effects of Learning and Technological Progress

Gustav Feichtinger, Richard F. Hartl, Peter M. Kort, and Vladimir M. Veliov

In this paper the effect of environmental policy on the composition of capital is investigated. By allowing for non-linearities it generalizes Xepapadeas and De Zeeuw (Journal of Environmental Economics and Management, 1999) and determines scenarios in which their results do not carry over. In particular, we show that the way acquisition cost of investment decreases with the age of the capital stock is of crucial importance. Also it is obtained that environmental policy has opposite effects on the average age of the capital stock in the case of either deterioration or depreciation. We also focus more explicitly on learning and technological progress. Among others we obtain that in the presence of learning, implementing a stricter environmental policy with the aim to reach a certain target of emissions reduction has a stronger negative effect on industry profits, which implies quite the opposite as to what is described by the Porter hypothesis.

Population Momentum for Real Population and Linear Fertility Transition

Nan Li

Population momentum is the ratio of a population's ultimate size after a demographic transition to its initial size before the transition. For stable population and instantaneous drop to replacement fertility, Keyfitz found a simple formula for the momentum. Keyfitz's formula has been extended to describe gradual demographic transitions. The stable initial population, however, still obstructs approaching reality. Extending the solution of Lotka equation with time-varying vital rates to negative values of time, this paper provides an approximate formula for the momentum of any initial population that undergoes linear fertility transition. Examples of using data from the United Nations indicate that this formula works well for all the three regions, be their age structures far away from or close to stable.

Optimal harvesting of age class structured forest resources

Olli Tahvonen

We study optimal harvesting of a multiple age class forest resources under discounting and strictly concave optimization criteria. The problem is analyzed as a discrete time, infinite horizon and any number of state and control variables optimization problem. The necessary and sufficient optimality conditions are obtained using the Karush-Kuhn-Tucker theorem in nonlinear programming. It is shown, that at the optimal stationary state, timber harvest may evolve cyclically over time. The cycles occur because in the forest age class model space is measured continuously and time in discrete units. Cycles disappear as period length moves toward zero, when the optimal rotation period is not unique or when the discount factor approaches unity. We develop an equivalent model that can be used to solve the age class problem numerically from any initial state. Numerical examples suggest that the optimal solution converges to the stationary cycle from any initial vintage structure. Next we consider three extensions to the prototype model and include any number of forest land quality classes, alternative use of bare forest land (i.e. agriculture) or alternative use of old growth forest (i.e. environmental conservation) and study their implication on the optimal forest harvesting. Finally, we discuss on applying continuous vs. discrete time in forest age class models.

Optimal taxation and governmental investment: an age-structured model

Tsvetomir Tsachev

Following a recent paper by Feichtinger, Prskawetz and Veliov we present a macroeconomic model in which the population and the capital are age-structured. The government's revenue consists of a single tax, modeled as a percentage of the GNP. The model is in the form of an optimal control problem. The control variables are the collected tax and the percentage of it which the government spends for investments. The discounted social welfare is to be maximized over a prescribed time interval.

Heterogeneous control systems: a general perspective and applications in population dynamics and economics

Vladimir .M. Veliov

We present the concept of a heterogeneous control system, which extends that of age-structured systems by allowing for parameters of heterogeneity other than age or duration. We briefly outline the basic theory, including necessary optimality conditions, second order sufficient conditions, and numerical approaches. Then we discuss some particular heterogeneous control models arising in demography, economics, and epidemiology.