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Abstract

The stylized model presented is an optimal control model of technology investment decision of a single product firm. The firm's technology investment does not have only a long-run positive effect but also a short-run adverse effect on its sales volume. We examine the case of high adverse investment effects where the firm finally leaves the market but we have observed different life cycles till this happens. Depending on the firm's initial technology stock and sales volume, we compute different firm's life cycles, which are driven by a trade-off between two strategies: technology versus sales focus strategy. Indifference curves, where managers are indifferent to apply initially technology or sales focus strategies, separate founding conditions of the firm to various classes distinguishable because of the firm's life cycle.

Keywords: Technology investment; Firm's life cycle; Optimal control; Non linear economic dynamics; Indifference curves; Computational economics

JEL Classification: D21; O30

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1 Introduction

A firm is not a static entity, but instead evolves in response to changes in internal conditions such as resource levels and competencies, and external conditions such as changes in competitive environment and overall economy. Changes in these founding conditions, including strategies taken by the firm, result in distinct and identifiable phases of the firm dynamics called "firm's life cycles".

The diversity across firm's life cycle seems to be significant for capturing a short-term effect called "adverse technology investment effect on sales". This effect characterizes a contraction of the firm's sales volume due to its effort on technology investment. The contraction of the firm's sales volume is the result of short-run behaviors of customers (Mansfield 1968): first, in short-run, rapid changes in product can discourage customers to buy the newest product for example for computer goods, because of the learning cost related to this newest product (Hall and Kan 2003). In this way, dragging sales of Microsoft's computer operating system Windows Vista are a perfect anecdotal example. Second, if customers know that a newer version of the product will arrive, they will prefer to wait for the upcoming version of the product in order to get a better product (to instance durable goods see Adda and Cooper 2000). These short-run customer's behaviors influence the long-run trajectory of the firm. That is why this paper relates firm's life cycle to firm's investment and customer's behavior: on one hand, the firm's investment shows the technology strategy of the firm. On the other hand, the customer behavior is caught by a key parameter of the stylized model which measures the adverse effect of investment on sales volume. The results are that different kinds of optimal long-run behaviors of the firm become noticeable and show the diversity of firm's life cycles. This paper, related on the firm's life cycle, focuses on the firm rather than on the market competition. But considering the firm's technology investment puts more interest on non-price competition.

While there has been a vast literature on entry and exit behavior of firms within an industry, little is known about the expansion and contraction paths of individual firm. In an overview article, Geroski (1995) points out with several stylized facts that entry in a market is a very common thing; for the firm, entry is not problematic. The challenge is rather to be able to survive once it has been introduced on the market. Exit is very likely to follow entry. From this, Geroski concludes that when a new firm is introduced in the market, it generally take several years to compete on equal terms with incumbents. This fact leads to focus on long-run analysis rather than on short-run studies. Moreover, empirical micro founded long-run studies are so uncommon due to the lack of data, that simulation analysis is needed and useful in order to have an idea on the qualitative behavior of the firm. Furthermore, most of this literature concentrates upon the effect of environmental conditions and very few studies have focused on the impact that strategic choices at founding time may have upon the survival prospects of firms. For this reason, knowledge of a firm's life cycle phase enhances the ability to explain and predict future profitability. In fact, within an industry, the life cycle phases of individual firms vary significantly due to differences in the firm's level of knowledge (know-how, competencies, technology stock), in the level of initial investment and re-investment in technology, and in the adaptability to the competitive environment (sales volume).

This paper presents a two dimensional optimal control model of a single product firm where technology investment is the control variable and technology stock and sales volume of the product are the state variables. The idea of this model is to fit some of the regularities shown in industry studies. Among these is the stylized fact that the firm tends to be affected in long-run by its founding conditions and its strategy in its early age (Geroski et al. 2003). In order to answer the following question: what characterizes the firm's life cycle with respect to its founding conditions and its strategy, the paper is organized as follows: in section 2, we present the stylized model, and in section 3, numerical analysis and the solution. The section 4 is devoted to the economic interpretations of the solution considering the founding conditions and the strategies of the firm and the diversity of the firm's life cycle. Finally, section 5 terminates with concluding comments.

2 Stylized model of firm's technology investment policy

We suppose an atomistic market driven by a perfect competition. The single product firm entries in the market as price taker with its founding conditions, viz., the firm's initial technology stock relative to the technology frontier of the market (T_0), and the firm's initial sales volume (C_0). At its entry in the market, the firm has to take a decision concerning its initial technology investment strategy. It should be obvious by now that the firm's initial strategy refers to its optimal technology investment decision: how much the firm should spend on technology investment over time in order to maximize its discounted profit flow. But the firm also needs to decide how much to invest at each time of its life considering that its investment is homogenous and irreversible. So, that the firm's technology investment is a sunk cost: $I_t \geq 0$.

In its (re)investment decision, the firm considers that technology investment will have opposite consequences on sales volume: a positive long-run and a negative short-run effect. For the firm, there is also a trade-off between short-run negative investment effect and long-run positive investment effect on sales which is governed by the goal of maximizing its discounted profit flow over time. Finally, this firm invests in such a way that the discounted profit flow is maximized. Hence, the firm's objective functional is given by

$$\max_{I_t \geq 0} \int_0^{\infty} e^{-rt} \left[\alpha\, C_t - a(I_t)\right] dt\,, \tag{1}$$

where t is time, r is the constant discount rate, α the revenue per sale. Profits decrease with investment costs and increase with revenue, where we model constant revenue (α > 0) per sale (C_t). Investment costs are the costs associated with carrying out technology investments. A convexly increasing function $a(I_t)$ (with $a_0 = 0,\, a' > 0,\, a'' > 0$) denotes investment costs at time t, where the convex shape reflects the fact that there are decreasing returns to effort at any point in time.

By T_t we denote the product's technology stock relative to the technology frontier of the market at time t. In general it holds that the greater the technology stock or the more modern the product, the more attractive it is to customers. The state variable T_t is also a measure for the level of know-how within the firm at time t.

$$\dot{T}_t = h\left(T_t\right) I_t - \delta\, T_t\,, \quad with \quad T_0 > 0 \tag{2}$$

where it is assumed that in this economy the technology frontier of the market increases with a constant rate δ (>0).

Firm's investment does not have only a long-run positive effect but we also model a short-run effect on sales volume. This effect characterizes the dynamics of the firm's sales volume as seen before, which results of short-run behaviors of customers: the "adverse technology investment effect on sales". The impact of this effect is driven by a key parameter of the model. Additionally, we assume difference in effectiveness of investment: the firm has a higher effectiveness of investment when its technology stock increases by investing. But the increase of effectiveness of investment is not sustainable for a certain level of the technology stock. This results in a convex concave model formulation.

In order to keep up with the technological development or even increase its own technology stock relative to the technology frontier, the firm makes technology investments I_t. We model that the effectiveness of these investments depends on the actual technology stock of the firm; the function $h(T_t)$ denotes the rate of effectiveness of technology investment. If the firm invests, it will have a higher level of the technology stock than if it refrains from investing; this implies that $h(T_t)>0$. Moreover, we assume that the investment effectiveness increases for a low technology stock, while it is decreasing for a higher technology stock. Furthermore, taking into account that technology investments will not be that effective, if the technology stock is small (almost no know-how), and that $h(T_t)$ is non-negative for very large level of the technology stock, it can be concluded, that a convex-concave-convex-bell shape for h is reasonable. Actually, Verspagen (1993) has empirically obtained a bell-shaped relationship between the technological distance of a firm from the technology frontier and the ability to integrate external knowledge through spillovers.

By C_t we denote the firm's sales volume at time t, sales driven by the customer's behavior:

$$\dot{C}_t = \gamma \left(f(T_t) - C_t \right) - k\, C_t\, I_t\,, \quad C_0 > 0. \tag{3}$$

The function $f(T_t)$ describes how much sales volume the firm would attract in a long run, if it held its technology stock at T_t forever. We assume $f(0)=0$, of course $f'(T_t)>0$, because the better the technology, the higher sales volume the firm would have, and non-increasing returns to technology level, $f''(T_t)\leq 0$. One could imagine that the rate of change in the number of sales positively depends on the difference between the current sales volume (C_t) and the number the current level of technology warrants ($f(T_t)$). We denote the speed at which customers adjust to the technology level of the firm's products by the positive factor γ. Negative short-run effects ($-C_t I_t$) model the adverse investment effect on sales volume and we denote the factor calibrating this negative effect of technology investment on the sales volume by the key parameter k.

This model documents firm's optimal investment policies and customer's behavior: On one hand, the firm's investment (I_t) induces an increase of the technology stock (T_t), see equation (2). The firm can choose between two strategies: The first called technology focus strategy is when the firm invests more than the technology frontier of the market (δ) and tries to push up this frontier by developing its technology stock. The second called sales focus strategy is when the firm keeps investment low and prefers to develop its sales at the actual product technology level.

On the other hand, the customer's behavior is caught by the dynamic equation for the sales volume (equation (3)). Firm's investment (I_t) will have two opposite consequences on the firm's sales volumes (C_t):

- first, a positive long-run effect on sales volume and we have modeled it by the function $f(T_t)$ (due to simplicity we have chosen a simple, symmetric sales adjustment to the actual sales volume C_t).
- Second, negative effects on sales volume, which result from short-run behaviors of customers.

3 Analysis of the model

The optimal control model of the firm comprises the objective (1) subject to the state equations (2) and (3). Functional and parameter specification used for our analysis one can find in the Appendix. To solve the model, we applied Pontryagin's maximum principle; see Haunschmied et al. (2005) for the derivations. When it was no longer possible to solve it analytically, we did it numerically in the same way as in Haunschmied et al. (2005). Prior results in Haunschmied et al. (2005) show that for a small adverse investment effect, it is optimal for the firm to converge to steady-state technology investment decisions. In case of a more pronounced adverse investment effect, periodic investment decisions appear to be optimal.

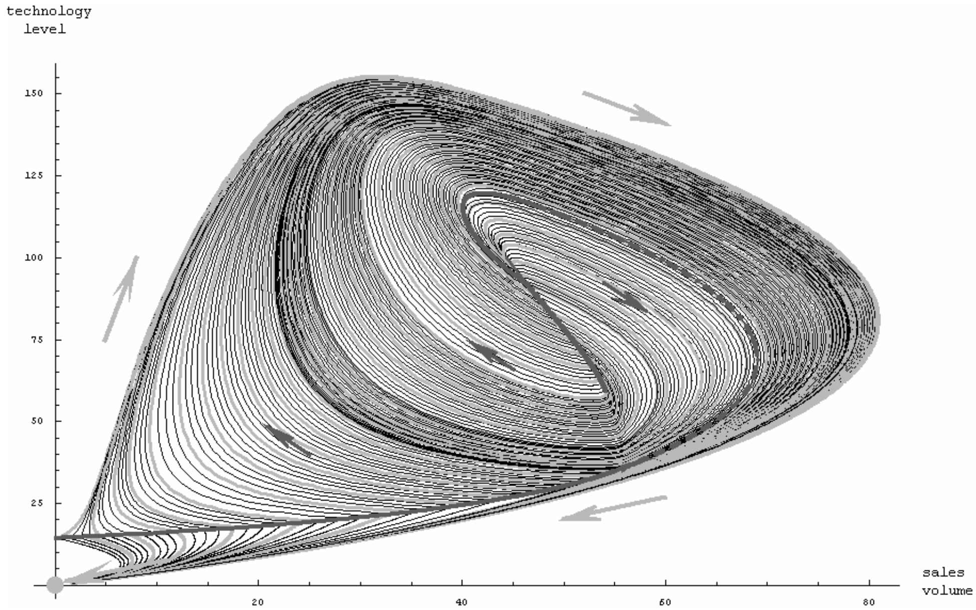

Figure1: The figure shows two indifference curves and a lot of exit paths of the firm starting at these curves and all of these paths converge to the origin; the paths are projected to the state space. One indifference curve is the cam segment between the points (54, 60) and (44, 100) approx., and the other indifference curve curls beginning at the point (0, 15) till it intersects the first mentioned indifference curve at approx. (46.4, 92.5). The adverse investment parameter value *k =1.3889%,* sales volume is in millions.

In this paper, we investigate the situation of distinctive adverse investment effect. A single trivial steady-state at the origin of the sales volume and technology space comes out of the resolution of the optimal control model. In the numerical analysis of the paper all the firm's trajectories are converging to the steady-state at the origin of the sales volume and technology space; confer Figure 1. The figure shows the diversity of firm's life cycles all ending when the firm exits the market (in the mathematical model this happens in the infinity – in practice of course earlier). We compute different firm's life cycles, which are driven by two strategies: technology focus and sales focus strategy. In summary, one key result of this paper is that, in case of distinct losses of sales due to adverse technology investment effects, in the long run the firm exits the market.

However, depending on its initial situation the firm follows substantially different optimal exit strategies. The second key result of this paper is that two indifference curves separate these strategies in the sales volume and technology space in different regions of optimal convergence.

Starting at these indifference curves, a firm can choose one out of two different optimal investment decisions, in the intersection of these curves one out of three; see Figure 2.

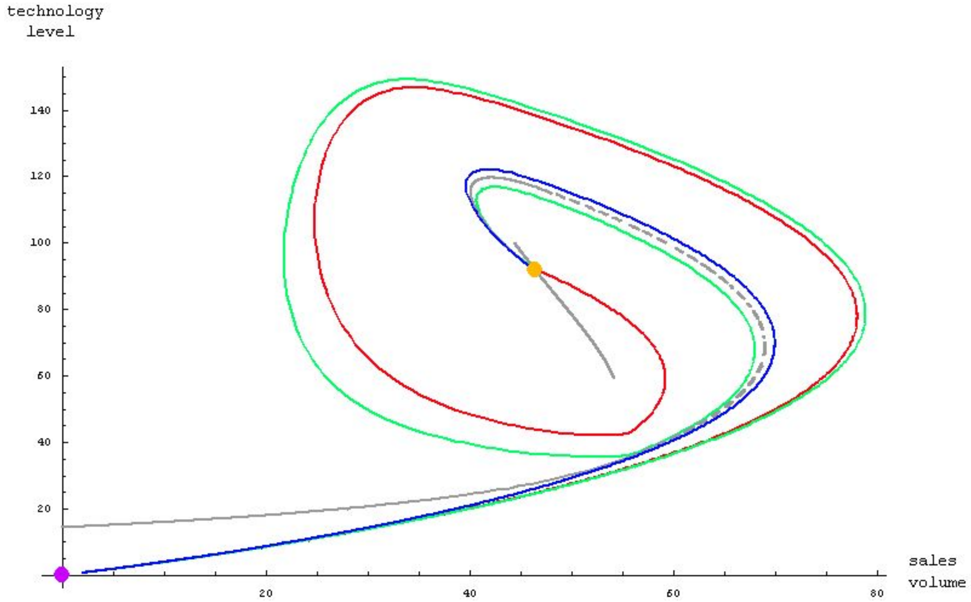

Figure 2: This figure shows three different optimal exit paths starting at the gray-shaded point at approx. (46.4, 92.5). One indifference curve is the cam segment between the points (54, 60) and (44, 100) approx., and the other indifference curve (partly dashed) curls beginning at the point (0, 15) till it intersects the first mentioned indifference curve at approx. (46.4, 92.5).

In Figure 1, the position on the two indifference curves characterizes the founding conditions of the firm, which are the initial technology stock and the initial sales volume as a proxy of the firm size. Starting from the same founding conditions (initial technology stock and initial sales volume), the firm can choose its optimal strategy; we categorize the firm's strategies in two different types:

- (mainly) technology focus (strategy on state variable T_t) the firm invests in know-how a lot more than the increase of the technology frontier of the market (δ).
- (mainly) sales focus (strategy on state variable C_t): the firm prefers to develop its sales volume. It may happen that the firm still invests in technology but less than the increase of technology frontier of the market (δ).

Basically the firm has always the choice between the two strategies: technology or sales focus. But for a very special case when the firm has a large size (sales volume > 40M) and very high initial technology stock, the only optimal strategy possible is the sales focus: the firm does not require developing its technology stock at the beginning of its life because the firm has already the capacity to develop its sales. Furthermore, depending on the firm's initial technology stock and on the firm's initial size, the firm's strategy drives its life cycle.

4 Diversity of firm's life cycle

Figure 1 tells the story of the firm's life cycle considering its founding conditions and strategies. Existing life cycle literature from industrial economics is largely analytical and often focuses on a single dimension such as investment behavior (Spence 1977; Spence 1979; Jovanovic 1982; Wernerfelt 1985), optimal production output (Spence 1977; Spence 1979; Spence 1981;

Wernerfelt 1985; Jovanovic and MacDonald 1994), experience curves (Spence 1981), or the competitive environment (Hannan and Freeman 1984). Most empirical studies concentrate on a single industry as opposed to a pooled sample firms in the economy (Jovanovic 1982; Caves 1998). One of the most interesting conjectures to emerge from the empirical literature is the argument that the conditions in which a firm is born may have a substantial effect on its performance and on its survival chances (Romanelli 1989). Cooper et al. (1994) found that the initial stocks of financial human capital were good predictors of firm performance, including survival. Kimberly (1979) goes further telling that environmental conditions, founder's personality, and initial strategic choices exert an enduring effect on the behaviour of organisations. The effect of initial strategy may also persist because strategic decisions frequently involve the deployment of resources that cannot be later reallocated, that is, which are sunk. When investment costs are sunk, there may be little point in inversing a decision, as costs cannot be recovered. Therefore, even if the firm realizes that one given decision was not a wise one the firm's best option will be to continue with it anyway (Dixit and Pindyck 1994). Our model shows that even when a firm is on a market exit strategy it can experience many years of a vital business life.

Life cycle theory suggests the existence of structural differences in the economics of the firm. Agarwal et al. (2002) define life cycle phases as discontinuous transformations of competitive conditions at particular points in time during an industry's evolution. An individual firm's life cycle phases can differ within a market because innovation is a continuing process with firms entering and exiting the market throughout the entire industry life cycle. (Industry's life cycle patterns occur because the technology level and intensity of competition change over the industry's life cycle.) Further, the life cycle phases of an individual firm within an industry vary significantly due to differences in a firm's level of knowledge acquisition (about core competencies, cost structure and production efficiencies), level of initial investment and re-investment of capital, and adaptability to the competitive environment. Our model shows that even when a firm is on a market exit strategy it may experience several different life cycle phases.
Figure 1 shows that these firm's life cycles are history dependant, because they are depending on its founding conditions and on its initial strategy chosen. Indeed, Geroski et al (2003) find with a nonlinear statistical model on Portuguese firms that founding conditions are important determinants of firm's exit of the market, and even are more important that current conditions. In most cases, founding conditions seem to persist without much of attenuation in their effect on survival for at least several years after the founding of the firm. For Frank (1988), different entry sizes signal diversity of firm's life cycle: a firm that enters at larger scales can endure poor performance for a longer time.

The general dynamics of the optimal trajectories in Figure 1 shows the common idea, that technology investments have positive impact on sales as Pegels (1991) empirically shows. Even though the results in Figure 1 are rather theoretical, it nevertheless serves as an illustration device for understanding the phases in which the firm goes through and the corresponding dynamics. We divide the trajectories in different phases with consequences on technology stock and sales volume of the firm as Abernathy and Utterback (1978) and Utterback (1996) emphasize in an industry's life perspective. Discussing this figure we divide it in two parts with respect to the initial sales volume of the firm: we are considering successively the life cycles of an initially small firm (0-40M sales volume) and the life cycles of an initially large firm (sales volume > 40M).

<table>
<tr><th></th><th>Firm's initial technology stock</th><th>Firm's initial strategy</th></tr>
<tr><td>Small firm
sales volume: 0-40M</td><td>Low</td><td rowspan="4">Technology focus

or

Sales focus</td></tr>
<tr><td rowspan="3">Large firm
sales volume > 40M</td><td>Low</td></tr>
<tr><td>Medium</td></tr>
<tr><td>High</td></tr>
</table>

Table 1: This table summarizes the firm's strategies (technologies focus or on sales focus) with respect to its founding conditions (initial technology stock and initial size).

4.1 Small firm's life cycle

In Figure 1, a small firm faces its strategy choice: sales focus or technology focus. Depending on its initial technology stock two different firm's life cycles appear.

In case of a small firm and low technology stock, we call the first possible life cycle of the firm: *out of market trajectory*. If at its entry in the market a small firm faces low technology, the firm will choose the sales focus strategy and disregards technology investment. Empirical literature shows that this is not an exception. Indeed, for Agarwal and Gort (2002), on average, roughly 5-10 percent of the firms in a given market leave that market over a single year. New start up firms display amazingly high failure rates (Romanelli 1989; Dunne et al. 1988; Dunne et al. 1989; Mata and Portugal 1994; Sharma and Kesner 1996). Firms with relative little capital have a higher risk of exiting the market than other firms (Doms et al 1995). Mata and Portugal (1994) shows a similar empirical analysis: 30% of the firms with a small initial size are forced to exist within the first year, and contrarily, only 5% large firms have to exit the market within the first year. So, small firm has a lower probability than large one to survive (Dunne et al. 1988; Dunne et al. 1989; Dunne and Hughes 1994; Audretsch and Mahmood 1994; Sharma and Kesner 1996).

In case of a small firm and a higher initial technology stock, the second possible life cycle for the firm, as can be seen in Figure 1, is a life cycle of three phases. At its entry in the market the small firm chooses to invest more than the increase of the technology frontier of the market: its life follows three phases which are the take-off, the maturity, and the out of market phases, where the length of each phase varies between firm's founding conditions.

The first phase is the take-off phase, which is aimed to build and develop firm's technology stock and so is focus on innovative activities on the product. So the firm is ready to loose sales volume in order to develop its product. The diminution of sales volume could be seen as an increase of the instability at the beginning of the firm life. The adverse effect on sales volume affects differently the firm depending on its founding conditions. After a certain time, the product becomes stable enough to have significant production volume.

In the second phase, called the maturity phase, the firm's investment still increases but less than the increase of the technology frontier of the market, this shows that the firm is no more focused on technology development but rather on sales development. This strategy change is occurring over time. In this phase, the sales volume increases significantly. Hall (1987) shows that small firms grow on average four percentage points faster than larger firms. Hence, the level of initial investment and reinvestment contribute to differences of specific life cycles of the firm within the market (Jovanovic 1982). The firm's founding conditions and strategy affect the firm's life cycle. At the end of the mature phase, the firm's profitability is eroded.

The third phase is the out of the market phase, which a firm can enter from the previous maturity phase. During this out of market phase, the erosion of the technology stock and sales volume that characterizes the out of market phase leads to an inevitable firm's exit of the market. Dunne and Hughes (1984), Evans (1987), and Evans (1987) show that firm age is an important factor when explaining the firm growth: for these authors, firm growth seems to decrease with age. Especially, Dunne and Hughes show a time dependant relationship between firm size and firm growth.

4.2 Large firm's life cycle

On the Figure 1, a large firm is faced to strategy choice: sales focus or technology focus. But with subject to its initial technology stock and its initial strategy, different life cycles of the large firm appear.

With low initial technology stock, the life cycles of a large firm are the same as for a small firm; two possibilities occur: on one hand a life *cycle of three* phases and on the other hand an *out of market* trajectory. Considering the *exit of large firm from the market*, in the prior subsection we have learnt that the empirical literature shows that a large firm experiences higher survival probability than a small one. Geroski et al (2003) find that firm that is larger in its initial year of founding will survive longer, and this effect is almost permanent (at least for the 2-8 years of life recorded by the firm). Variation in the level of founding conditions interacts with the time effect of firm mortality (Stinchcombe 1965; Freeman et al. 1983; Amit and Schoemaker 1993). This interaction between founding conditions and time determines how quickly a firm is able to identify and develop its core competencies and adapt to the competitive environment (Fichman and Levinthal 1991). Furthermore, for Geroski et al. (2003), any subsequent increases in firm size improve its survival. The impact on firm survival of initial human capital formation such as know-how or technology stock seems also to be both important and nearly permanent. Founding conditions contributes, for these authors, significantly to explain the variation in firm's survival. Although, for Geroski et al (2003), the empirical effect of initial conditions is not strictly permanent, many factors such as the firm size and the human capital seem to have relatively long lived effects on survival possibly for at least 10 years after they are set up.

With high and medium initial technology stock, a firm is in front of six different life cycles depending on its initial size and its initial strategy. When the large firm owns a high initial technology stock, the firm does not require developing its technology stock at the beginning of its life because the firm has already a huge technology capacity to use. In the case of very high initial technology stock for the firm, the only optimal strategy possible is the sales focus, because the firm has already the capacity to develop its sales, and if the firm chose nonetheless the technology focus strategy at the beginning of its life, technology development would be costly and sophisticated. If the firm chooses to develop its technology after a sales focus strategy, then the technology take-off will appear to be aimed in order to afford new potential firms entrance in the market. We call this phase: *technology take-off strategy against new potential entrance*. In this phase, as the technology frontier increases over time, the barrier to new entry in the market is increasing as well. But as time goes by the technological opportunities tend to become fewer and fewer as an exhaustion of technological opportunities. By making additional technological investments early in the life cycle, the firm can erect barriers to entry against new potential entrance (Spence 1977; 1979; 1981). Hence, Spence states preemptive investments are made by new firms to dissuade entry. These investments (initial and additional) include not only financial and tangible assets, but also organizational capital such as investments in distribution systems, manufacturing infrastructure, and technological capabilities (Levinthal 1991). However, preemptive investments diminish with maturity (Wernerfelt 1985).

5 Conclusion

This paper relates firm's life cycle to firm's optimal investment policies and customer's behavior. We use an optimal control model of a single product firm technology investment, where technology investment does not have only a long-run positive effect but also a short-run adverse effect on sales volume. The stylized model shows that firm's life cannot only be characterized as a stochastic process as Gibrats' Law states (Jovanovic 1982). The life pattern of the firm is strongly depending on its founding conditions and its first strategy and so is firm specific, but also history matters.

Two empirical implications of our model appear: First in the presence of a distinct adverse effect of technology investment on customer's behavior, the single product firm's is always on an exit strategy. Second, however, this exit strategy can be essentially different. Depending on the firm's initial distance to the technology frontier and its initial sales volume different optimal technology investment strategies result for the firm to a wide range of outcomes varying from a quick market exit to a long prosperous life.

For the firm, the optimal technology investment strategy depends on its initial distance to the technology frontier. When it is close to the frontier, it has to choose a sales focus and runs an optimized short-term exit strategy. Otherwise, the results explain the life cycle of different types of firms: a first small firm which needs to build its technology at the beginning of its life, second a large firm which focuses on sales and take advantage of high know-how (Schumpeter 1942). In the latter case, a technology focus strategy may have little rewards and high pains, when the adverse effect of technology investment is pronounced; hence a sales focus strategy is preferable.

From optimal control theoretical viewpoint the most interesting result is the threshold behavior (indifference curves), where managers are indifferent to apply technology focus or sales focus strategies. These thresholds separate founding conditions of the firm to various classes, where managers apply basically different initial optimal technology investment strategies.

Analyzing the model, we compute numerically two different indifference curves, which intersect in a triple indifference point, where managers are indifferent to apply a sales focus or one out of two specifiable technology focus strategies. These indifference curves differ from DNS curves e.g. computed in Haunschmied et al. (2003) or Caulkins et al. (2006), because here different optimal policies result in the same long run outcome; in this way we follow the Definition 5.5 in Grass et al. (2008) .

Policy considerations have two dimensions: one dimension for policy makers and another for managers. First, policy makers can often affect the market conditions in which a firm operates. But it is never possible to go back in history and modify the conditions under which a firm was born. The importance of founding conditions means that they are inherent limits to what policy makers can do for firms. Second, when managers are going to set up a new firm, it is important to establish it properly from the beginning. Firm's founding conditions and initial strategies have on the long-run permanent effects upon firm's life cycle, and subsequent reversal of the initial strategies later on may be insufficient to change the firm's life cycle.

Appendix

Functional and parameter specification for numerical analysis

Due to the complexity of the analysis we have to rely on numerical tools. The numerical computations were done by MATHEMATICA (Mathematica is a registered trademark by Wolfram Research).

For the investment costs we choose the following function

$$a(I) = a_1 I + a_2 I^2, \tag{3}$$

and for the function *f(T)* we simply take

$$f(T) = f_1 T . \tag{4}$$

and the functional form for *h(T)* is

$$h(T) = \frac{h_1 T^2}{1 + \left(\frac{T}{h_2}\right)^3} . \tag{5}$$

The physical dimension of *C*, *T* and *I* are sales in million, technology units, and investment in million \$, respectively; the unit of time is one year. We use the values

$$r = 5\%, \quad \delta \approx 4.17\%, \quad \alpha = \$2, \tag{6}$$

$$\gamma = 0.0625, \quad f_1 = 1, \quad a_1 = 1, \quad a_2 = \frac{1}{3} \tag{7}$$

$$h_1 = \frac{1}{2700}, \quad h_2 = 75, \tag{8}$$

and the parameter ***k*** varies. For ***k***, the situation when the loss of sales volume due to \$1M investment per year is less than≈1.3 02% is thoroughly discussed in Haunschmied et al. (2005). In case of a loss higher than approximately 1.302%, it is always optimal to converge to the origin (exit the market). However, the behavior is surprisingly complex and it is the topic of this actual article.

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Figure captions (2 Figures):

Figure1: The figure shows two indifference curves and a lot of exit paths of the firm starting at these curves and all of these paths converge to the origin; the paths are projected to the state space. One indifference curve is the cam segment between the points (54, 60) and (44, 100) approx., and the other indifference curve curls beginning at the point (0, 15) till it intersects the first mentioned indifference curve at approx. (46.4, 92.5). The adverse investment parameter value $k = 1.3889\%$, sales volume is in millions.

Figure 2: This figure shows three different optimal exit paths starting at the gray-shaded point at approx. (46.4, 92.5). One indifference curve is the cam segment between the points (54, 60) and (44, 100) approx., and the other indifference curve (partly dashed) curls beginning at the point (0, 15) till it intersects the first mentioned indifference curve at approx. (46.4, 92.5).

Table caption (1 Table):

Table 1: This table summarizes the firm's strategies (technologies focus or on sales focus) with respect to its founding conditions (initial technology stock and initial size).