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Individual Firm and Market Dynamics of CSR Activities

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Abstract

This paper investigates how a firm should plan its *Corporate Social Responsibility* (short CSR) activities. The dynamic analysis starts with the firm's intertemporal optimization problem, and proceeds then to include interactions with other firms. Integration of these 'social' interactions is crucial: if CSR is profitable for firm A then it is most likely also profitable for competitors B and C, and these simultaneous decisions affect the gain for each from trying to advance its own position. We find that multiple equilibria exist, irrespective of whether interactions with other firms are taken into account. In particular, including interactions can eliminate or create additional steady states. Moreover, it can lead to a situation where history is insufficient to choose one of multiple longrun outcomes, so that coordination between the firms is required.

Keywords: CSR, dynamics, firms' interactions, stability, history and expectation dependence.

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1 Introduction

The term *Corporate Social Responsibility* (CSR) came into common use in the early 1970s and it refers to a form of corporate self-regulation. This approach accounts for stakeholders interests (advanced in a book by Freeman (1984)) in addition to the shareholders' focus on profits and includes thus a number of ethical, social, and environmental targets, where the green aspect is the most visible one nowadays. It is presumably the hottest management fad and it is not short of prophets. Michael Porter praises the profitability (win-win) of socially and in particular environmentally responsible behavior since a long time, e.g., already in Porter and van der Linde (1995a, b), regularly in the annual *Global Competitiveness Reports* des *World Economic Forum* (Porter, Sachs, Cornelius, McArthur and Schwab (2004)), in Porter and Kramer (2006), and most recently in Porter and Kramer (2011). In the latest paper it is argued that taking a broad view on societal needs is nothing less than a mean to 'reinvent capitalism and unleash a wave of innovation and growth'. Palmer, Oates and Portnoy (1995) is an early critique of this 'promise' of CSR prophets (like Porter), while Lyon and Maxwell (2008) and Reinhardt, Stavins, and Vietor (2008) are part of a more recent debate about the pros and cons (in this order). The public opinions about CSR also vary greatly across countries[1].The importance of Corporate Social Responsibility is also documented by the appearance in articles in the public press. We only mention here the surveys in *The Economist,* first in 2005 ("The good company", January 20th) followed in 2008 (in "Just good business", January 19th).

The academic literature about CSR is dominated by empirical investigations. Orlitzky, Schmidt and Rynes (2003) find in their meta-study some support for the optimistic claims. Similarly, Nishitani (2011) finds that environmental management adds value for a sample of Japanese firms. In Hull and Rothenberg (2008) innovation and the level of differentiation in the industry serve as moderators for a positive relationship between corporate social and financial performance, which is thus strongest in low-innovation firms in industries with little differentiation. Of course, correlation is not causation, i.e. the found patterns may be due to reverse causality since only profitable firms can afford CSR or are spurious. And indeed, Surroca,

[1]Milton Friedman goes on tour. A survey of attitudes to business turns up some intriguing national differences, *The Economist*, Jan 27th 2011.

Tribó and Waddock (2010) find merely an indirect relationship that relies on the mediating effect of a firm's intangible resources. Fernández-Kranz and Santaló (2010) studies the link between competition and social performance also empirically and find that competition fosters CSR. Krishna and Rajan (2009) links CSR with 'cause marketing' by analyzing a game of competing firms and run corresponding experiments. Eichholtz, Kok and Quigley (2010) find that green office buildings fetch higher rents and selling prices. Chatterji, Levine, and Toffel (2009) finds that published rankings provide 'fairly good summaries of past environmental performance'. Siegel and Vitaliano (2007) verify empirically that firms selling credence goods are more likely to be socially responsible than firms selling search goods. This is economically intuitive and in line with Heyes (2005) that suggests signalling as motive for CSR. Baron (2007) presents an agency theoretic framework of corporate social responsibility and shows inter alia (i) that CSR is costly when it is an imperfect substitute and (ii) that the entrepreneurs, but not the shareholders bear its cost. In a related paper, Baron (2008) ventures the theory that corporate social responsibility is a form of private provision of public goods. And recently, Baron (2010) advances how self regulation and the existence of social labels and certification organizations mitigate free riding using a matching model. Cespa and Cestone (2007) draws attention to the possibility that inefficient CEOs may abuse social activism as an effective entrenchment strategy. Hence, introducing stakeholder protection explicitly can help to deprive managers this option.

The dynamics and evolution, one can even speak of a wave, of CSR activities can be seen e.g. by considering how many firms subjected themselves voluntarily over time to (environmental and social) standards like EMAS ("eco-management and audit scheme" within the European Union) and ISO14001 (it shares the same objective of good environmental management) as shown in Figure 1. Figure 2 shows the composition by countries. Understanding an evolution as the one shown in Figure 1 requires an explicit dynamic analysis of CSR activities complementary to the above quoted static investigations. However, such dynamic analyses are missing. In particular there is a need to explain the international spread of CSR activities, to address the question whether these activities are sustainable, and explain how and why they came about. This paper tries to fill this gap.

[Insert Fig. 1 approximately here]
[Insert Fig. 2 approximately here]

Starting point is the by and large optimistic management literature that encourages firms to pick up this win-win. Recently, Unruh and Ettenson (2010) even warn managers: "Don't let your competitors control what "sustainable" means in your industry." What this quote and many other management proposals often ignore, however, is that one must take the reaction of other firms into account, because they face similar incentives. Therefore, this paper investigates also how such interactions among firms affect outcomes. This integration allows to address the following questions: Can we explain a CSR-wave? Is it a transient phenomenon or is it sustainable in the longrun? Can multiple equilibria arise such that an individual firm's activities depend on its history? Does this history dependence extend to entire industries? Does a history dependent outcome for the firm's isolated profit maximization problem vanish if we include the interaction with the other firms in our analysis? Or is the converse possible: a firm's decision fixing the other firms' CSR activities leads to a unique and stable outcome, whereas including interactions with other firms yields a history dependent, maybe even expectation driven, outcome?

The paper is organized as follows: The model is introduced in Section 2, while Section 3 derives the first order conditions for the optimal expansion of CSR activities. Section 4 characterizes the firm's optimal expansion strategy holding the other firms' activities constant. Section 5 integrates the decisions of the other firms within a competitive setting in order to address some of the above raised questions. The consequences of some extensions are addressed in Section 6.

2 Model

Starting point is a firm that considers to expand (or reduce) its CSR activities. Its aim is to maximize its net present value of profits accounting for all related costs and benefits. The following notation is introduced: x stands for a firm's number of CSR projects, X denotes the industry wide CSR activity per firm, whereas the control variable u represents the change in a firm's CSR activities. To allow for new or terminated projects, the variable u can be either negative or positive.

Costs consist of, firstly, running existing projects ($K(x)$) that may include increasing or decreasing returns, $K'' \gtreqless 0$. Secondly, there are costs for adding new CSR projects, $C(u, x)$, including scrapping if $u < 0$. With respect to expansion it makes sense to assume diminishing returns, $C_{uu} > 0$. The dependence of these costs, and in particular of the marginal costs (C_u), on the state x, is presumably nonlinear due to two opposite effects. On the one hand we have scale economies and experience, thus $C_{ux} \leq 0$ for x small. On the other hand, diseconomies of scale occur as new projects are getting more and more costly if already many projects have been launched (because the cheaper ones will be selected first), thus $C_{ux} \geq 0$ for x 'large'. Although one might also include industry-wide spillovers, $C(u, x, X)$ and $C_X < 0$, i.e., a firm may benefit from the experience in other companies, this presumably small effect is neglected in the analysis.

The benefit for an individual firm from CSR is

$$B(u, x, X) = b(u, x) + f(x, X).$$

B has positive derivatives with respect to both private variables (u, x), but depends presumably negatively on X. We include the possibility of (at least locally) increasing returns. For simplicity, the benefits from a firm's position vis a vis its competitors (f) are separable from the direct benefits (b). If only differences in positions, absolute $(x - X)$ or relative (x/X), matter as argument of f, then f is a constant in the symmetric case of $X = x$. Furthermore, we impose that $f_X < 0$, since the firm's benefit from its CSR activities is reduced if its competitors expand. One could extend the benefits B for an interaction between the firm's (u) new projects and those of its competitors (denoted again by a capital letter, U), which has however no significant impact on qualitative results.

Combining benefits and costs yields the firm's compound instantaneous objective (from CSR activities)

$$\pi(u, x, X) = b(u, x) + f(x, X) - C(u, x) - K(x), \tag{1}$$

and its dynamic optimization problem

$$\max_{\{u(t),\, t\in[0,\infty)\}} \int_0^\infty e^{-rt}\pi(u(t), x(t), X(t))\, dt, \tag{2}$$

$$\dot{x}(t) = u(t), \; x(0) = x_0, \; x \geq 0. \tag{3}$$

The following analysis is carried out in terms of a general instantaneous payoff function $\pi(u, x, X)$ while commenting on the consequences of the simplifying and partially separable structure introduced in (1), which is further complemented below by suggesting corresponding specifications of b, f, C, and K. However, even in the general case of $\pi(u, x, X)$, the following plausible assumptions are made:

1. $\pi(u, x, X)$ is strictly concave with respect to u.
2. Increasing returns of π with respect to the firm's variables (u, x), if existing, are limited to small and intermediate values of x.
3. Similarly, if $\pi_u > 0$ at all, then it is restricted to small or intermediate values of x such that $\pi_u < 0$ at least for large expansions and many projects.
4. Competitive effects among the firms' CSR activities dominate, i.e., $\pi_{xX} < 0$.

Assumption 1 ensures an interior optimal policy and Assumption 2 prevents that "trees grow into the sky". The last two assumptions characterize CSR. Additional projects' setup costs exceed the immediate benefit at least if many projects are run by the company (Assumption 3) and the competitive element dominates among firms (Assumption 4). Complementarity between own and industry-wide CSR, $\pi_{xX} > 0$, which may hold (again locally) if the positive spillovers in costs and benefits dominate, are unlikely as additional source for complexity and is thus excluded from our analysis.

2.1 Example

An example of a payoff function is constructed bottom up. Costs for CSR projects may be linear for a start, but more plausible seems to include both increasing returns at small and diminishing returns at high levels, e.g.:

$$K(x) = k_1 x - \frac{k_2}{2} x^2 + \frac{k_3}{3} x^3. \tag{4}$$

Costs for expanding CSR must satisfy (strictly) the law of diminishing returns with respect to expansion (the linear-quadratic relation in (5)) but may include experience effects if x is 'small', e.g., $C_{ux} < 0$, but the opposite, $C_{ux} \geq 0$, for x 'large'; a simple representation is

$$C(u, x) = c_1 u + \frac{c_2}{2} u^2 - c_3 ux + c_4 ux^2. \tag{5}$$

The direct benefits can be approximated by a simple linear-quadratic specification

$$b(u,x) = b_1 u + b_2 x + b_3 ux - \frac{1}{2} b_4 u^2 - \frac{1}{2} b_5 x^2. \quad (6)$$

Furthermore, a linear relation,

$$f(x,X) = \varphi \frac{x}{X}, \quad (7)$$

describes how a firm benefits from its position relative to its competitors.

Combining (4) - (7) yields the following instantaneous profit

$$\pi(u,x,X) = \alpha_1 u + \alpha_2 x + \alpha_3 ux - \frac{1}{2}\left(\alpha_4 u^2 + \alpha_5 x^2\right) - c_4 ux^2 - \frac{k_3}{3} x^3 + \varphi \frac{x}{X}, \quad (8)$$

where the signs of most coefficients in (8) are indeterminate,

$$\alpha_1 = b_1 - c_1, \alpha_2 = b_2 - k_1, \alpha_3 = b_3 + c_3 > 0, \alpha_4 = b_4 + c_2 > 0, \alpha_5 = b_5 - k_2.$$

Nevertheless, a few restrictions will be imposed in the numerical examples below. First, we impose that $\alpha_1 < 0$, i.e., the cost element dominates with respect to new projects, second, the benefit dominates for all CSR activities, $\alpha_2 > 0$, whereas, third, the negative effect of x^2 on the benefit dominates the experience effect within the cost specification, i.e. $\alpha_5 \geq 0$.

3 First order conditions

Setting up the current value Hamiltonian of the firm's optimal control problem, $H = \pi(u,x,X) + \lambda u$, we obtain the following first order conditions,

$$H_u = \pi_u + \lambda = 0, \quad (9)$$
$$H_{uu} = \pi_{uu} < 0, \quad (10)$$
$$\dot{\lambda} = r\lambda - \pi_x. \quad (11)$$

The maximum principle implies that 'marginal costs' for expansion, $(-\pi_u)$, must equal the net present value of marginal benefit from an increase in CSR activities given by the shadow price λ. The Legendre-Clebsch condition (10) holds globally by assumption 1,

$$u = u^*(x,\lambda,X), \quad (12)$$
$$u_x^* = -\frac{\pi_{ux}}{\pi_{uu}},\ u_\lambda^* = -\frac{1}{\pi_{uu}} > 0,\ u_X^* = -\frac{\pi_{uX}}{\pi_{uu}}, \quad (13)$$

which implies for the example that

$$u^* = \frac{\lambda + \alpha_1 + x(\alpha_3 - c_4 x)}{\alpha_4}. \tag{14}$$

4 Individual CSR

Individual CSR refers herewith to a firm that takes the industry-wide activity X as given (and for simplicity constant). Defining the rate of substitution between already existing (x) and new (u) CSR activities at $u = 0$, by

$$\rho\left(x, X\right) := -\frac{\pi_x\left(0, x, X\right)}{\pi_u\left(0, x, X\right)}, \tag{15}$$

the above first order conditions (9) - (11) imply the following steady state condition:

Proposition 1: *Any (interior) steady state, x_∞, must satisfy:*

$$\rho\left(x_\infty, X\right) = r. \tag{16}$$

I.e., the marginal cost for expanding CSR net of immediate benefits, $(-\pi_u)$, must equal the net present value of the marginal benefit from a higher level of CSR $\left(\frac{\pi_x}{r}\right)$; if $\pi_u > 0$, i.e., the benefit from adding projects exceeds the associated costs, one must expand the total set of CSR activities up to a level where the corresponding marginal benefit is negative, $\pi_x < 0$.

If $\rho\left(x, X\right)$ is non-monotonic in x then multiple steady states are possible. Its (partial) is,

$$\rho_x = -\frac{\pi_{xx} - \frac{\pi_x}{\pi_u}\pi_{ux}}{\pi_u} \Longrightarrow \rho_x = \left(-\frac{1}{\pi_u}\right)\left(\pi_{xx} + r\pi_{ux}\right) \text{ at a steady state.} \tag{17}$$

Therefore, the slope of ρ at a steady state is linked to two characteristics of firm specific CSR: (i) the sign of (π_u) and (ii) the interaction term times the discount rate $(\pi_{ux} r)$ relative to degree of concavity (or nonconcavity) with respect to the state (π_{xx}). Assuming $\pi_u < 0$ at a steady state, a positive slope requires either $\pi_{ux} > 0$ and thus interaction and/or $\pi_{xx} > 0$, i.e., increasing returns with respect to x. For $\pi_u > 0$ the converse holds.

It is important for the firm to know how the activities of its competitors (X) affect its marginal rate of substitution, because that determines how the locations of the steady states shift as a response to an increase of its competitors' CSR activities (see the dashed lines in Figure 1). Differentiating ρ with respect to X and substituting the steady state condition yields

$$\rho_X = \left(-\frac{1}{\pi_u}\right)(\pi_{xX} + r\pi_{uX}) = \left(-\frac{1}{\pi_u}\right)\pi_{xX} \text{ at a steady state} \tag{18}$$

The expression after the first equality sign holds for general $\pi(u, x, X)$ and the one on the right hand side is the result after dropping those interactions that are absent in the specification (1).

Proposition 2: *Assuming $\pi_{uX} = 0$, $\pi_{xX} < 0$ as in (1), and $\pi_u < 0$, then any increase in the competitors' CSR activities lowers ρ; if $\pi_u > 0$ then ρ is increased. In both cases, the impact on the steady states can be in- or decreasing.*

Phase diagram analysis. Taking X as given (and constant), the slope of the $\dot{x} = 0$ isocline equals $(-\pi_{ux})$ due to (13) and this isocline is thus downward sloping in the (x, λ) plane for the dominant case of $\pi_{ux} > 0$. The costate dynamics

$$\dot{\lambda} = r\lambda - \pi_x(u^*(x, \lambda, X), x, X), \tag{19}$$

implies

$$\frac{\partial\dot{\lambda}}{\partial x} = -\pi_{xx} + \frac{\pi_{ux}^2}{\pi_{uu}} = -\frac{1}{\pi_{uu}}\left(\pi_{uu}\pi_{xx} - \pi_{xu}^2\right), \frac{\partial\dot{\lambda}}{\partial\lambda} = r + \frac{\pi_{ux}}{\pi_{uu}}.$$

Hence, the $\dot{\lambda} = 0$ isocline can have a non-trivial shape. If $\pi_{ux} > 0$ and $\pi_{uxx} \neq 0$ then $\frac{\partial\dot{\lambda}}{\partial\lambda}$ can switch sign. At such a point we have $\frac{\partial\dot{\lambda}}{\partial\lambda} = 0$ implying that there the $\dot{\lambda} = 0$ isocline has a pole that separates two parts of the isocline in the (x, λ)-plane. The numerator of the implicitly given slope of this isocline $\left(-\frac{\partial\dot{\lambda}}{\partial x}\right)$ may change signs too, but along the already addressed and expected lines: it is definitely negative in the domain of non-diminishing returns, but positive in the case of decreasing returns combined with concavity of π with respect to (u, x). The Jacobian of the dynamic system $\binom{\dot{x}}{\dot{\lambda}}$, and its determinant are

$$j = \begin{pmatrix} -\frac{\pi_{ux}}{\pi_{uu}} & -\frac{1}{\pi_{uu}} \\ -\frac{\pi_{uu}\pi_{xx} - \pi_{xu}^2}{\pi_{uu}} & r + \frac{\pi_{ux}}{\pi_{uu}} \end{pmatrix} \Longrightarrow \det j = -\frac{1}{\pi_{uu}}\left[r\pi_{ux} + \pi_{xx}\right]. \tag{20}$$

Therefore, saddlepoint stability is equivalent to

$$\det j < 0 \Longleftrightarrow r\pi_{ux} + \pi_{xx} < 0, \tag{21}$$

and concavity of π coupled with $\pi_{ux} \leq 0$ provides thus a sufficient condition. Two properties follow immediately. The first one links the above derived stability properties to geometric properties of the marginal rate of substitution (ρ) and the second one addresses the economic routes for instability and implied history dependent outcomes.

Proposition 3: *Since combining (17) and (20) implies*

$$\det j = \frac{\pi_u}{\pi_{uu}} \rho_x \text{ at any steady state,} \tag{22}$$

instability and stability of a steady state are directly linked to two signs: of π_u *and of the slope* ρ_x*. Therefore, assuming* $\pi_u < 0$*, instability of a steady state is equivalent to* $\rho_x > 0$*, while for* $\pi_u > 0$ *the converse holds.*

Proposition 4 *The inequality* $r\pi_{ux} + \pi_{xx} > 0$ *is necessary for an unstable interior steady state to occur (and sufficient if evaluated at such a steady state), and thus for an outcome in which the firm's long-term strategy depends on its own history* (x_0)*. There are two distinct routes for such history dependent outcomes: (i) (locally) convex objective, i.e.,* $\pi_{xx} \geq 0$*, (ii) if the law of diminishing returns holds* ($\pi_{ux}^2 < \pi_{uu}\pi_{xx}$)*, then the interactions between new (= control) and existing CSR (= state) activities must be positive and sufficiently,*

$$r\pi_{ux} > -\pi_{xx} > 0.$$

The first route is economically very intuitive and therefore it is no surprise that literally hundreds of papers pursue this avenue (i) following the pioneering works of Sethi (1977), Skiba (1978) and Dechert and Nishimura (1983). The claim that increasing returns are necessary for any kind of history dependency can be also found in the works of Arthur, e.g., Arthur (1989, p 116), and of Krugman, e.g. in Krugman (1991), which accounts for firms' interactions similar to the extension introduced in Section 4. Figure 3 shows the phase diagram of a corresponding numerical example based on (8). The intersections of ρ (shown on top in Fig. 5 by the dashed line) with r determine two steady states in the nonconcave domain of which the unstable one is a focus and the larger steady state is (saddlepoint) stable.

The second route assumes that the firm operates under the law of diminishing returns. Contrary to the impression created by the above quoted large volume of literature, this still allows for unstable steady states to occur. According to (**??**), this route requires π_{ux} being positive and sufficiently large (compare Wirl and Feichtinger (2005) which gives a second mechanism). It is surprisingly easy to construct a corresponding example drawing on the specification (8), which is shown in Figure 4 again in the form of phase diagram. The crucial differences between the unstable steady states in Figures 3 and 4 are: if the objective is concave at the unstable steady state, as in Figure 4, then the policy function is continuous and passes through the unstable steady state (x_∞), which is simultaneously the threshold separating domains of attraction, here either $x \to x^\infty$, or quitting, $x \to 0$. In the case of Figure 3, where the unstable steady state is situated in the non-concave domain, this unstable steady state is neither optimal, nor is the policy function continuous in this case[2]. Here the determination of the threshold requires an explicit computation for all initial conditions (locally around the unstable steady state, x_∞), to see where the firm is indifferent between the paths $x \to x^\infty$ or $x \to 0$.

Although various parameter variations and corresponding comparative statics (with respect to steady states) and comparative dynamics are possible, the discussion is here restricted to the parameter α_4 that accounts for adjustment costs and which does not affect the steady states but the concave domain: the larger α_4 the larger is the concave domain. Therefore, lowering α_4 can shift (the unchanged) unstable steady state into the non-concave domain, including the possibility that the unstable node turns into an unstable focus such that the local dynamics are like in Figure 3.

[Insert Fig. 3 approximately here]
[Insert Fig. 4 approximately here]

The upshot of both scenarios is that the decision whether to continue, possibly to expand CSR, or to leave this field can depend on the initial conditions. Furthermore, exogenous events re-shaping a firm's benefits or costs may alter the longrun outcome in a discontinuous way. Indeed equipped with the above derived properties, it is possible to eliminate or generate an outcome with CSR including the possibility of threshold effects. Assuming $\pi_u < 0$ globally, and existence of an unstable steady state at the starting point (i.e., $\rho_x > 0$ at one intersection with r), then any change in parameters

[2] This is for sure in this example, because the unstable steady state is a focus. However, and contrary to an impression again created in many papers, the policy function can be continuous and thus passing through the unstable state, which is an unstable node, even if this steady state is in the non-concave domain. Hartl et al. (2004) provides a corresponding example for a firm's investment problem facing relative adjustment costs.

that increase the marginal benefit from CSR (i.e., π_x) raises the inverted U representation of ρ shown in Fig. 5 (below) leaving the stable steady state as the unique positive longrun outcome, e.g., setting $\alpha_2 = 3$ in the example in Figure 3 yields a unique and stable outcome (of $x_\infty = 5.74$).

On the other hand, parameters that increase the costs for (new) CSR projects or decrease the marginal benefit (an example is an increase of X), lower the inverted U curve of ρ in Figure 5. This first raises the unstable steady state (thus enlarges the basin of attraction of $x \to 0$, i.e., no CSR in the longrun) and lowers the stable steady state until no intersection exists. Then, no CSR will occur in the longrun, e.g., reducing α_2 below ≈ -5.21 for the example in Figure 3 drives ρ below r.

5 Equilibrium among interacting firms

A large part of the management literature provides recommendations to single firms without drawing attention that a similar logic applies to its competitors as mentioned in the introduction. In contrast to these often firm specific advises by consultants, it must recognize that its own incentives to expand CSR are similar to those of its competitors. Although it cannot decide upon the competitors' actions, it can (rationally) predict the future path of $\{X(t)\}$ from its own profitability considerations. Assuming a perfectly symmetric setting for reasons of analytical tractability, competitors will simply choose the same investment, implying

$$X = x. \tag{23}$$

The crucial point is that in spite of this identity, X cannot be part of the firm's optimization, and must therefore be treated as exogenous when deriving the optimality conditions as outlined in the previous section. However, the equilibrium condition (23) can be substituted into the firm's first order optimality conditions, implying that the firm has 'rational' expectations about its competitors' current and future actions. This yields the following characterization of the evolution of firm and of industry-wide CSR, in which each firm correctly anticipates how its competitors will react to the incentives including the interactions among the firms,

$$\dot{x} = u^*(x, \lambda, x), \tag{24}$$
$$\dot{\lambda} = r\lambda - \pi_x(u^*(x, \lambda, x), x, x). \tag{25}$$

Krugman (1991) argues that increasing returns are nevertheless needed to obtain history dependency even after including firms' interactions. However, this is not true in general and not even within the specific framework that Krugman (1991) presents, see Wirl and Feichtinger (2006) for a counterexample.

This dynamic system (24) and (25) implies the same steady state condition (15), but with the difference that ρ is now a scalar function after substituting (23)

$$r = \rho(x) := -\frac{\pi_x(0,x,X)}{\pi_u(0,x,X)}\,|_{X=x}\,.$$

Its (total) derivative is

$$\begin{aligned}\rho' &= \frac{-\pi_u(\pi_{xX}+\pi_{xx})+\pi_x(\pi_{uX}+\pi_{ux})}{\pi_u^2}\,|_{X=x,u=0} \\ &= -\frac{1}{\pi_u}\left[(\pi_{xX}+\pi_{xx})+(\pi_{uX}+\pi_{ux})\,r\right] \text{ at a steady state.} \qquad (26)\end{aligned}$$

The Jacobian is

$$J = \begin{pmatrix} -\frac{\pi_{ux}+\pi_{uX}}{\pi_{uu}} & -\frac{1}{\pi_{uu}} \\ \frac{(\pi_{uX}\pi_{ux}-\pi_{xX}\pi_{uu})-\left(\pi_{uu}\pi_{xx}-\pi_{xu}^2\right)}{\pi_{uu}} & r+\frac{\pi_{ux}}{\pi_{uu}} \end{pmatrix}.$$

Since the determinant of the Jacobian is

$$\det(J) = -\frac{1}{\pi_{uu}}\left[(\pi_{ux}+\pi_{uX})\,r+(\pi_{xx}+\pi_{xX})\right], \qquad (27)$$

it can be linked to the slope ρ' at a steady state as given in (26) in a way similar to Proposition 4.

Proposition 5: $\det(J) = \frac{\pi_u}{\pi_{uu}}\rho'$ *at any steady state of (24) and (11). Therefore, assuming* $\pi_u < 0$*, then* $\rho' < 0 \Longleftrightarrow \det(J) < 0$. *The non-standard outcome,* $\det(J) > 0$*, allows in general, and in contrast to the firm's isolated optimization problem, for two subcases depending the trace of the Jacobian,*

$$tr(J) = r - \frac{\pi_{uX}}{\pi_{uu}}. \qquad (28)$$

(i) if $tr(J) > 0$*, the steady state is unstable and gives rise to a threshold. (ii) if* $tr(J) < 0$*, the steady state is (locally) stable leading to indeterminacy (any local choice of* $\lambda(0)$ *for a given* x_0 *is attracted). Assuming* $\pi_u > 0$ *then* $\rho' > 0 \Longleftrightarrow \det(J) < 0$.

We have $\pi_{uX} = 0$ in the specification (1). In this case, instability requires that π_{ux} is positive and sufficiently large. In the firm's optimization problem of the previous section the corresponding criterion is $r\pi_{ux} > -\pi_{xx}$. However, π_{ux} must be even larger after accounting for the interaction with competitors, because from (27) it follows that instability requires that

$$r\pi_{ux} > -(\pi_{xx} + \pi_{xX}),$$

where we should note that $\pi_{xX} < 0$. This suggests that market interactions help to eliminate unstable steady states that result in the firm's isolated optimization problem of the previous section. Figure 5 (top) shows a corresponding case based on the example from Figure 3. The unique longrun outcome shown in the top of Figure 5 is close to the unstable steady state of the firm's isolated optimization problem. Hence, it seems that this interaction eliminates the high and second steady state and turns the low and unstable steady state into a stable one. However, both conjectures do not hold in general even within the restrictions imposed by the particular example (8). First, the example corresponding to Figure 4, with both positive steady states in the concave domain (i.e., within an extremely well-behaved setup), can lead to three steady states after accounting for social interactions, see Figure 5 (bottom). Second, interactions can also eliminate the low and unstable steady state, which happens if in Figure 5 (bottom) the curve for ρ corresponding to the interacting firms' case, intersects the line $\rho = r$ only once such that the two smaller intersections do not exist.

[Insert Fig. 5 approximately here]

Interaction among firms can introduce another degree of indeterminacy about longrun outcomes if the unstable steady state is a focus. Then the basins of attractions of two saddlepoint stable steady states overlap, implying that, when x_0 is situated in the overlapping area, x_0 itself (i.e., history alone) is insufficient to determine the longrun outcome. In this case firms must coordinate about whether they will perform high or low CSR activities in the long run. This scenario follows if the eigenvalues are complex, which requires a sufficiently large determinant of J. Expression (27) suggests that a low value of $|\pi_{uu}|$ can achieve this. Indeed, applying comparative dynamics in the context of the example in Figure 4 leads to the conclusion that variations in the adjustment cost parameter α_4 ($= -\pi_{uu}$ in the example (8)) are crucial. As in the firm's isolated problem of the previous section, this parameter does not affect the steady states x_∞ in (24) and (25), but it can change the stability properties. In particular, reducing α_4 can turn the unstable node into an unstable focus such that history is insufficient and coordination among the competitors is necessary in order to attain one of the two stable longrun outcomes. Hence, low adjustment costs, which allow for a fast expansion of CSR, require coordination. On the other hand, a large α_4 slows the process sufficiently such that no coordination is needed[3]. Since one has to be fast in today's markets (i.e., adjustment costs are small relative to the incremental benefit from CSR), one of the crucial rules of labelling and certification organizations and of international consultants could be to assist in this process of coordination. Figure 6 shows a phase diagram including the numerical determination of the overlap, i.e., for initial conditions $x_0 \in (1.11,\ 1.175)$, firms can choose between both saddlepoint paths, one heading to the low and the other heading to the high equilibrium.

[Insert Fig. 6 approximately here]

The consideration of interactions among firms retains the possibility of multiple equilibria and adds the complexity that history can be insufficient to determine the future of CSR. This scenario occurs whenever the unstable steady state is a spiral. Furthermore, drawing on the comparative dynamics sketched for the firm (how increased benefits shift ρ), this framework allows to explain how exogenous events, like, e.g., a greening of stakeholders in particular of consumers, changes the firms' incentives. This has the potential of triggering a wave of activities across firms even if they know that relative positions will not change. The possibility of multiple equilibria suggests a considerable variation of activities across industries due to differences in benefits from CSR (e.g. for branches more or less dependent on green minded consumers) or even history. Late comers may attain low levels, while other industries that already run a variety of programs, will opt for a high level of CSR in the longrun (of course ceteris paribus, i.e., unless preferences change, e.g. if CSR, essentially a credence good, is discredited by cheating and free riding). Finally, even the attainment of a high level of CSR may require coordination between firms. Therefore, coordination can be an additional reason for the crucial role of labelling and certification institutions to foster CSR complementary to mitigate free riding (Baron (2010)).

[3]Karp and Paul (2005, 2007) show in a related framework (but with adjustment externalities and two states) that this monotonicity with respect to adjustment costs does not hold in general.

6 Extensions

6.1 Interaction in (u, U)

The analysis so far assumed that consumers and other stakeholders are concerned about the firm's level of CSR activities vis a vis its competitors, i.e., this comparison is linked to the state variable. What is the consequence if this comparison is linked to the announcements of new projects, u, with capital letter U denoting the competitors' average choice? In order to focus on this kind of interaction, $f(x, X)$ is replaced by $g(u, U)$ satisfying similar properties as f, i.e., $\pi_{uU} = g_{uU} < 0$. This modifies the Hamiltonian, $H = \pi(u, x, U) + \lambda u$, but leaves the first order conditions unchanged, except for the optimal control,

$$\begin{aligned} u &= u^*(x, \lambda, U), \\ u_x^* &= -\frac{\pi_{ux}}{\pi_{uu}},\ u_\lambda^* = -\frac{1}{\pi_{uu}} > 0,\ u_U^* = -\frac{\pi_{uU}}{\pi_{uu}}. \end{aligned}$$

As a consequence, after setting $\pi_{xX} = 0$ this outcome corresponds to the ones characterized in Propositions 1 - 5, including the possibility of multiple steady states for the already addressed reasons related to the state ($\pi_{ux} > 0$, or $\pi_{xx} > 0$).

In the competitive equilibrium, the firm's choice is the fixed point of the implicit relation,

$$\begin{aligned} u &= u^*(x, \lambda, u), \\ \frac{\partial}{\partial u} &= 1 + \frac{\pi_{uU}}{\pi_{uu}} = \frac{\pi_{uu} + \pi_{uU}}{\pi_{uu}}, \end{aligned}$$

and denoted by

$$u^{**}(x, \lambda),\ u_x^* = -\frac{\pi_{ux}}{\pi_{uu} + \pi_{uU}},\ u_\lambda^* = -\frac{1}{\pi_{uu} + \pi_{uU}},$$

with the given partial derivatives. Substitution yields the dynamics of the competitive equilibrium,

$$\begin{aligned} \dot{x} &= u^{**}(x, \lambda), \\ \dot{\lambda} &= r\lambda - \pi_x(u^{**}(x, \lambda), x), \end{aligned}$$

and differentiation the Jacobian,

$$J=\begin{pmatrix} -\frac{\pi_{ux}}{\pi_{uu}+\pi_{uU}} & -\frac{1}{\pi_{uu}+\pi_{uU}} \\ -\frac{\left(\pi_{uu}\pi_{xx}-\pi_{xu}^{2}\right)}{\pi_{uu}+\pi_{uU}} & r+\frac{\pi_{ux}}{\pi_{uu}+\pi_{uU}} \end{pmatrix}.$$

Hence, the outcome is the same as in the isolated firm's case if we substitute $\pi_{uu}+\pi_{uU}$ for π_{uu}. What we see is that the outcome with interaction is *observationally equivalent* to the one where firms ignore this interaction while having a different (and anyway not observable) preference structure (see Hof and Wirl (2008) for a corresponding discussion). As a consequence, the Jacobian retains the usual symmetry so that the eigenvalues are symmetric around $r/2$. In conclusion, this kind of interaction between u and U cannot add further complexity.

6.2 Complementarity, $\pi_{xX}>0$

Departing from a framework with diminishing returns, the above analysis shows that a positive interaction term, i.e. $\pi_{ux}>0$, is crucial to obtain multiple steady states including an unstable one. This holds for the isolated firm model, as well as for the model where firms' interactions are taken into account. In the latter case, an additional property helpful to create an unstable steady state is complementarity between own and industry-wide CSR activities, i.e., $\pi_{xX}>0$, due to (27). The specification of f in (8) stresses however the competitive nature such that $\pi_{xX}=f_{xX}<0$. Complementarity is implied by a different but also plausible specification

$$\begin{aligned} f(x,X) &= -(\alpha/\beta)\,e^{-\beta(x-X)}, \qquad \alpha>0,\beta>0. \\ f_x &= \alpha e^{-\beta(x-X)}, f_{xx}=-\alpha\beta e^{-\beta(x-X)}, f_{xX}=\alpha\beta e^{-\beta(x-X)}. \end{aligned} \tag{29}$$

However, obtaining an unstable steady state due to this property requires that this complementarity is strong. This is because

$$\det J=-\frac{1}{\pi_{uu}}\left[(\pi_{xx}+\pi_{xX})\right]>0\Longleftrightarrow-\frac{\pi_{xX}}{\pi_{xx}}>1,$$

after assuming $\pi_{ux}=0$ (and $\pi_{uX}=0$ by assumption). This condition is not met by (29) , because there it holds that $-\frac{f_{xX}}{f_{xx}}=1$. After substituting (29) into (1) and ruling out increasing returns ($\pi_{xx}<f_{xx}$), this implies that

$-\frac{\pi_{xX}}{\pi_{xx}} < 1$. We conclude that complementarity is an unlikely source on its own to explain multiple equilibria in the context of CSR.

An indeterminate outcome, i.e. a stable steady state with both eigenvalues having negative real parts[4], needs first of all a positive determinant, (negative) control-externality interaction, $\pi_{uX} < 0$, and 'small' discount rates, $tr(J) < 0 \Longrightarrow r < \frac{\pi_{uX}}{\pi_{uu}}$ due to (28). The specification in (1), more precisely $\pi_{uX} = 0$, rules out a negative trace of J. Therefore, an indeterminate (transiently only) coevolution of firms requires negative spillovers of aggregate activity on a firm's individual costs and benefits. Such an interaction, $\pi_{uX} < 0$, is not entirely implausible: Given many CSR projects already around, the firm's (marginal and net) benefit from an additional project is reduced. This can be caused by, e.g., insignificant exposure, there is no good project available anymore, or the costs are increased due to competing for experienced CSR managers who are then short in supply.

7 Concluding remarks

This paper focussed on dynamic aspects associated with Corporate Social Responsibility (CSR), which is complementary to the so far published papers with all their emphasis on empirics and static models. More precisely, a dynamic model of a firm's CSR activities is introduced in order to analyze how a firm should pursue its CSR activities over time, while taking into account that the relative position with respect to its competitors is an important determinant for its reputation and thus long run profit. The actions of its competitors are of course an exogenous datum to the firm, and was considered as constant in the first set up. However, one must account how other firms will react facing similar incentives, a point that many management advises ignore. Therefore, this paper considers also how such interactions among firms affect outcomes, e.g., whether they trigger an industry standard.

The upshot of the paper is that CSR involves multiple long run equilibria in both instances, i.e. either when firms account for interactions among firms, or ignore them. In addition to parameters relating to benefits and costs, the occurrence of multiple equilibria implies that a firm's engagement into CSR can also depend on initial conditions (i.e., its history). This adds an

[4]This possibility is extensively discussed in the theoretical endogenous growth literature. Examples are Benhabib and Farmer (1994), Benhabib and Perli (1994), and more recently Benhabib, Nishimura, and Shigoka (2008).

additional degree of variability (and sustainability) of these activities across firms and sectors. Finally, reaching and sustaining a high level of CSR may not only require a lucky history, but also coordination between firms could be needed. This provides an additional and maybe crucial role for labelling and certification institutions.

A simplifying assumption was that of perfect competition in CSR activities, i.e., each firm treats the industry average X as exogenous. Weakening this assumption, and thus allowing for explicit strategic interactions, suggests a dynamic game version of this paper (see Dockner et al. (2000)) and He, Prasad, Sethi, Genaro (2007) for a survey of applications of differential games to managerial issues (supply and marketing channels) and Jørgensen, Martín-Herrán, and Zaccour (2010) for a survey of differential games dealing with environmental issues. This, as well as the other addressed extensions, are left for future research.

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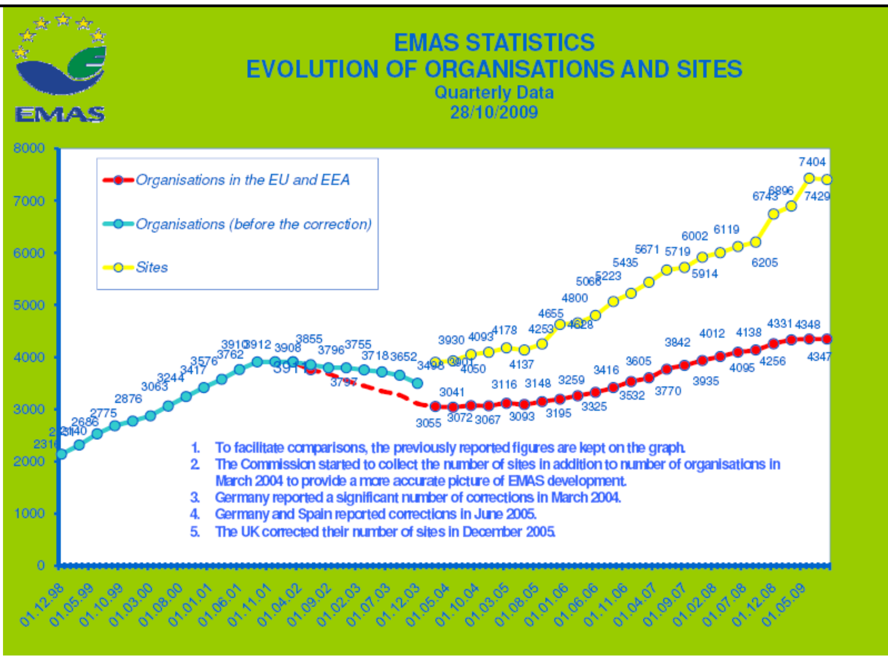

Fig. 1: Evolution of EMAS certifications over time.
http://ec.europa.eu/environment/emas/pdf/5_5articles_en.pdf

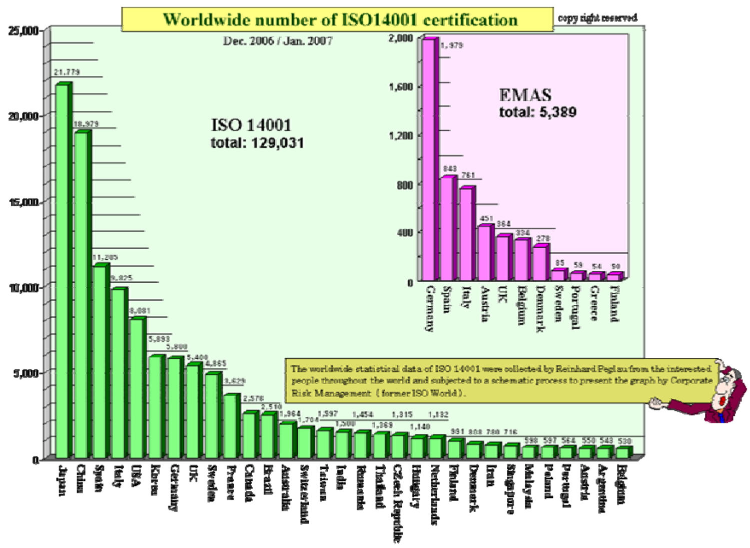

Fig. 2: Regional distribution of ISO 14001 and EMAS certifications
http://www.ecology.or.jp/isoworld/english/analy14k.htm

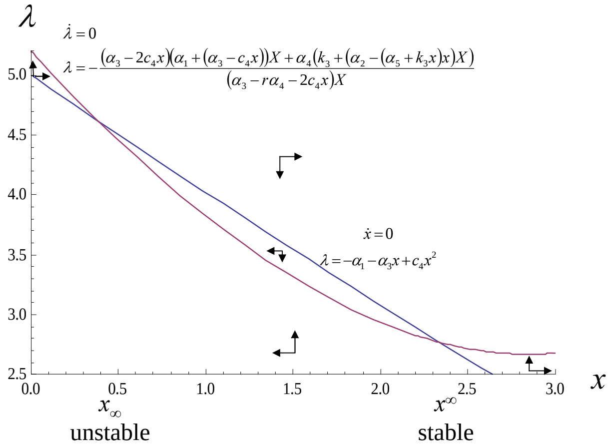

Fig. 3: Example of firm's optimization problem with steady states in the **non-concave** domain
α_1 = -5, α_2 = 1, α_3 = 1, α_4 = 2, α_5 = $^1/_{10}$, c_4 = 0.02, r = ¼, k_3 = $^1/_{20}$, φ = $^1/_{10}$, X = ½.

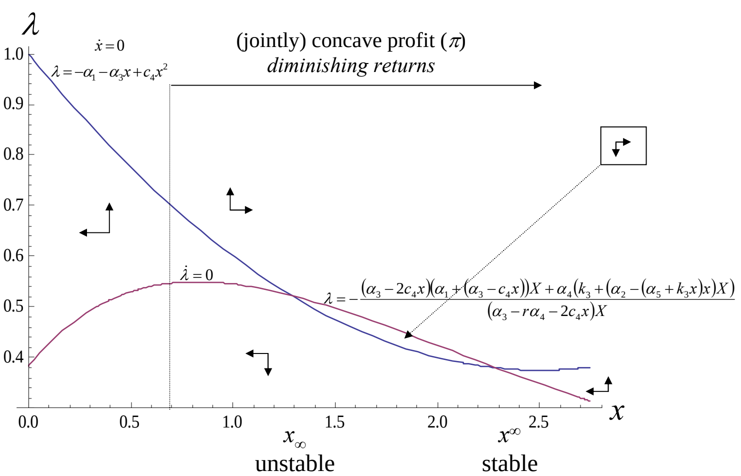

Fig. 4: Example of firm´s optimization problem with all interior steady states (incl. unstable one) in the **concave** domain
α_1 = -1, α_2 = $^4/_3$, α_3 = ½, α_4 = ½, α_5 = $^1/_3$, c_4 = $^1/_{10}$, r = 2, k_3 = $^1/_{100}$, φ = $^1/_{10}$, X = 2.

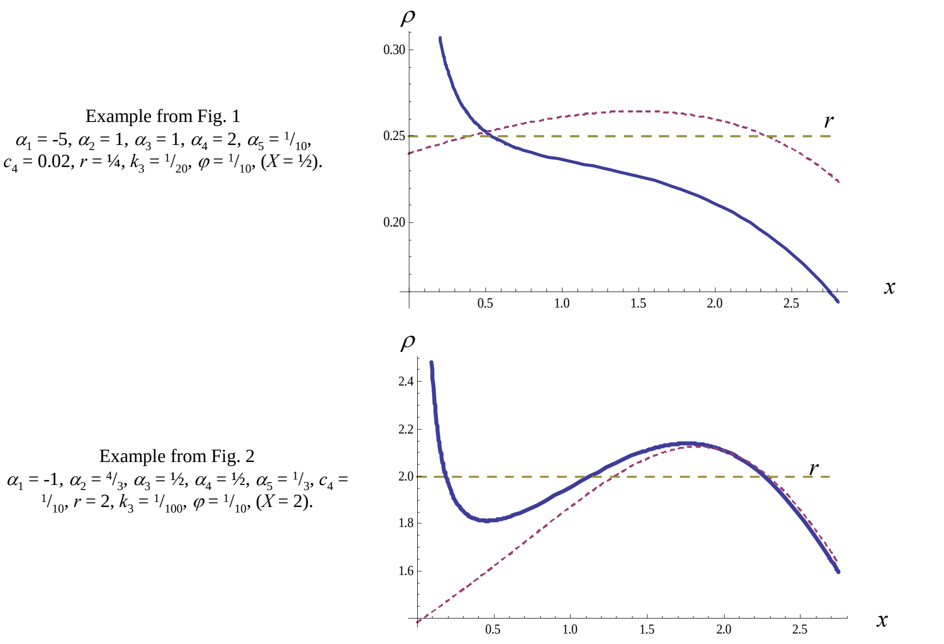

Fig. 5: Steady states (where ρ cuts r) with and without (dashing, at indicated X) firms´ interacting

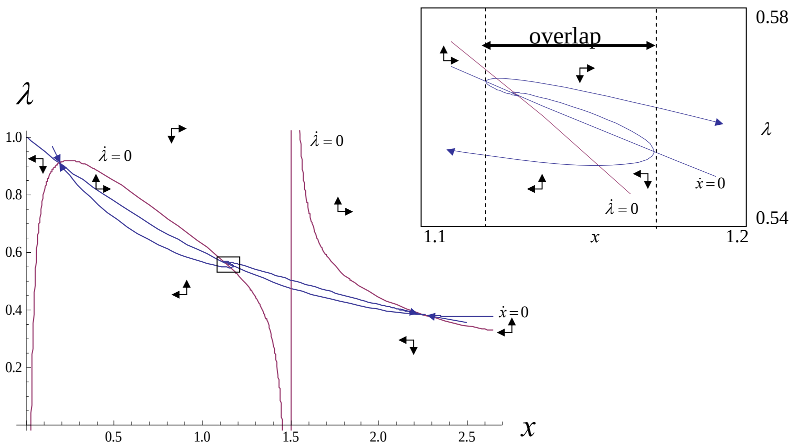

Fig. 6: Phase diagram of parameters of Fig. 2 but with small adjustment costs (α_4 = $^1/_{10}$) leading to an overlap from which the low and high saddlepoint stable steady states are reachable and thus both are potential longrun outcomes.

9 Appendix for Referees

This appendix for referees present additional examples to stress the robustness of our findings and to provide a brief list of references that pursue the 'turnpike' of convex-concave optimal control problems in order to obtain tresholds.

9.1 Additional literature on convex-concave objectives

A first wave of papers appeared in the 1980-ies, e.g. Lewis and Schmalensee (1982) on renewable resources, Brock (1983) on lobbying, Dechert (1984) on regulated firms, and Brock and Dechert (1985) on dynamic Ramsey pricing. Recent applications, like, e.g., Mäler (2000), Mäler et al. (2003), and Wagener (2003), build on the shallow lake model of Brock and Starrett (2003). An early example is Clark (1971) that considers a renewable resource model.

Brock, W. A., 1983, Pricing, predation and entry barriers in regulated industries, In: D. S. Evans (Ed.), *Breaking up Bell*, Amsterdam: North-Holland, 191-229.

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9.2 Additional Examples

Both steady states are in the non-concave domain in the example in Fig. 3. Of course, one can find many examples where the stable steady state is then in the concave domain (in which case the sufficiency criteria hold at least locally). Fig. R13 is a corresponding example where the steady state is in the concave domain; the bold line includes the interactions among the firms.

[Insert Fig. R1 approximately here]

Fig. R2 shows the phase digram corresponding to example from Fig. 4 in which leads to three steady states after accounting for firms' interactions.

[Insert Fig. R2 approximately here]

In the paper it is cliamed that interactions can also eliminate the low and unstable steady state, if the curve for ρ corresponding to the interacting firms' case, intersects the line $\rho = r$ only once such that the two smaller intersections do not exist. Fig. R3 provides a corresponding example including the phase portraits where the dashing lines refer to the firm's isolated control problem.

[Insert Fig. R3 approximately here]

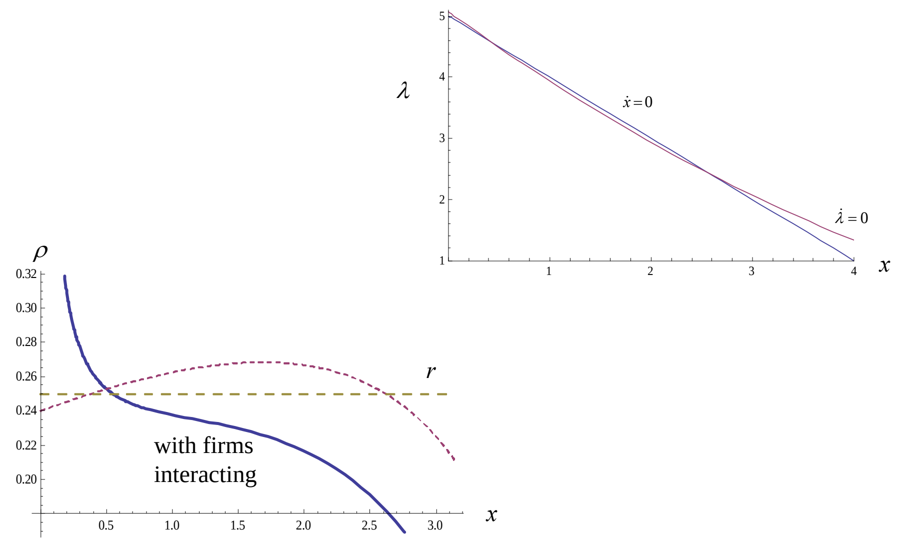

Fig. R1: Examples of firm‘s optimization problem with lower and unstable steady state in the **non-concave** and larger and stable steady state in concave domain α_1 = -5, α_2 = 1, α_3 = 1, α_4 = 1, α_5 = $^1/_{10}$, c_4 = 0, r = ¼, k_3 = $^1/_{20}$, φ = $^1/_{10}$, X = ½.

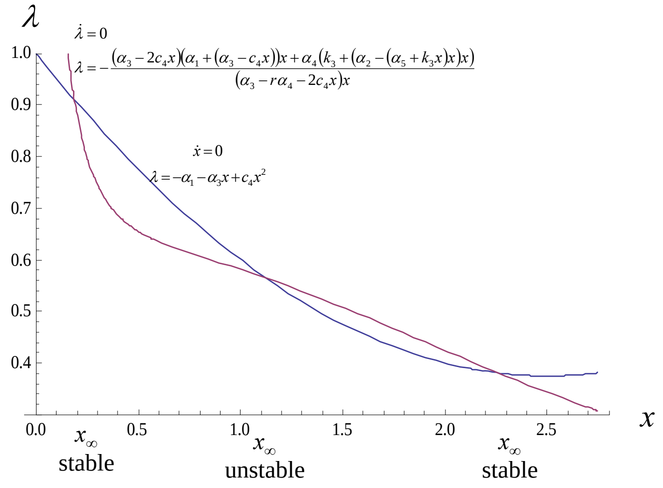

Fig. R2: Phase diagram of Example in Fig. 5 but with firms interacting α_1 = -1, α_2 = $^4/_3$, α_3 = ½, α_4 = ½, α_5 = $^1/_3$, c_4 = $^1/_{10}$, r = 2, k_3 = $^1/_{100}$, φ = $^1/_{10}$.

Fig. R3: Example similar to Fig. 5 (dashing = without interactions)
(lower steady state in the **non-concave** and larger and stable steady state in **concave** domain)
but where interaction leads to the high level as unique longrun outcome:
$\alpha_1 = -1$, $\alpha_2 = {}^3/_2$, $\alpha_3 = ½$, $\alpha_4 = ½$, $\alpha_5 = ¼$, $c_4 = {}^1/_{10}$, $r = 2$, $k_3 = 0$, $\varphi = {}^1/_5$, ($X = 4$).