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**Research Report 2012-10**

December, 2012

**Operations Research and Control Systems**  
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# Individual Firm and Market Dynamics of CSR Activities

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November 20, 2012

## Abstract

This paper investigates how firms should plan *Corporate Social Responsibility* (short CSR). This dynamic analysis starts with a firm's intertemporal optimization problem, and proceeds to analyze interactions with other firms, which are crucial: if CSR is profitable for firm A then it is most likely also profitable for competitors B and C, and these simultaneous decisions affect the gain each would achieve from trying to advance its own position. We find that multiple equilibria exist, irrespective of whether interactions with other firms are taken into account. Interactions can eliminate or create additional steady states and can lead to a situation in which history is insufficient to determine the longrun outcome among multiple steady states, so that coordination is beneficial.

Keywords: CSR, dynamics, firms' interactions, stability, history and expectation dependence.

JEL codes: C61, D21, D92, L21.

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§We all are grateful for the very helpful comments from an associate editor and three anonymous referees. Of course, the usual caveat applies.

# 1 Introduction

The term *Corporate Social Responsibility* (CSR) came into common use in the early 1970s and it refers to a form of corporate self-regulation. This approach accounts for stakeholders' interests (advanced in a book by Freeman (1984)) in addition to the shareholders' focus on profits and includes thus a number of ethical, social, and environmental targets, where the green aspect is the most visible one currently. Michael Porter has for some time praised the profitability (win-win) of socially and, in particular, environmentally responsible behavior, e.g., in Porter and van der Linde (1995a, 1995b), in the annual *Global Competitiveness Reports* des *World Economic Forum* (Porter et al. (2004)), in Porter and Kramer (2006), and most recently in Porter and Kramer (2011). In the last paper, it is argued that taking a broad view on societal needs is nothing less than a means to 'reinvent capitalism and unleash a wave of innovation and growth'. Another broader view is given in Aoki (2011) that mentions CSR as a part in the role of organizations, which is to cognitively mediate agents' strategic interactions. Lyon and Maxwell (2008) and Reinhardt et al. (2008) are part of a recent debate about the pros and cons (in that order). The importance of CSR is also documented by its appearance in many articles in the public press. We only mention here the surveys in *The Economist*, first in 2005 ("The good company", January 20th) followed in 2008 (in "Just good business", January 19th) and a very recent and more optimistic article ("Schumpeter: Good business; nice beaches", May 17th 2012).

The evolution, one can even speak of a wave, of CSR activities can be seen, e.g., by considering how many firms subjected themselves voluntarily over time to (environmental and social) standards like EMAS ("eco-management and audit scheme" within the European Union) and ISO14001 (the same objective of good environmental management), as shown in Figure 1. Figure 2 shows the composition by countries. A survey of attitudes to business turns up some intriguing differences about CSR: first across nations, where Friedman's view ('the business of business is business') dominates in developing and emerging economies according to *The Economist* (Jan 27th 2011), whereas Germany and other northern and western European companies are much more concerned about CSR. Second, CSR activities differ not only across nations but also widely across industries. For example, all major oil companies are very CSR conscious (if not always successful, as BP's failures document) while much less is heard of other mining industries. Understand-

ing an evolution as the one shown in Figure 1 requires an explicit dynamic analysis of CSR activities complementary to the many static investigations. In particular, a dynamic analysis is needed that allows for a dynamic pattern compatible with Figs. 1 and 2, implying different evolutions and long run outcomes across countries, regions and industries.

[Insert Fig. 1 approximately here]

[Insert Fig. 2 approximately here]

The aim of our model is to address these aspects. In general and more formal terms, two things will be shown: First, positive interactions between a firm's current investment and its stock, which seems to be of particular importance for CSR. For the reasons given, it can make a firm's long run choice dependent on initial conditions even if the environment remains constant and the firm operates under diminishing returns. Second, the interactions among firms, which all invest in CSR, can lead to history dependent, and to even more complex, market outcomes.

The starting point is by and large the optimistic management literature that encourages firms to pick up this win-win situation associated with CSR. Unruh and Ettenson (2010) even warn managers: "Don't let your competitors control what "sustainable" means in your industry." What this quote and many other management proposals often ignore, however, is that one must take the reactions of other firms into account, because they face similar incentives. Therefore, this paper investigates also how such interactions among firms affect outcomes. This integration allows us to address the following questions: Can we explain a CSR-wave? Is it a transient phenomenon or is it sustainable in the long run? Can multiple equilibria arise such that an individual firm's activities depend on its history? Does this history dependence extend to entire industries? Does a history dependent outcome for the firm's isolated profit maximization problem vanish if we include the interaction with the other firms in our analysis? Or is the converse possible: where a firm's decision to fix the other firms' CSR activities leads to a unique and stable outcome? Does including interactions with other firms yield a history dependent, maybe even expectation driven, outcome?

The paper is organized as follows: The model is introduced in Sections 3 and 4, after a brief literature review (Section 2). Section 5 derives the necessary optimality conditions for the optimal expansion of a firm's CSR activities. Section 6 characterizes a single firm's optimal expansion strategy

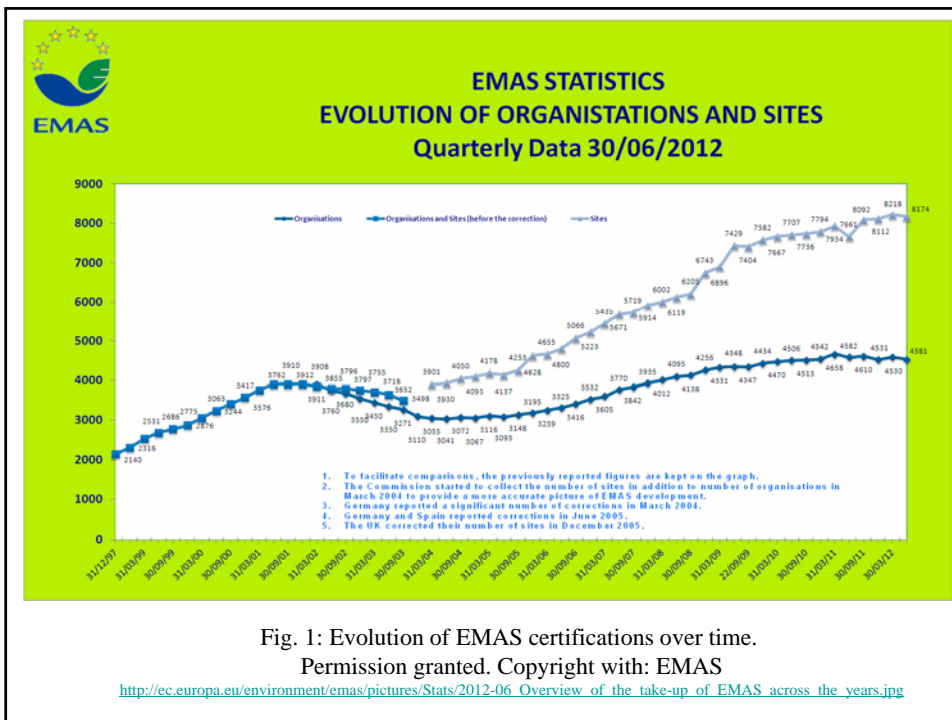


Fig. 1: Evolution of EMAS certifications over time.

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<http://ec.europa.eu/environment/emas/pictures/Stats/2012-06 Overview of the take-up of EMAS across the years.jpg>

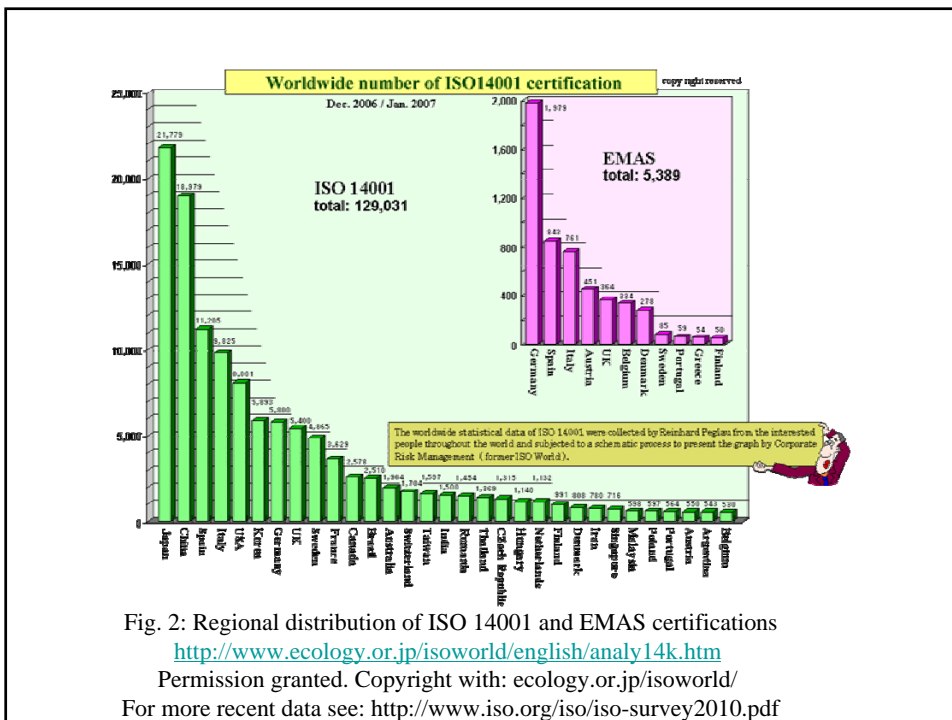


Fig. 2: Regional distribution of ISO 14001 and EMAS certifications

<http://www.ecology.or.jp/isoworld/english/analy14k.htm>

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For more recent data see: <http://www.iso.org/iso/iso-survey2010.pdf>

holding the other firms' activities constant. Section 7 integrates the decisions of the other firms within a competitive setting in order to address the above raised questions. Section 8 concludes.

## 2 Literature review

A large volume of empirical papers investigates the benefits from CSR. Most of them focus on particular initiatives in isolation, which may overestimate the effect of particular initiatives if the agents care about the whole CSR portfolio. The following sample is somewhat selective and focuses on those that bear some connection to our paper. Orlitzky et al. (2003) find in their meta-study some support for the optimistic claims about CSR. Similarly, Nishitani (2011) obtains that environmental management adds value for a sample of Japanese firms. This is confirmed in Becchetti et al. (2007) by looking at how additions and deletions to the Domini Social 400 index affect share prices. Of course, correlation is not causation, i.e. the found patterns may be due to reverse causality since only profitable firms can afford CSR, or they are spurious. And indeed, Surroca et al. (2010) merely find an indirect relationship that relies on the mediating effect of a firm's intangible resources. Fernández-Kranz and Santaló (2010) study the link between competition and social performance, also empirically, and obtain that competition fosters CSR. Krishna and Rajan (2009) link CSR with cause marketing by analyzing a game of competing firms, and they run corresponding experiments. Eichholtz et al. (2010) establish that green office buildings fetch higher rents and selling prices. Chatterji et al. (2009) find that published rankings provide 'fairly good summaries of past environmental performance'. Siegel and Vitaliano (2007) verify empirically that firms selling credence goods are more likely to be socially responsible than firms selling search goods. This is economically intuitive and in line with Heyes (2005) that suggests signalling as a motive for CSR.

A subclass of the theory papers uses signaling games. Heyes (2005) and Goyal (2006) investigate signaling in connection with FDI. Recently, Heijnen and van der Made (2012) consider signaling from the other side, more precisely they show consumers boycotting a firm if it is non-CSR. Baron (2007) presents an agency theoretic framework of corporate social responsibility and shows, among other things, (i) that CSR is costly when it is an imperfect substitute and (ii) that the entrepreneurs, but not the shareholders, bear its cost.

In a related paper, Baron (2008) suggests that corporate social responsibility is a form of private provision of public goods. Bagnoli and Watts (2003) link CSR to the sales of their private goods. And more recently, Baron (2010), by using a matching model, advances how self regulation and the existence of social labels and certification organizations mitigate free riding.

Another branch treats CSR as a kind of product differentiation. Arora and Gangopadhyay (1995) explain overcompliance as a result of willingness to pay for green goods depending on income and resulting market segmentation. Becchetti et al. (2003) show in a Hotelling framework that a socially concerned entrant generates a Pareto improvement. Manasakis et al. (2007) consider CSR in a two-stage game of product differentiation and delegation. In Alves and Pinto (2008) CSR is linked to goods being complements if consumers prefer CSR firms. Kopel (2009) employs a product differentiation framework to study the conditions under which socially responsible firms can develop a first-mover advantage. Toolsema (2009) designs a duopoly framework to investigate the role of switching costs in a setting with green and nongreen products.

In contrast, the analysis of CSR within a dynamic framework is rare. There exist papers that investigate dynamic versions of the above sketched product differentiation games, but they do not consider CSR (e.g., Piga (1998), Cellini and Lambertini (2002), Hsu and Wang (2004), and, more recently, Lee et al. (2012)). The evolutionary but descriptive approach of CSR in Chen et al. (2009) applied to emerging economies (as well as the multi-period supply chain networks combined with multi-criteria decision optimization in Cruz (2008) and in Cruz and Walkobinger (2008)) are complementary to our approach. The following papers choose a similar focus and method: Becchetti et al. (2010) assume that consumer tastes for CSR increase with a monopoly's CSR activities. This is extended to an investigation of the open loop Nash equilibrium among oligopolists in Becchetti and Solferino (2005). Lundgren (2011) focuses on intertemporal consumer goodwill, which is similar to our model but without competitors. However, all these dynamic models are structured in a way such that unique and monotonic saddlepoint paths result, which cannot explain the variance in CSR activities internationally and across industries. As said before, our aim is to develop a framework that can explain this variance of CSR activities, where at the same time we can address the question of whether these activities are sustainable.

### 3 Model

The starting point is similar to Lundgren (2011): a firm considers whether to expand (or to reduce) its CSR activities, with the aim to maximize its net present value of profits. The following notation is used:

$$\begin{aligned}
 x &= \text{a firm's number of CSR projects,} \\
 u &= \text{change in a firm's CSR activities,} \\
 X &= \text{average industry wide CSR activity,} \\
 \pi(u, x, X) &= \text{instantaneous payoff for the firm.}
 \end{aligned}$$

Examples of CSR projects ( $x$ ) are: environmental reports, philanthropic supports and sponsoring, energy and environmental management, mentoring and educational programs for workers, family friendly workplaces and alike. Further examples are higher wages, improvements of working conditions and a switch to local sourcing. The control variable  $u$  can be either negative or positive in order to allow for additions and terminations of such activities and projects. Assuming a competitive environment, it follows that the firm cannot influence the level of the average industry-wide activity,  $X$ .

The firm's expenditures for CSR consist of, firstly, running existing projects ( $K(x)$ ) that may include increasing or decreasing returns,  $K'' \stackrel{\geq}{\leq} 0$ . Secondly, there are costs for adding new CSR projects,  $C(u, x)$ , including scrapping if  $u < 0$ . With respect to expansion it makes sense to assume diminishing returns,  $C_{uu} > 0$ . The dependence of these costs, and in particular of the marginal costs ( $C_u$ ), on the state  $x$ , is presumably nonlinear due to two opposite effects. On the one hand we may have scale economies and experience, thus  $C_{ux} \leq 0$  for  $x$  small. On the other hand, diseconomies of scale occur as new projects are getting more and more costly after many projects have already been launched (because the cheaper ones will be selected first), thus  $C_{ux} \geq 0$  for  $x$  'large'. Although one might also include industry-wide cost spillovers,  $C(u, x, X)$ , where either  $C_X < 0$ , i.e., a firm may benefit from the experience in other companies, or  $C_X > 0$ <sup>1</sup>. Their presumably small effect is neglected in the analysis.

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<sup>1</sup>This could be caused by diminishing returns, e.g., due to resource constraints. A concrete example is the cacao industry and protection of the biosphere. The more the industry pursues this CSR-strategy - to grow cacao beans in the shadow of the jungle instead of on cleared agricultural plots - the more costly become additional locations, because the best places have been already taken.



The benefit for an individual firm from CSR is

$$B(u, x, X) = b(u, x) + f(x, X).$$

$B$  has positive derivatives with respect to both private variables  $(u, x)$ , but depends presumably negatively on  $X$ . At this abstract level, benefits  $B$  can include many aspects of CSR. Here we think of, e.g., higher revenues by increasing consumers' willingness to pay for a CSR-promoted good, or for all goods of a firm investing in CSR, less risks and liabilities in the case of active environmental management and therefore less insurance premia, better access to capital, less sick leaves, and more effort due to better treatment of workers and providing a family-friendly environment<sup>2</sup>. In addition, CSR allows the manager to obtain private and personal gains from running a green and socially concerned company by accruing public admiration, invitations, and honors for the firm's philanthropic engagement and/or support of local industry.

The benefits from a firm's position in relation to its competitors ( $f$ ) are separated from the direct benefits ( $b$ ) only for reasons of simplicity, which is by no means crucial for the results. Moreover, it is also not entirely out of line with classifying CSR benefits. For example, let  $b$  account for all the firm's internal benefits, like more effort, less sick leave, less disruption due to local supplies, less risks and liabilities, etc. and  $f$  the gain in the market. Such a gain can be accomplished, e.g., in a Hotelling setting by catering to customers with special needs, which will only work out if the firm can put a 'distance' between its own and its competitors' positions. Relative positions matter in the sense that a more CSR-active firm can increase its market share at the expense of less CSR-active competitors for reasons given above, such as higher valuation of the firm's goods. Concerning the differences in positions, this can be modeled in an absolute  $(x - X)$  or relative  $(x/X)$  way. The function  $f$  will be based on either one of them in such a way that  $f(x, X)$  has the same value whenever  $X = x$ . Furthermore, we impose that  $f_X < 0$ , i.e. the firm's benefit from its CSR activities is reduced if its competitors expand.

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<sup>2</sup>There is a large amount of literature, theoretical, empirical and experimental, that supports this: on efficiency wages e.g., Shapiro and Stiglitz (1984), Salop (1979), Malcomson (1981); on gift exchanges Akerlof (1982) is the seminal paper; on intrinsic motivation we have Ryan et al. (1991), Frey and Oberholzer-Gee (1997), and Kreps (1997); and for positive reputation effects, see Myers (2009).

Combining benefits and costs yields the firm's compound instantaneous objective (from CSR activities)

$$\pi(u, x, X) = b(u, x) + f(x, X) - C(u, x) - K(x). \quad (1)$$

The corresponding dynamic optimization problem is represented by

$$\max_{\{u(t), t \in [0, \infty)\}} \int_0^{\infty} e^{-rt} \pi(u(t), x(t), X(t)) dt, \quad (2)$$

$$\dot{x}(t) = u(t), \quad x(0) = x_0, \quad x \geq 0. \quad (3)$$

Although, the spirit of this model - CSR buys something (e.g., goodwill of consumers but also of its workers, of politicians and regulators) - is partially similar to Becchetti et al. (2005) and Lundgren (2011), it differs (a) in the complexity that allows for history dependent outcomes and (b) in its extension for interactions with other firms. Only Becchetti and Solferino (2005) accounts for firm interactions in a dynamic setting. It computes the open loop Nash equilibrium between duopolists that are differentiated according to their CSR position along the Hotelling line  $[0, 1]$ .

## 4 Assumptions and example

This analysis is carried out in terms of the general instantaneous payoff function  $\pi(u, x, X)$ . This is complemented by deriving the consequences for the partially separable structure introduced in (1) and by numerical examples based on specifications of  $b$ ,  $f$ ,  $C$ , and  $K$  introduced below. The following assumptions are made:

1.  $\pi(u, x, X)$  is strictly concave with respect to  $u$ .
2. Decreasing returns of  $\pi$  hold at least for large values of  $x$ . Increasing returns, if existing, are thus limited to small and intermediate values.
3.  $\pi_u < 0$ .
4. Competitive effects among the firms' CSR activities dominate, i.e.,  $\pi_{xX} < 0$ .

Assumption 1 allows for an interior optimal policy and Assumption 2 prevents that "trees grow to the sky". Assumption 3 states that setup costs for additional projects exceed the immediate benefit, while Assumption 4 stresses that the competitive element dominates among firms.

**Remarks:**

1. The model and the subsequent analysis assume that consumers and other stakeholders are concerned about the firm's level of CSR activities in relation to its competitors, i.e., this comparison is linked to the state. It turns out that if, alternatively, this comparison is related to new CSR projects, i.e. with respect to the control variable  $u$ , the competitive outcome is qualitatively the same as in the isolated firm's case. This is the case, because it is *observationally equivalent* to the one where firms ignore this interaction after modifying the firms' preference structure (but not observable, see Hof and Wirl (2008) on this concept and the online appendix for a corresponding discussion).
2. If Assumption 4 does not hold, we have complementarity between own and industry-wide CSR. This implies that, at least locally, it holds that  $\pi_{xX} > 0$ , so that positive spillovers dominate. However, complementarity is an unlikely additional source for complexity and is thus excluded from our analysis, see the online appendix for further details.
3. An interesting extension could be to incorporate consumer goodwill. This is typically a stock, and, like our state variable  $x$ , it is influenced by the firm's past initiatives. However, introducing goodwill explicitly, as is done in many advertising papers (e.g. in Cellini and Lambertini (2002)), requires introducing a second state variable called 'goodwill'. This will lead to a different kind of analysis and presumably allows for even more complex (cyclical?) patterns. This extension is left for future research.

An example of a payoff function is constructed bottom up and will be used for numerical illustrations of the major results that hold of course for general payoff functions  $\pi$ . Costs for CSR projects may be linear for a start, but may include both increasing returns at small and diminishing returns at high levels, e.g.:

$$K(x) = k_1x - \frac{k_2}{2}x^2 + \frac{k_3}{3}x^3, \quad k_i \geq 0. \quad (4)$$

Costs for expanding CSR must satisfy (strictly) the law of diminishing returns with respect to expansion (the linear-quadratic relation in (5)) but may include experience effects if  $x$  is ‘small’,  $C_{ux} < 0$ , and the opposite,  $C_{ux} \geq 0$ , if  $x$  is ‘large’; a simple representation is

$$C(u, x) = c_1u + \frac{c_2}{2}u^2 - c_3ux + c_4ux^2, \quad c_i \geq 0. \quad (5)$$

The direct benefits are approximated by a simple linear-quadratic specification

$$b(u, x) = b_1u + b_2x + b_3ux - \frac{1}{2}b_4u^2 - \frac{1}{2}b_5x^2, \quad b_i \geq 0. \quad (6)$$

Finally, the ratio

$$f(x, X) = \varphi \frac{x}{X}, \quad \varphi > 0, \quad (7)$$

describes how a firm benefits from its position relative to its competitors.

Combining (4) - (7) yields the instantaneous profit,

$$\pi(u, x, X) = \alpha_1u + \alpha_2x + \alpha_3ux - \frac{1}{2}(\alpha_4u^2 + \alpha_5x^2) - c_4ux^2 - \frac{k_3}{3}x^3 + \varphi \frac{x}{X}, \quad (8)$$

where the signs of most coefficients in (8) are indeterminate,

$$\alpha_1 = b_1 - c_1, \alpha_2 = b_2 - k_1, \alpha_3 = b_3 + c_3 > 0, \alpha_4 = b_4 + c_2 > 0, \alpha_5 = b_5 - k_2.$$

The model allows for many scenarios. Therefore, for reasons of tractability a few parameter restrictions will be imposed on the numerical examples below to focus on specific scenarios. First, we impose that  $\alpha_1 < 0$ , i.e. the cost element, dominates with respect to new projects in line with Assumption 3. Second, the benefit dominates for all CSR activities,  $\alpha_2 > 0$ . Third, the negative coefficient of  $x^2$  in the benefit function dominates the experience effect within the cost specification, i.e.  $\alpha_5 \geq 0$ .

## 5 Necessary optimality conditions

Setting up the current value Hamiltonian of the firm’s optimal control problem,  $H = \pi(u, x, X) + \lambda u$ , we obtain the following necessary optimality

conditions (see e.g. Grass et al. (2008)),

$$H_u = \pi_u + \lambda = 0, \quad (9)$$

$$H_{uu} = \pi_{uu} < 0, \quad (10)$$

$$\dot{\lambda} = r\lambda - \pi_x \implies \lambda(t) = \int_t^{\infty} e^{-r(v-t)} \pi_x dv. \quad (11)$$

The maximum principle (9) implies that the marginal net costs for expanding CSR,  $(-\pi_u)$ , must equal the shadow price  $\lambda$  that stands for the net present value of expanding CSR at period  $t$  by an infinitesimal unit. This economically intuitive condition follows after integrating the costate differential equation, as is shown in expression (11). The second order necessary optimality condition (10), i.e., concavity of the Hamiltonian with respect to the control (also known as the Legendre-Clebsch condition), holds globally by Assumption 1. This implies that the optimal change in a firm's CSR activities,  $u^*$ , satisfies (9), which in turn implies that

$$u = u^*(x, \lambda, X), \quad u_x^* = -\frac{\pi_{ux}}{\pi_{uu}}, \quad u_\lambda^* = -\frac{1}{\pi_{uu}} > 0, \quad u_X^* = -\frac{\pi_{uX}}{\pi_{uu}}. \quad (12)$$

Hence, a higher shadow price increases the optimal current expansion,  $u_\lambda^* > 0$ , while the signs of the qualitative dependence on the own and economy-wide CSR activities equal the signs of the corresponding mixed derivatives,  $\pi_{ux}$  and  $\pi_{uX}$ , respectively. In the case of example (8) we have

$$u^* = \frac{\lambda + \alpha_1 + x(\alpha_3 - c_4x)}{\alpha_4}. \quad (13)$$

Note that  $u^*$  does not directly depend on the average industry wide CSR activities,  $X$ . This is because there is no interaction term between  $u$  and  $X$  in (8). However, there is still an indirect effect, because  $X$  affects  $\lambda$ , which in turn has a positive effect on the optimal change in the firm's research activities  $u^*$ .

## 6 Individual CSR

In this section we study the scenario where the firm takes the industry-wide CSR activity as given. This implies that  $X$  is treated as a parameter. The

resulting optimal control model has the existing CSR activities,  $x$ , as state variable, and setting up the new CSR activities,  $u$ , as control variable. We start out by determining a condition that a steady state of the model should satisfy. It turns out that the rate of substitution between existing and new CSR activities is a crucial element here. We further derive a condition that tells us when a steady state is stable or unstable. Occurrence of an unstable steady state together with existence of a stable steady state points to history dependence (see, e.g., Skiba (1978)), i.e. it depends on the initial value of existing CSR activities whether CSR is sustainable in the long run or not. The analysis at the end of this section is devoted to this topic.

The rate of substitution between already existing ( $x$ ) and new ( $u$ ) CSR activities at  $u = 0$ , is given by

$$\rho(x, X) := -\frac{\pi_x(0, x, X)}{\pi_u(0, x, X)}. \quad (14)$$

The necessary optimality conditions (9) - (11) imply the following steady state condition, where the symbol  $\infty$  is used to identify steady states throughout the paper:

**Proposition 1:** *Any (interior) steady state,  $x_\infty$  (unstable), or  $x^\infty$  (stable), must satisfy:*

$$\rho(x_\infty, X) = \rho(x^\infty, X) = r. \quad (15)$$

Expression (15) says that at a steady state the marginal cost for expanding CSR net of immediate benefits,  $(-\pi_u)$ , must equal the net present value of the marginal benefit from a higher level of CSR  $(\frac{\pi_x}{r})$ .

The following proposition provides a condition that determines the stability of the steady state.

**Proposition 2:** *If at a steady state it holds that*

$$r\pi_{ux} + \pi_{xx} < 0,$$

*then the corresponding steady state is a saddle point. A steady state is unstable in the opposite case, thus if at this steady state it holds that*

$$r\pi_{ux} + \pi_{xx} > 0. \quad (16)$$

**Proof.** *See Appendix.*

Taking the (partial) derivative of  $\rho(x, X)$  we obtain

$$\rho_x = -\frac{\pi_{xx} - \frac{\pi_x}{\pi_u}\pi_{ux}}{\pi_u} \implies \rho_x = \left(-\frac{1}{\pi_u}\right) (\pi_{xx} + r\pi_{ux}) \text{ at a steady state.} \quad (17)$$

Since Assumption 3 states that  $\pi_u < 0$ , then from (17) and Proposition 2 it follows that a positive slope of  $\rho$  at the steady state implies that this steady state is unstable, while a negative slope points to saddle point stability. The latter either requires  $\pi_{ux} > 0$ , and thus a corresponding positive interaction, and/or  $\pi_{xx} > 0$ , i.e. increasing returns with respect to  $x$ .

It is important for the firm to know how the activities of its competitors ( $X$ ) affect its marginal rate of substitution, because that determines how the locations of the steady states shift as a response to an increase of its competitors' CSR activities (see the dashed lines in Fig. 3). Differentiating  $\rho$  with respect to  $X$  and substituting the steady state condition yields

$$\rho_X = \left(-\frac{1}{\pi_u}\right) (\pi_{xX} + r\pi_{uX}) = \left(-\frac{1}{\pi_u}\right) \pi_{xX} \text{ at a steady state.} \quad (18)$$

The expression after the first equality sign holds for general  $\pi(u, x, X)$  and the one on the right hand side is the result after dropping the interaction that is absent in the specification (1), i.e.  $\pi_{uX} = 0$ .

**Proposition 3:** *Assuming  $\pi_{uX} = 0$ ,  $\pi_{xX} < 0$  as in (1), it holds that any increase in the competitors' CSR activities lowers  $\rho$ . The impact on the steady states is such that  $x$  has a higher value at an unstable steady state while it has a lower value at a stable steady state.*

Returning to Proposition 2, we obtain that there are two economic routes for instability. From the literature (e.g., the references stated right below Proposition 4), we know that the occurrence of an unstable steady state could imply history dependent outcomes, especially when it jointly occurs with another steady state being a saddle point.

**Proposition 4** *The inequality  $r\pi_{ux} + \pi_{xx} > 0$  is necessary for an unstable interior steady state (and sufficient if evaluated at such a steady state), and thus for an outcome in which the firm's long-term strategy depends on its own history ( $x_0$ ). There are two distinct routes for such history dependent outcomes: (i) a (locally) convex objective, i.e.  $\pi_{xx} \geq 0$ , (ii) if the law of diminishing returns holds ( $\pi_{ux}^2 < \pi_{uu}\pi_{xx}$ ), then the interactions between new (= control) and existing CSR (= state) activities must be positive and sufficiently large,  $r\pi_{ux} > -\pi_{xx} > 0$ .*

The route (i) from Proposition 4 is economically very intuitive and therefore it is no surprise that hundreds of papers pursue this route following the pioneering works of Sethi (1977), Skiba (1978) and Dechert and Nishimura (1983). The claim that increasing returns, i.e.  $\pi_{xx} \geq 0$ , are necessary for any kind of history dependency can also be found in the works of Arthur, e.g., Arthur (1989, p 116), and of Krugman, e.g. Krugman (1991), which accounts for firms' interactions similar to the extension introduced in Section 7. Figure 3 shows the phase diagram of a corresponding numerical example based on (8). The intersections of  $\rho$  (shown on top in Fig. 5 by the dashed line) with  $r$  determine two steady states in the nonconcave domain of which the unstable one is a focus and the larger steady state is (saddlepoint) stable.

Route (ii) assumes that the firm operates under the law of diminishing returns (see also Wirl and Feichtinger (2005) for a similar scenario). Although concavity of an optimal control problem does imply uniqueness of maximizers for given initial conditions, it does not imply uniqueness of steady states. This possibility is in contrast to the impression created by the above referred large volume of literature. According to (16), this route requires  $\pi_{ux}$  to be positive and sufficiently large. It is surprisingly easy to construct a corresponding example drawing on the specification (8), which is shown in Figure 4, again in the form of a phase diagram. The crucial differences between the unstable steady states in Figures 3 and 4 are: if the objective is concave at the unstable steady state, as in Figure 4, then the policy function is continuous and passes through the unstable steady state ( $x_\infty$ ), being an unstable node. The important implication is that then this unstable steady state is the threshold separating the domains of attraction of either  $x \rightarrow x^\infty$ , or quitting CSR,  $x \rightarrow 0$ . Also, if  $x(0) = x_\infty$ , then the optimal policy is to stay at the unstable steady state, i.e. the unstable steady state is optimal.

In the case of Figure 3, where the unstable steady state is situated in the non-concave domain, this unstable steady state is neither optimal, nor is the policy function continuous in this case<sup>3</sup>. Moreover, the determination of the threshold separating the domains of attraction requires an explicit computation of the objective value for initial values of the state  $x$  around the unstable steady state,  $x_\infty$ . Then the threshold is that value of  $x$  for which

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<sup>3</sup>This is for sure in this example, because the unstable steady state is a focus. However, and contrary to an impression created in many papers, the policy function can be continuous even if this steady state is in the non-concave domain and a node. Hartl et al. (2004) provides a corresponding example for a firm's investment problem facing relative adjustment costs.



the firm is indifferent between the paths  $x \rightarrow x^\infty$  or  $x \rightarrow 0$ .

Although various parameter variations and corresponding comparative statics (with respect to steady states) and comparative dynamics are possible, the discussion is here restricted to the parameter  $\alpha_4$ . We made this choice because  $\alpha_4$  does not affect the locations of the steady states, while, on the other hand, a larger value of  $\alpha_4$  increases the concave domain of the objective in the example (8). In particular, increasing  $\alpha_4$  helps to obtain a positive determinant of the Hessian of  $\pi$  with respect to  $(u, x)$ , which at the steady state is equal to

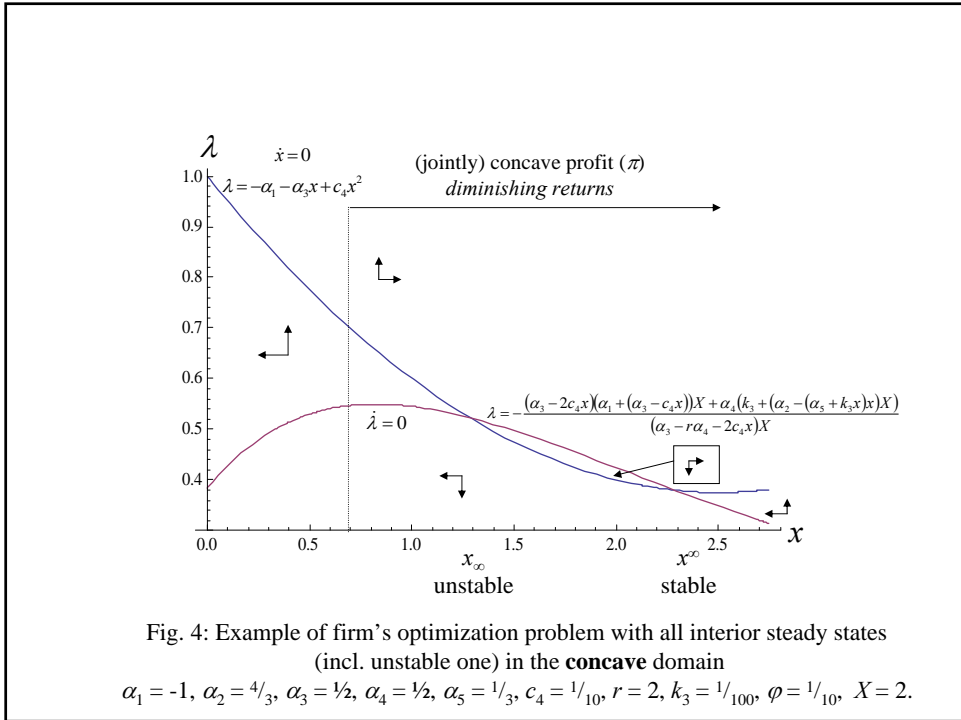
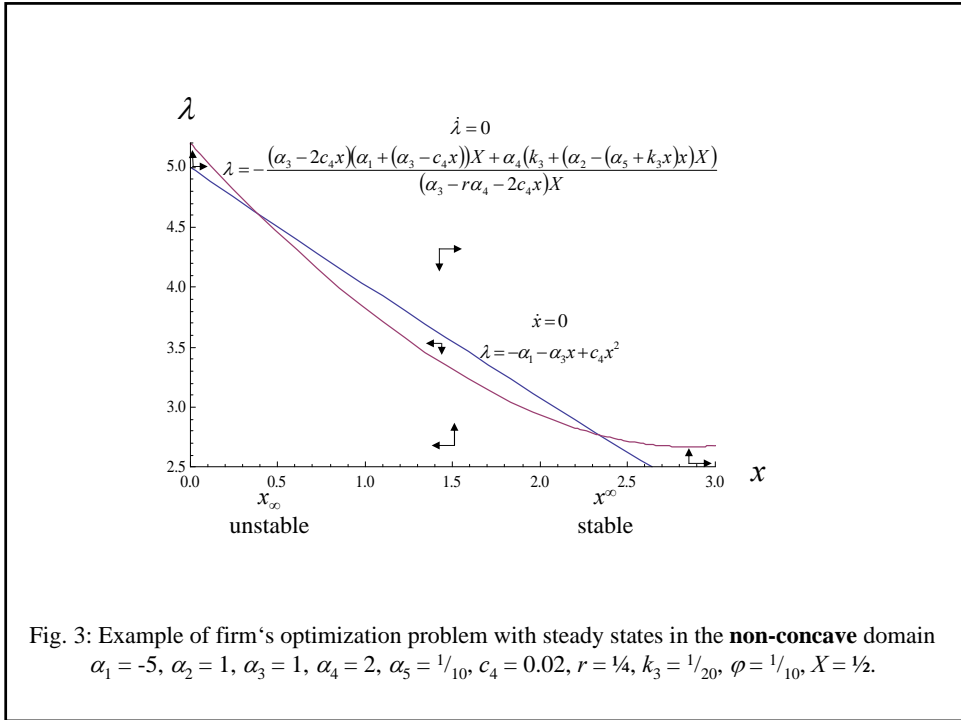
$$\begin{pmatrix} \pi_{uu} & \pi_{ux} \\ \pi_{ux} & \pi_{xx} \end{pmatrix} = \begin{pmatrix} -\alpha_4 & \alpha_3 - 2c_4x \\ \alpha_3 - 2c_4x & -\alpha_5 - 6k_3x \end{pmatrix}.$$

Note that concavity of the Hamiltonian requires that this Hessian is negative definite. The implication is that lowering  $\alpha_4$  can shift (the unchanged) unstable steady state into the non-concave domain, including the possibility that the unstable node turns into an unstable focus such that the local dynamics are like those in Figure 3.

[Insert Fig. 3 approximately here]

[Insert Fig. 4 approximately here]

The upshot of both scenarios is that the decision whether to continue, possibly to expand CSR, or to leave this field, can depend on the initial conditions. Furthermore, exogenous events re-shaping a firm's benefits or costs may alter the long run outcome in a discontinuous way. Indeed, equipped with the above derived properties, it is possible to eliminate or generate an outcome with CSR including the possibility of threshold effects. Assuming existence of an unstable steady state at the starting point (i.e.,  $\rho_x > 0$  at one intersection with  $r$ ), then any change in parameters that increases the marginal benefit from CSR (i.e.,  $\pi_x$ ) raises the inverted U representation of  $\rho$  shown in Fig. 5 (below). A raise of  $\rho$  results in a decrease of the value of the state that corresponds to the unstable steady state. Hence, for a large enough raise of  $\rho$  the unstable steady state ceases to exist, implying that the stable steady state is left as the unique positive long run outcome. For example, setting  $\alpha_2 = 3$  in the example in Figure 3 yields a unique and stable outcome (of  $x_\infty = 5.74$ ). Summarizing, increased marginal benefits from CSR can lead to the emergence of CSR with or without dependence on initial conditions.



On the other hand, parameters that increase the costs for (new) CSR projects or decrease the marginal benefit (an example would be an increase of  $X$ ; see Proposition 3), lower the inverted U curve of  $\rho$  in Figure 5. Small changes raise the unstable steady state (thus enlarges the basin of attraction of  $x \rightarrow 0$ , i.e., no CSR in the long run) and lowers the stable steady state. However, if these changes are large enough, the intersections between  $\rho$  and  $r$  will disappear. Then, no CSR will occur in the long run, e.g., reducing  $\alpha_2$  below  $\approx -5.21$  for the example in Figure 3 drives  $\rho$  below  $r$ .

## 7 Equilibrium among interacting firms

A large part of the management literature provides recommendations to single firms without drawing attention to the fact that a similar logic applies to its competitors, as mentioned in the Introduction. In contrast to this often firm-specific advice by consultants, a firm must recognize that its own incentives to expand CSR are similar to those of its competitors. The framework in this section takes this into account. To do so, we consider a competitive situation implying that no firm has a strategic leverage, which in turn implies that a differential game analysis is not necessary. Nevertheless, each firm still has still the incentive to get ahead of competitors, which creates an externality among all firms. Although a firm cannot decide upon the competitors' actions, it can (rationally) predict the future path of  $\{X(t)\}$  from its own profitability considerations. For tractability reasons we assume a perfectly symmetric setting so that it makes most sense to develop a symmetric equilibrium. In such a setting it holds that competitors will simply choose the same investment, implying

$$X = x. \tag{19}$$

This approach is adopted from the endogenous growth literature and is used as well in Krugman (1991). In spite of the identity (19),  $X$  cannot be part of the firm's optimization, and must therefore be treated as exogenous when deriving the optimality conditions as outlined in Section 5. However, the equilibrium condition (19) can be substituted into the firm's necessary optimality conditions, implying that the firm has rational expectations about its competitors' current and future actions. This yields the following characterization of the evolution of firm and of industry-wide CSR, in which each firm correctly anticipates how its competitors will react to the incentives,

including the interactions among the firms,

$$\dot{x} = u^*(x, \lambda, x), \quad (20)$$

$$\dot{\lambda} = r\lambda - \pi_x(u^*(x, \lambda, x), x, x). \quad (21)$$

Krugman (1991) inter alia argues that increasing returns are needed to obtain history dependency even after including firms' interactions. However, this is not true in general, and not even within the specific framework that Krugman (1991) presents, see Wirl and Feichtinger (2006) for a counterexample.

The dynamic system (20) and (21) implies the same steady state condition (14), but with the difference that  $\rho$  is now a scalar function after substituting (19)

$$r = \rho(x) := -\frac{\pi_x(0, x, X)}{\pi_u(0, x, X)} \Big|_{X=x}. \quad (22)$$

Straightforward calculations lead to the following proposition.

**Proposition 5:** . *The determinant of the Jacobian (capital letter to differentiate from the firm's  $j$ ) of the dynamic system  $(x, \lambda)$  is*

$$\det(J) = -\frac{1}{\pi_{uu}} [(\pi_{ux} + \pi_{uX})r + (\pi_{xx} + \pi_{xX})]. \quad (23)$$

*If at a steady state it holds that*

$$(\pi_{ux} + \pi_{uX})r + \pi_{xx} + \pi_{xX} < 0,$$

*then the corresponding steady state is a saddle point. A steady state is unstable in the opposite case, thus if at this steady state it holds that*

$$(\pi_{ux} + \pi_{uX})r + \pi_{xx} + \pi_{xX} > 0.$$

*The non-standard outcome,  $(\pi_{ux} + \pi_{uX})r + \pi_{xx} + \pi_{xX} > 0$ , or  $\det(J) > 0$ , allows in general, and in contrast to the firm's isolated optimization problem, for two subcases depending on the trace of the Jacobian,*

$$\text{tr}(J) = r - \frac{\pi_{uX}}{\pi_{uu}}. \quad (24)$$

- (i) if  $\text{tr}(J) > 0$ , the steady state is unstable and gives rise to a threshold.*
- (ii) if  $\text{tr}(J) < 0$ , the steady state is (locally) stable leading to indeterminacy (any local choice of  $\lambda(0)$  for a given  $x_0$  is attracted).*

The derivative of  $\rho(x)$  is

$$\begin{aligned}\rho' &= \frac{-\pi_u(\pi_{xX} + \pi_{xx}) + \pi_x(\pi_{uX} + \pi_{ux})}{\pi_u^2} \Big|_{X=x, u=0} \\ &= -\frac{1}{\pi_u} [(\pi_{xX} + \pi_{xx}) + (\pi_{uX} + \pi_{ux})r] \text{ at a steady state.}\end{aligned}\quad (25)$$

If we combine this result with Proposition 5, we conclude that, analogous to the previous section, positivity of  $\rho'$  at a steady state implies an unstable steady state, while a negative slope means that the corresponding steady state is a saddle point.

We have  $\pi_{uX} = 0$  in the specification (1). In this case, instability requires that  $\pi_{ux}$  is positive and sufficiently large. In the firm's optimization problem of the previous section the corresponding criterion is  $r\pi_{ux} > -\pi_{xx}$ . However,  $\pi_{ux}$  must be even larger after accounting for the interaction with competitors, because from Proposition 5 it follows that instability requires that

$$r\pi_{ux} > -(\pi_{xx} + \pi_{xX}), \quad (26)$$

where  $\pi_{xX} < 0$  by Assumption 4. This suggests that market interactions help to eliminate unstable steady states that result in the firm's isolated optimization problem of the previous section. Figure 5 (top) shows a corresponding case based on the example from Figure 3. The unique long run outcome shown in the top of Figure 5 is close to the unstable steady state of the firm's isolated optimization problem. Hence, it seems that this interaction eliminates the high and second steady state and turns the low and unstable steady state into a stable one. However, both conjectures do not hold in general even within the restrictions imposed by the particular example (8). First, the example from Figure 4 with both positive steady states in the concave domain (i.e., within an extremely well-behaved setup) can lead to three steady states after accounting for social interactions, see Figure 5 (bottom). Second, interactions can also eliminate the low and unstable steady state, which happens if in Figure 5 (bottom) the curve for  $\rho$  corresponding to the interacting firms' case, intersects the line  $\rho = r$  only once, such that the two smaller intersections do not exist. Hence, the following proposition can be formulated.

**Proposition 6:** *Adding market interactions to the firm's isolated optimization problem of the previous section can eliminate or create steady states.*

[Insert Fig. 5 approximately here]

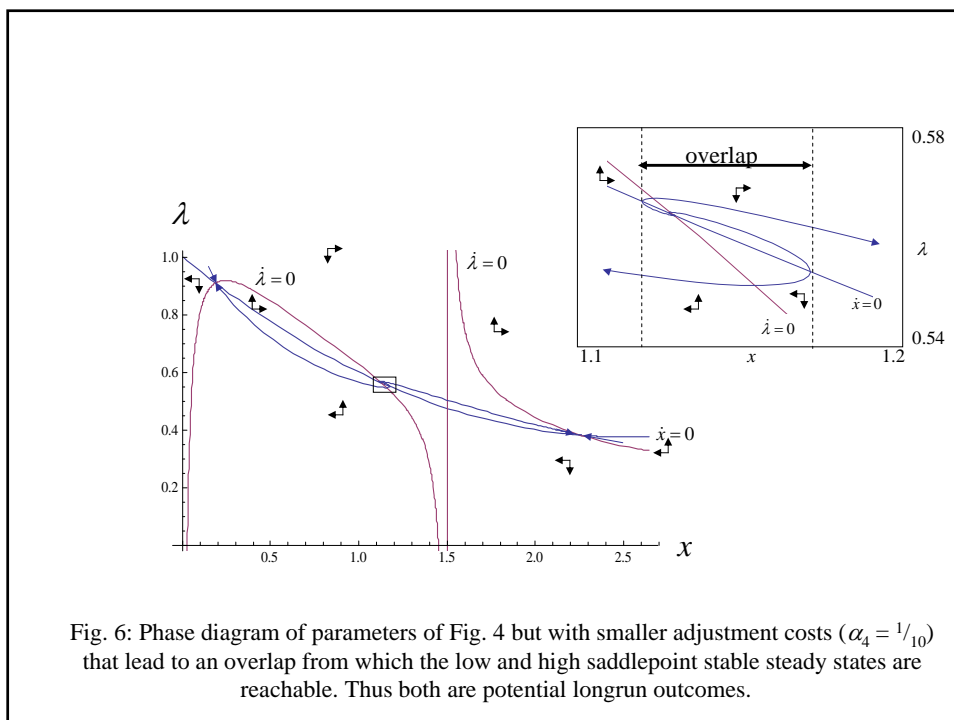
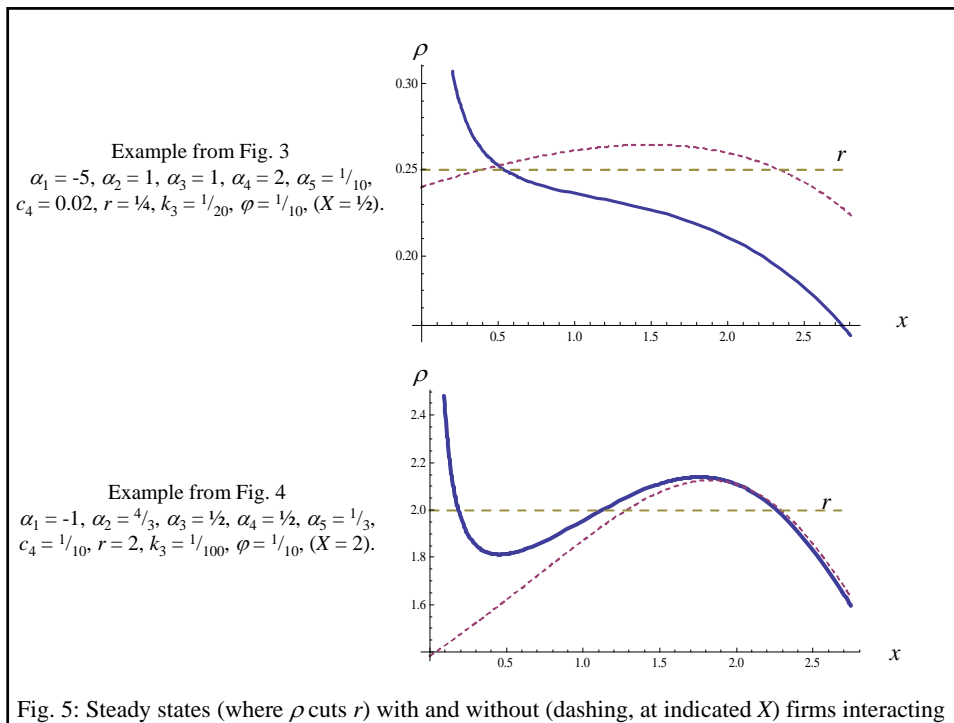
Interaction among firms can introduce another degree of indeterminacy about long run outcomes if the unstable steady state is a focus. Then the basins of attractions of two saddlepoint stable steady states overlap, implying that, when  $x_0$  is situated in the overlapping area,  $x_0$  itself (i.e., history alone) is insufficient to determine the long run outcome. In this case firms must coordinate about whether they will perform high or low CSR activities in the long run. This scenario follows if the eigenvalues are complex, which requires a sufficiently large determinant of  $J$ . Expression (23) suggests that a low value of  $|\pi_{uu}|$  can achieve this. Indeed, applying comparative dynamics in the context of the example in Figure 4 leads to the conclusion that variations in the adjustment cost parameter  $\alpha_4$  ( $= -\pi_{uu}$  in the example (8)) are crucial. As in the firm's isolated problem of the previous section, this parameter does not affect the steady states in (20) and (21), but it can change their stability properties. In particular, reducing  $\alpha_4$  can turn the unstable node into an unstable focus such that history is insufficient and coordination among the competitors is necessary in order to attain one of the two stable long run outcomes. Hence, low adjustment costs, which allow for a fast expansion of CSR, require coordination. On the other hand, a large  $\alpha_4$  slows the process sufficiently such that no coordination is needed<sup>4</sup>. Since one has to be fast in today's markets (i.e., adjustment costs are small relative to the incremental benefit from CSR), one of the crucial rules of labeling and certification organizations and of international consultants could be to assist in this process of coordination. Figure 6 shows a phase diagram including the numerical determination of the overlap, i.e., for  $x_0 \in (1.11, 1.175)$ , firms have to coordinate between either to head to the low or to the high equilibrium along the corresponding saddlepoint path.

[Insert Fig. 6 approximately here]

The consideration of interactions among firms retains the possibility of multiple equilibria and adds the complexity that history can be insufficient to determine the future of CSR. This scenario occurs whenever the unstable steady state is a spiral. Low adjustments cost, and thus fast reactions of firms foster this scenario. Furthermore, drawing on the comparative dynamics sketched for the firm (how increased benefits shift  $\rho$ ), this framework allows

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<sup>4</sup>Karp and Paul (2005, 2007) show in a related framework (but with adjustment externalities and two states) that this monotonicity with respect to adjustment costs does not hold in general.



to explain how exogenous events, like, e.g., a greening of stakeholders in particular of consumers, changes the firms' incentives. This has the potential of triggering a wave of activities across firms even if they know that relative positions will not change. The possibility of multiple equilibria suggests a considerable variation of activities across industries due to differences in benefits from CSR (e.g., branches vary in their dependency on green minded consumers) or even history. Late comers may attain low levels, while other industries that already run a variety of programs, will opt for a high level of CSR in the long run (of course *ceteris paribus*, i.e., unless preferences change, e.g. if CSR, essentially a credence good, is discredited by cheating and free riding). Finally, even the attainment of a high level of CSR may require coordination between firms. Therefore, coordination can be an additional reason for the crucial role of labeling and certification institutions to foster CSR complementary to mitigate free riding (Baron (2010)).

## 8 Concluding remarks

This paper focussed on dynamic aspects associated with Corporate Social Responsibility (CSR), which is complementary to the recently published papers with their emphases on empirical issues and static modeling. More precisely, a dynamic model of a firm's CSR activities is introduced in order to analyze how a firm should pursue its CSR activities over time, while taking into account that the relative position with respect to its competitors is an important determinant for its reputation, and thus long run profit. The actions of its competitors are of course an exogenous datum to the firm, and this was considered as a constant in the first set up. Within this scenario we found that history dependence can occur, i.e. it depends on the initial level of CSR activities whether CSR will survive as an optimal strategy in the long run. In particular, we obtained that occurrence of history dependence can happen, either due to increasing returns of CSR activities, or due to sufficiently large positive dependencies in profits of existing and new CSR activities.

In a second scenario we take into account reactions of other firms in a completely symmetric setting, where we adopt an equilibrium condition from the endogenous growth literature (see also Krugman (1991)). Studying this second scenario is particularly important, because many management advisors ignore that other firms face similar incentives, and act accordingly.



Compared to the previous scenario it turns out that the condition for history dependence is more strict, i.e. taking into account reactions of other firms may eliminate occurrence of history dependent solutions. However, we show that history dependence can still occur in such situations. We argue that coordination is necessary for an industry to converge to an overall CSR-active outcome. Such coordination can be achieved by labeling and certification institutes like EMAS and ISO14001.

Dependence of the long run outcome on initial conditions may also explain why CSR activities differ widely across nations and industries. For example, international oil companies strive to be seen as being socially active corporations and in particular being environmentally responsible (e.g., Chevron regularly publishes advertisements with slogans like "Oil companies should support the communities they're a part of" or "Big oil should support small business" in *the Economist*). However, at the same time other mining companies do not care.

A simplifying assumption was that of perfect competition in CSR activities, i.e. each firm treats the industry average  $X$  as exogenous. Weakening this assumption, and thus allowing for explicit strategic interactions, suggests a dynamic game version of this paper (see Dockner et al. (2000)), He et al. (2007) for a survey of applications of differential games to managerial issues (supply and marketing channels), and Jørgensen et al. (2010) for a survey of differential games dealing with environmental issues. Becchetti et al. (2010) consider a CSR motivated differential game with open loop strategies (only). Another related starting point is the differential game in Cellini and Lambertini (2002) where firms invest in technologies producing differentiated products. This product differentiation could be caused by CSR activities. This, as well as the other addressed extensions (e.g. accounting explicitly for the dynamics of consumer goodwill for CSR active companies), are left for future research.

## 9 Appendix

### 9.1 Proof of Proposition 2

In Section 6  $X$  is a parameter. Then the slope of the  $\dot{x} = 0$  isocline in the  $(x, \lambda)$  plane equals  $(-\pi_{ux})$  due to (12). This isocline is downward sloping if

$\pi_{ux} > 0$ . The costate dynamics

$$\dot{\lambda} = r\lambda - \pi_x(u^*(x, \lambda, X), x, X), \quad (27)$$

implies

$$\frac{\partial \dot{\lambda}}{\partial x} = -\pi_{xx} + \frac{\pi_{ux}^2}{\pi_{uu}} = -\frac{1}{\pi_{uu}} (\pi_{uu}\pi_{xx} - \pi_{xu}^2), \quad \frac{\partial \dot{\lambda}}{\partial \lambda} = r + \frac{\pi_{ux}}{\pi_{uu}}.$$

Hence, the  $\dot{\lambda} = 0$  isocline can have a non-trivial shape. If  $\pi_{ux} > 0$  and  $\pi_{uux} \neq 0$  then  $\frac{\partial \dot{\lambda}}{\partial \lambda}$  can switch sign at  $\frac{\partial \dot{\lambda}}{\partial \lambda} = 0$ . This point (the  $\dot{\lambda} = 0$  isocline diverges to  $\pm\infty$ ) separates the isocline into two parts in the  $(x, \lambda)$ -plane. The numerator of the implicitly given slope of this isocline  $\left(-\frac{\partial \dot{\lambda}}{\partial x}\right)$  may change signs too, but along the already addressed and expected lines: it is definitely negative in the domain of non-diminishing returns, but positive in the case of decreasing returns combined with concavity of  $\pi$  with respect to  $(u, x)$ .

The Jacobian ( $j$ ) of the dynamic system  $\begin{pmatrix} \dot{x} \\ \dot{\lambda} \end{pmatrix}$  and its determinant are

$$j = \begin{pmatrix} -\frac{\pi_{ux}}{\pi_{uu}} & -\frac{1}{\pi_{uu}} \\ -\frac{\pi_{uu}\pi_{xx} - \pi_{xu}^2}{\pi_{uu}} & r + \frac{\pi_{ux}}{\pi_{uu}} \end{pmatrix} \implies \det j = -\frac{1}{\pi_{uu}} [r\pi_{ux} + \pi_{xx}]. \quad (28)$$

Therefore, saddlepoint stability is equivalent to

$$\det j < 0 \iff r\pi_{ux} + \pi_{xx} < 0, \quad (29)$$

while the steady state is unstable if these signs are reversed.

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## 11 Online Appendix - Two Extensions

### 11.1 Interaction in $(u, U)$

The analysis so far assumed that consumers and other stakeholders are concerned about the firm's level of CSR activities in relation to its competitors, i.e., this comparison is linked to the state variable. What is the consequence if this comparison is linked to the announcements of new projects,  $u$ , with capital letter  $U$  denoting the competitors' average choice? In order to focus on this kind of interaction,  $f(x, X)$  is replaced by  $g(u, U)$  satisfying similar properties as  $f$ , i.e.,  $\pi_{uU} = g_{uU} < 0$ . This modifies the Hamiltonian,  $H = \pi(u, x, U) + \lambda u$ , but leaves the first order conditions unchanged, except for the optimal control,

$$\begin{aligned} u &= u^*(x, \lambda, U), \\ u_x^* &= -\frac{\pi_{ux}}{\pi_{uu}}, \quad u_\lambda^* = -\frac{1}{\pi_{uu}} > 0, \quad u_U^* = -\frac{\pi_{uU}}{\pi_{uu}}. \end{aligned}$$

As a consequence, after setting  $\pi_{xX} = 0$  this outcome corresponds to the ones characterized in Propositions 1 - 5, including the possibility of multiple steady states for the already addressed reasons related to the state ( $\pi_{ux} > 0$ , or  $\pi_{xx} > 0$ ).

In the competitive equilibrium, the firm's choice is the fixed point of the implicit relation,

$$\begin{aligned} u &= u^*(x, \lambda, u), \\ \frac{\partial}{\partial u} &= 1 + \frac{\pi_{uU}}{\pi_{uu}} = \frac{\pi_{uu} + \pi_{uU}}{\pi_{uu}}. \end{aligned}$$

The solution is now identified by two asterisks (to differentiate from above) by with the given partial derivatives given below,

$$u^{**}(x, \lambda), \quad u_x^{**} = -\frac{\pi_{ux}}{\pi_{uu} + \pi_{uU}}, \quad u_\lambda^{**} = -\frac{1}{\pi_{uu} + \pi_{uU}}.$$

Substitution yields the dynamics of the competitive equilibrium,

$$\begin{aligned} \dot{x} &= u^{**}(x, \lambda), \\ \dot{\lambda} &= r\lambda - \pi_x(u^{**}(x, \lambda), x), \end{aligned}$$



and differentiation the Jacobian,

$$J = \begin{pmatrix} -\frac{\pi_{ux}}{\pi_{uu} + \pi_{uU}} & -\frac{1}{\pi_{uu} + \pi_{uU}} \\ -\frac{(\pi_{uu}\pi_{xx} - \pi_{xu}^2)}{\pi_{uu} + \pi_{uU}} & r + \frac{\pi_{ux}}{\pi_{uu} + \pi_{uU}} \end{pmatrix}.$$

Hence, the outcome is the same as in the isolated firm's case if we substitute  $\pi_{uu} + \pi_{uU}$  for  $\pi_{uu}$ . What we see is that the outcome with interaction is *observationally equivalent* to the one where firms ignore this interaction while having a different (and anyway not observable) preference structure (see Hof and Wirl (2008) for a corresponding discussion). As a consequence, the Jacobian retains the usual symmetry so that the eigenvalues are symmetric around  $r/2$ . In conclusion, this kind of interaction between  $u$  and  $U$  cannot add further complexity.

## 11.2 Complementarity, $\pi_{xX} > 0$

Departing from a framework with diminishing returns, the above analysis shows that a positive interaction term, i.e.  $\pi_{ux} > 0$ , is crucial to obtain multiple steady states including an unstable one. This holds for the isolated firm model, as well as for the model where firms' interactions are taken into account. In the latter case, an additional property helpful to create an unstable steady state is complementarity between own and industry-wide CSR activities, i.e.,  $\pi_{xX} > 0$ , due to (23). The specification of  $f$  in (8) stresses however the competitive nature such that  $\pi_{xx} = f_{xx} < 0$ . Complementarity is implied by a different but also plausible specification

$$\begin{aligned} f(x, X) &= -(\alpha/\beta) e^{-\beta(x-X)}, & \alpha > 0, \beta > 0. \\ f_x &= \alpha e^{-\beta(x-X)}, f_{xx} = -\alpha\beta e^{-\beta(x-X)}, f_{xX} = \alpha\beta e^{-\beta(x-X)}. \end{aligned} \quad (30)$$

However, obtaining an unstable steady state due to this property requires that this complementarity is strong. This is because

$$\det J = -\frac{1}{\pi_{uu}} [(\pi_{xx} + \pi_{xX})] > 0 \iff -\frac{\pi_{xX}}{\pi_{xx}} > 1,$$

after assuming  $\pi_{ux} = 0$  (and  $\pi_{uX} = 0$  by assumption). This condition is not met by (30), because there it holds that  $-\frac{f_{xX}}{f_{xx}} = 1$ . After substituting (30) into (1) and ruling out increasing returns ( $\pi_{xx} < f_{xx}$ ), this implies that

$-\frac{\pi_{xX}}{\pi_{xx}} < 1$ . We conclude that complementarity is an unlikely source on its own to explain multiple equilibria in the context of CSR.

An indeterminate outcome, i.e. a stable steady state with both eigenvalues having negative real parts<sup>5</sup>, needs first of all a positive determinant, (negative) control-externality interaction,  $\pi_{uX} < 0$ , and ‘small’ discount rates,  $tr(J) < 0 \implies r < \frac{\pi_{uX}}{\pi_{uu}}$  due to (24). The specification in (7), more precisely  $\pi_{uX} = 0$ , rules out a negative trace of  $J$ . Therefore, an indeterminate (transiently only) coevolution of firms requires negative spillovers of aggregate activity on a firm’s individual costs and benefits. Such an interaction,  $\pi_{uX} < 0$ , is not entirely implausible: Given many CSR projects already around, the firm’s (marginal and net) benefit from an additional project is reduced. This can be caused by, e.g., insignificant exposure, there is no good project available anymore, or the costs are increased due to competing for experienced CSR managers who are then short in supply.

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<sup>5</sup>This possibility is extensively discussed in the theoretical endogenous growth literature. Examples are Benhabib and Farmer (1994), Benhabib and Perli (1994), and more recently Benhabib et al. (2008).

## 12 Appendix for Referees - Additional Examples

Both steady states are in the non-concave domain in the example in Fig. 3. Of course, one can find many examples where the stable steady state is then in the concave domain (in which case the sufficiency criteria hold at least locally). Fig. R1 is a corresponding example where the steady state is in the concave domain; the bold line includes the interactions among the firms.

[Insert Fig. R1 approximately here]

Fig. R2 shows the phase diagram corresponding to example in Fig. 4 but one that leads to three steady states after accounting for firms' interactions.

[Insert Fig. R2 approximately here]

In the paper it is claimed that interactions can also eliminate the low and unstable steady state, if the curve for  $\rho$  corresponding to the interacting firms' case, intersects the line  $\rho = r$  only once such that the two smaller intersections do not exist. Fig. R3 provides a corresponding example including the phase portraits where the dashed lines refer to the firm's isolated control problem.

[Insert Fig. R3 approximately here]

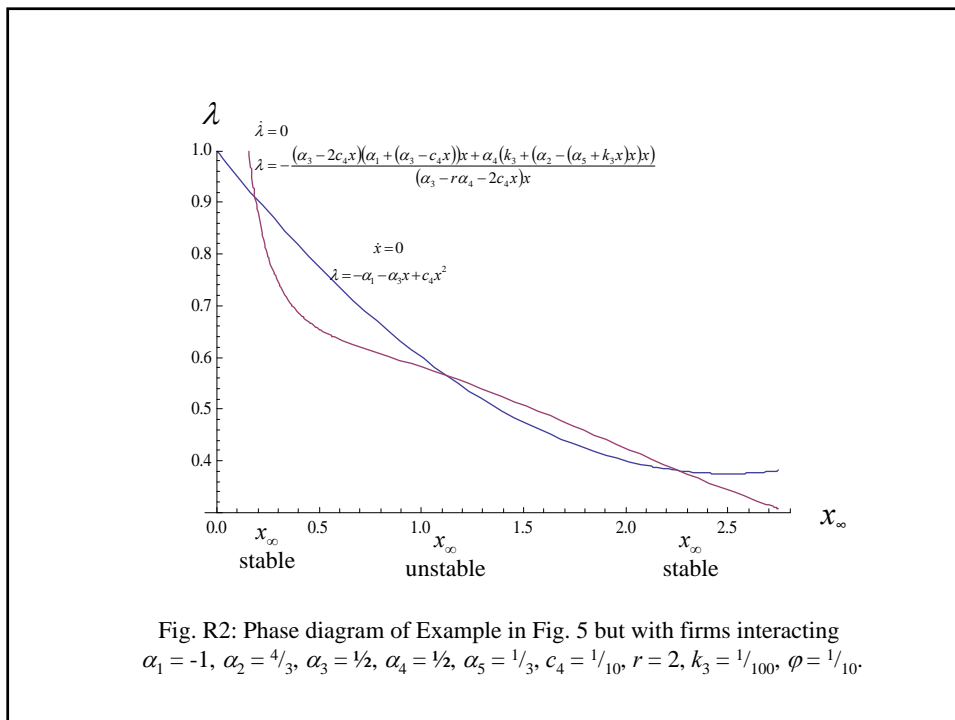
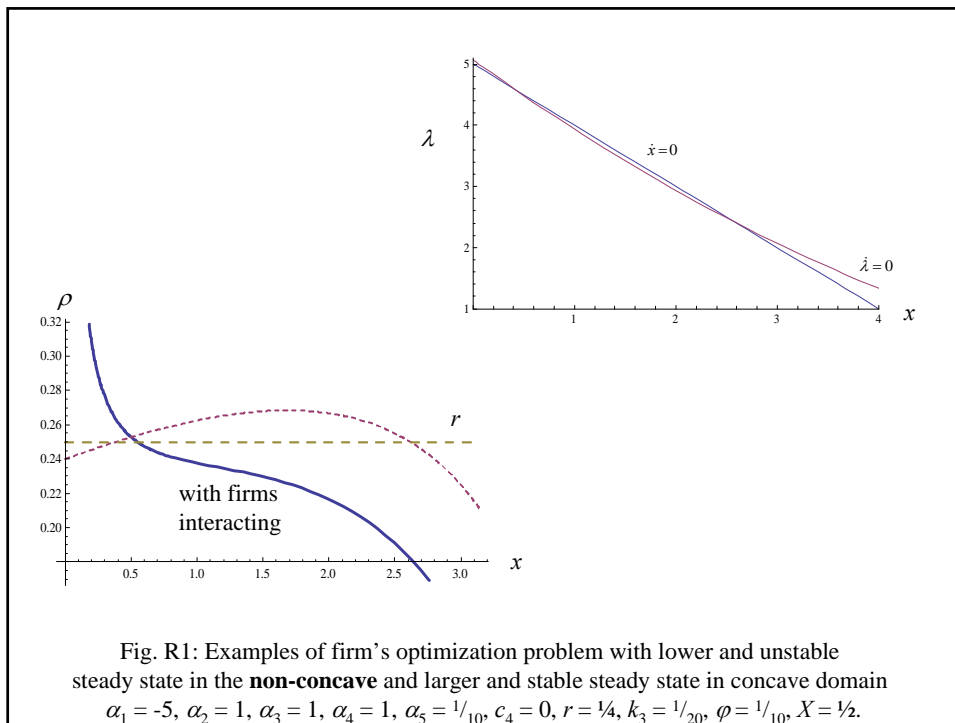


Fig. R3: Example similar to Fig. 5 (dashing = without interactions)  
 (lower steady state in the **non-concave** and larger and stable steady state in **concave** domain)  
 but where interaction leads to the high level as unique longrun outcome:  
 $\alpha_1 = -1, \alpha_2 = 3/2, \alpha_3 = 1/2, \alpha_4 = 1/2, \alpha_5 = 1/4, c_4 = 1/10, r = 2, k_3 = 0, \varphi = 1/5, (X = 4)$ .

