Capital stock management during a recession that freezes credit markets

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Abstract

This paper considers the problem of how to price a conspicuous product while maintaining liquidity during a recession, which both reduces demand and freezes credit markets. Reducing price would help maintain cash flow, but low prices can erode brand image and, hence, long-term sales. The paper extends earlier work of the same authors by explicitly deriving a firm's optimal cash management behavior, taking into account that a too low cash level results in bankruptcy.

There are different sets of initial conditions for which qualitatively different solution trajectories are optimal. Along a single trajectory, it can be optimal to throttle forward then back how aggressively one spends down cash reserves, with the associated state constraint alternately being non-binding, binding, non-binding, then binding in such a way that the firm ceases operation.

1 Introduction

Dynamic optimization addresses the problem of how a decision maker should manage a system's operations as that system moves around within some state space. Two-stage problems are a natural extension to situations when at some (possibly random) point in time, there is an abrupt change in problem parameters or in the objective or state equations. The state is continuous at the transition from stage one to stage two, but nothing else need be.

This paper addresses a problem in which there is a "forbidden region" of state space in stage I, a prohibition that disappears at the moment of transition to stage II. Equivalently, one could see the stage transition as involving a sudden expansion in the feasible region. In stage I, one of the state variables can be interpreted as representing distance from the forbidden region; that variable is dropped in stage II since it is no longer relevant.

Many circumstances fit within this broad concept. Obvious ones come from military or conflict applications; e.g., until an enemy force or capability has been eliminated, it is not safe to enter certain physical regions. Likewise, changes in laws, rules, or government policies could define transitions between stages; e.g., until a competitor's patent expires, the decision maker cannot operate using the intellectual property protected by that patent.

Here we explore an interesting application motivated by the recent global recession, that even found its attention in the popular press, see The Economist, November 20, 2008; January

22, 2009; The New York Times, September 2, 2009. From a firm's perspective, a recession is typically characterized by a reduction in demand. But the recent global recession was atypical in the sense that it also involved a major disruption ("freezing") of capital markets. Even firms with sound fundamentals found it essentially impossible to raise financing in the form of either bank loans or new equity. So for the duration of the recession, firms' feasible regions were restricted by an additional constraint that current obligations had to be covered by current operating revenue plus cash on hand. (Here and throughout we will use the term "cash" to mean assets that are liquid even during a severe recession.) During the recession liquidity mattered, not just the firm's discounted expected future stream of net operating profits, because there was no way to convert a future profit stream into cash while the capital markets were not functioning. Complicating firms' planning, it was unclear in the midst of the recession how long it would be before the financial institutions returned to their customary practices of providing short-term financing to firms with sound fundamentals but a short-term need for liquidity.

We conclude that managing cash reserves is an important aspect of firm behavior in the recent global recession. The model presented in this paper determines the firm's optimal cash management policy during this recession and is as such an extension to Caulkins et al. (2011). In the latter paper the firm does also operate during a recession with a not functioning capital market, but the so crucial cash management issues are not taken into account there. Where our paper offers a theoretical approach, empirical works regarding the effects of financial factors on real firm (investment) behavior include Fazzari et al. (1988) and Bayer (2008). Like us, Mittnik and Semmler (2013) focus on the influence of a recession, but they take a more macroeconomic approach.

During the recession firms do not just passively look on as cash burn takes them dangerously close to illiquidity. Rather, they begin to draw down other assets. This can be done actively, by selling or liquidating some of that asset. Or it can be done by investing at below replacement rates, e.g., deferring maintenance on physical capital or not hiring replacements when human capital leaves the firm. Similar ideas apply not only to physical and human capital, but also to technological capital or brand reputation/awareness.

The last is perhaps the least familiar, but brand reputation is indeed a form of capital; it is built up over time, has inertia, and enhances profitability. We make it the focus below because it produces particularly interesting dynamics. Since reputation is influenced directly by price, the decision variable (price) affects not only cash flow (by determining demand and revenues) but also changes in the capital stock.

We use the term "capital stock" to refer to the stock that can be drawn down, regardless of whether it is a physical, human resources, intellectual, or reputational stock. But since we use the adjective "capital" to describe those stocks, we will use the term "credit markets' rather than "capital markets" to refer to sources of financing, even though we mean to encompass not only credit markets in the narrow sense but also equity financing.

Uncertainty about how long it will be before the firm can negotiate normal financing greatly complicates decision making. If the firm is too passive, and forgoes relatively low cost ways of spending down capital early on, it might be forced into slash and burn tactics later or even simply be forced out of business. But almost by definition, any draw down of a capital stock, even a reduction in the rate by which that stock would otherwise have increased, that is motivated by the (temporary) lack of access to financing is suboptimal relative to a hypothetical situation in which the firm could borrow against future revenues. So if the firm makes plans that would see it through an extended capital drought, and then it turns out that the transition to stage II happens sooner than expected, the firm will have made sacrifices that become unnecessary in retrospect. In short, management has to balance the risks of two types of errors: acting with insufficient decisiveness and acting too aggressively when it comes to protecting liquidity.

These decisions are not one-shot, so a two-period discrete time model would be overly simplistic. Rather, on a day-by-day if not continuous basis, the firm can adjust the aggressiveness with which it spends down its capital stock in order to delay deterioration in its cash position. So this is a sequential decision problem. Furthermore, there are no natural milestones; strategy can be adjusted at any moment, not just at the beginning of each week or month. So this problem is perhaps best thought of within the framework of continuous dynamic optimization.

Naturally the particular functional forms and parameter values will be capital-type-, industry- and firm-specific. We focus on one particular type of capital (brand reputation for selling goods expensive enough to be exclusive) but make no attempt to model any particular firm based on its unique data and operating parameters. Rather, we create a stylized model that captures the essence of the problem and expresses it in the precise and parsimonious language of mathematics. Doing so clarifies the nature of the problem and provides by example that the optimal strategies can be complex and counter-intuitive.

We describe this problem by setting up a two-stage dynamic model. During Stage 1 (the recession), the firm has to price so that its operations are self-financing. The firm takes into account that the recession's duration is unknown. In Stage 2 the recession is over, demand returns to its normal level, and there are normal (perfect) credit markets, implying that the firm can borrow as much as it wants at a fixed interest rate.

As already stated, the model considered here is an extension of Caulkins et al. (2011) who look at the impact of a recession freezing capital markets on a firm with a conspicuous product, however, without the possibility of building up and spending down cash reserves. The characteristic of a conspicuous product is that its demand increases in brand reputation, which itself increases in price. Examples include luxury cars, jewelry, fancy hotel rooms, fashion goods, etc. This so-called Veblen effect has found much attention in literature, see Bagwell and Bernheim (1996). The question why it is advantageous for consumers to be attracted by this higher price has been studied within a large number of papers; see, e.g., Bikhchandani et al. (1992); Coelho and McClure (1993); Frijters (1998); Corneo and Jeanne (1999); Bianchi (2002). Papers studying the resulting implications for firms particularly concerning strategic decisions such as the question of how to price a conspicuous product include Amaldoss and Jain (2005b,a); Kort et al. (2006); Caulkins et al. (2007); Amaldoss and Jain (2008, 2010); Huschto and Sager (2012).

Huschto et al. (2011) consider the impact of delays in a similar model. However, while the paper put its main focus on the underlying methods, the present paper will provide more information on the economic implications.

The paper is organized as follows. Section 2 introduces the model. Section 3 presents the main results, including sensitivity analysis with respect to the severity of the recession. Section 4 elaborate on one particular parameter set that produces a surprising non-monotonicity. This non-monotonicity demonstrates the richness that models of this sort can produce. Section 5 elaborates.

2 The Model

2.1 Brand image and demand

During the recent global recession, a number of articles in popular outlets such as the New York Times and the Economist described the difficult pricing dilemmas confronted by purveyors of luxury brand good such as fashion goods, high-end hotel rooms, and marquee car models (The Economist, December 11, 2008; The New York Times, October 28, 2008; June 28, 2009). Demand fell particularly steeply, in percentage terms, for these pricey goods, leaving insufficient operating revenues. Cutting prices increased sales, reducing the short-term risk of default, but discounting erodes the brand's reputation for exclusivity and, hence, potential for generating revenues in the long run, after the recession ends. Hence, for these elite brands, cutting prices is a mechanism for spending down a capital stock (brand image) in order to improve short-run cash flow.

For a literature review and a more detailed explanation of the demand and brand image equations, see Caulkins et al. (2011). The key ideas, though, are that demand at any given price is increasing in the current brand image, $X(t)$, and brand image adjusts over time, but not instantaneously, when prices are changed. In particular, we imagine that the gap between actual brand reputation and the reputation that would be earned by holding forever the current price point, $X(t)$, decays exponentially over time via simple adjustment dynamics.

$$\dot{X} = \kappa \left(\gamma p - X \right),$$

where p is price, γ and κ are positive constants, so brand-image is always non-negative. The parameter γ is just a scaling parameter, but parameter κ determines how quickly brand image adjusts to gaps between the current image and the image that would be warranted based on current price. Industries in which reputations rise and fall quickly would be characterized by a larger κ than would industries in which reputations are hard to earn and slow to erode. Such differences could arise because of differences in the frequency of purchase. (Most people buy a new luxury car less often than business travelers sample hotels' quality.) Other factors might include the prominence of public reviews of quality (contrast circulation and viewership of reviews of clothing designers' new lines vs. reviews of hotel quality) and or the presence of absence of loyalty clubs (routine for hotels, rare to non-existent for perfumes).

For any given brand image, we assume demand is linearly decreasing in price with an intercept m that is reduced by a constant, α, during the recession. We want the demand curve's slope with respect to price to be flatter for goods with an exclusive image than for run of the mill goods. We achieve this my making the slope an inverse function of brand image, specifically:

$$Q = (m - \alpha) - p/X^{\beta}, \tag{1}$$

where Q is the firm's produced quantity. We assume $\beta < 1$ so demand is increasing in brand image, but at a decreasing rate.

2.2 Structure of the two stages

The model has two stages, a recession stage of unknown length followed by a stage with normal business conditions that persist indefinitely. The objective throughout is maximizing shareholder value as embodied in a discounted stream of dividend payments. We presume no dividends are paid during stage 1; the firm hoards cash to weather the recession. In stage 2 there are no financing constraints, so the firm remits net operating profits to the share-holders on an on-going basis. There is no reason not to; if the firm ever found itself cash poor in stage 2, it could always raise additional financing. For the same reason, if the firm enters stage 2 with cash on hand, that cash is turned over to the owners at that time.

Implicitly this places conditions on the owner's discount rate, r, relative to the interest rate on cash, δ. In stage 2 we follow the common convention of having a single rate for both (and denote it by r). This prevents our industrial/manufacturing firm from acting like a bank because it cannot lend at rates superior to the owner's time value of money; conversely, if net operating revenues happen ever to be negative, the firm effectively borrows at that same interest rate. This indifference as to the location of cash is a corollary of having access to perfect financial markets.

In stage 1, we make the interest rate the firm earns on cash slightly greater than the owner's discount rate. That is realistic given that during the most recent recession, interest rates share holders could have obtained by depositing dividends in conventional time deposits were near zero, whereas firms with big war chests could acquire other firms at firesale prices. It also makes the mathematics consistent; if the owner's discount rate r exceeded the interest rate on the firm's cash δ then having the firm hoard cash until the end of stage 1 would not necessarily be optimal. So in stage 1 we make the firm's interest rate on cash (δ) slightly greater than firm owner's discount rate (r).

It is customary to model decision makers as having a constant rate of time preference (i.e., r should be the same in both stages). So, during the recession the firm values cash at a rate slightly higher than typical discount rates (0.05 vs. 0.04 with the parameters introduced below). Via compounding interest, this can create trajectories that head off toward having infinite cash balances if the recession lasts infinitely long. However, the probability of remaining on those trajectories becomes vanishingly small because the hazard rate for the recession ending is an order of magnitude greater than is this interest rate.

Translating these considerations into state variables, we track separately two state variables during stage 1, brand image, $X(t)$, and cash on hand, $Y(t)$, both of which are non-negative. Cash on hand can be negative in stage 2, effectively expanding the feasible region, but setting $r = \delta$ makes tracking cash on hand superfluous, so we omit the 2nd state variable in stage 2.

Since the second stage determines the salvage value function for the first stage, we begin the model statement with stage 2.

The firm maximizes its discounted cash flow stream, by solving the following model:

$$\begin{aligned} S(X_0) &= \max_{p(\cdot)} \int_0^\infty e^{-rt} \left(p \left(m - p/X^\beta \right) - C \right) \mathrm{d}\, t, \\ \dot{X} &= \kappa (\gamma p - X), \quad X(0) = X_0, \\ Q &= m - p/X^\beta \geq 0, \quad p \geq 0, \end{aligned}$$

Variable costs per unit produced are assumed to be reflected within the revenue term (i.e., price is net of per unit production cost), but we allow explicitly for a fixed cost C.

Note: p and Q should obviously never be negative. It will turn out that p will always be strictly positive, but sometimes it will be optimal to let Q fall to zero. We interpret that as selling the minimum amount necessary for the price p to be observed and so influence image, a level insufficient to generate meaningful revenues.

Caulkins et al. (2011) show this stage 2 model has a unique interior steady state that is a saddle. If the brand image is initially sufficiently small, the optimal trajectory might involve charging maximum prices so sales volume is essentially zero for a time, but the equilibrium always involves sales volumes that are strictly positive.

Although the value function $S(X)$ of the Stage 2 problem cannot be written explicitly, it is easy to compute and so serves as a suitable salvage value function for the Stage 1 problem.

The Stage 2 problem also offers insight as to which initial conditions for Stage 1 are of greatest interest. If the firm had been operating under normal conditions for an extended period before the recession struck, then its brand reputation at the beginning of stage 1 would likely be close to the steady state value from Stage 2.

Likewise, firms manage their current and quick ratios (loosely, ratios of cash and other current assets to current liabilities), with typical values of around 2.0 and 1.0, respectively. Hence, we will be most interested in initial states for the Stage 1 problem for which $Y(0)$ does not exceed a modest multiple of revenues in the no-recession steady state.

In Stage 1 the recession reduces demand by the constant α, and we must track explicitly the firm's cash on hand, $Y(t)$. The dynamics of the cash balance comprise sales revenue, the fixed operating cost (C), and interest earned at rate δ on cash on hand, i.e.,

$$\dot{Y} = p \left(m - \alpha - p/X^\beta \right) - C + \delta Y.$$

We assume the firm is too small to influence how long the recession lasts. In particular, we assume the duration of the recession is exponentially distributed with mean $1/\mu$ independent of the firm's actions. Given that the recession ends at time $t = \tau$, its value to the shareholder is

$$V \mid_{t=\tau} = e^{-r\tau} \left(Y(\tau) + S(X(\tau)) \right).$$

The overall objective function is the weighted integral of these conditional valuations, weighting by their relative likelihood. If it is both feasible and optimal for the firm to manage itself in such a way that it would never go bankrupt, no matter how long the recession lasts, this reduces simply to

$$
\begin{aligned}
V &= \max_{p(\cdot)} \int_0^\infty e^{-rt} \left[Y(t) + S(X(t)) \right] \mu e^{-\mu t} \, \mathrm{d}\, t \\
&= \max_{p(\cdot)} \int_0^\infty e^{-(r+\mu)t} \mu \left[Y(t) + S(X(t)) \right] \mathrm{d}\, t.
\end{aligned}
$$

However, it can be necessary or simply preferable for the firm to manage itself in such a way that if the recession does not end by a time T, then the firm will go out of business at that time. In that case the overall optimization problem is actually a free end point problem, with the firm optimizing not only over prices but also over the time T:

$$
\begin{aligned}
& V = \max_{p(\cdot),T} \int_0^T e^{-(r+\mu)t} \mu \left[Y(t) + S(X(t)) \right] \mathrm{d}\, t \\
\text{s.t.} \quad & \dot{X}(t) = \kappa \left(\gamma p(t) - X(t) \right) \\
& \dot{Y}(t) = p(t) Q(p(t), X(t)) - C + \delta Y(t) \\
& X(0) = X_0 \geq 0, \qquad Y(0) = Y_0 \geq 0, \\
& Y(t) \geq 0 \quad \text{for all } t, \\
& Q(p(t), X(t)) \geq 0 \quad \text{for all } t, \\
& p(t) \geq 0 \quad \text{for all } t, \\
& Y(T) = 0.
\end{aligned}
$$

In general, the firm should consider both possibilities (planning to live forever regardless of the recession's length and pursuing a strategy that leaves a positive probability of bankruptcy) and choose whichever yields the greater expected value. As we will see shortly, two interesting situations can arise: (1) Sometimes which strategy is optimal depends on the initial state, with threshold values for the initial conditions such that the firm is indifferent between two quite different solution trajectories and (2) Sometimes even if it is feasible to manage in such a way that the probability of bankruptcy is zero, it is nonetheless optimal to pursue a strategy that entails a positive probability of bankruptcy; maximizing the probability of firm preservation may not maximize expected shareholder value.

3 Analysis

The analysis of the Stage 2 problem is an easy exercise and gives the following result (see also Caulkins et al., 2011). The optimal dynamic price satisfies

$$
p^* = \frac{1}{2} X^\beta (m + \kappa \lambda_1 \gamma)
$$

in the second stage, where λ_1 denotes the costate corresponding to state variable X. There is a unique steady state in Stage 2, i.e.

$$\hat{X} = \gamma \left(\frac{\kappa + r + \frac{1}{2}\kappa\beta}{\frac{1}{2}\gamma^{\beta} m(\kappa + r)} \right)^{1/(\beta-1)}, \qquad \hat{p} = \left(\frac{\kappa + r + \frac{1}{2}\kappa\beta}{\frac{1}{2}\gamma^{\beta} m(\kappa + r)} \right)^{1/(\beta-1)},$$

which is admissible for $\beta < 2(1 + r/\kappa)$ and a saddle point if $\beta < 1$. The optimal strategy in the second stage is as follows. If the initial brand reputation is below its steady state value, then increase the price, leading to a higher brand image, which consequently means a higher future demand. On the other hand, if the initial state value is above its steady state value, the optimal strategy is to sacrifice brand reputation to strengthen immediate demand.

The Hamiltonian of Stage 1 is

$$\mathcal{H} = \mu(Y + S(X)) + \lambda_1 (\kappa (\gamma p - X)) + \lambda_2 (pQ(p, X) - C + \delta Y),$$

where λ_2 denotes the adjoint variable corresponding to Y. Thus, the Lagrangian is

$$\mathcal{L} = \mathcal{H} + \pi_1 Q(p, X) + \pi_2 p + \nu_1 Y,$$

with π_i, $i = 1, 2$ and ν_1 being the Lagrange multipliers.

In the interior of the control region, we find by applying Pontryagin's maximum principle (see, e.g., Grass et al., 2008)

$$p^* = \frac{X^{\beta}}{2} \left(\frac{\lambda_1}{\lambda_2} \kappa\gamma + m - \alpha \right), \qquad \pi_1 = \pi_2 = \nu_1 = 0.$$

Thus, we see that the control constraint $p \geq 0$ cannot become active since $m \geq \alpha$ and $\lambda_i > 0$, $i = 1, 2$. However, the constraint $Q(p, X) = 0$ can become active. Then we have

$$\bar{p} = (m - \alpha) X^{\beta}, \qquad \pi_1 = (\kappa\gamma\lambda_1 - \lambda_2 (m - \alpha)) X^{\beta}.$$

When we consider the state constraint $Y = 0$, we find that it holds that $\dot{Y} = 0$ for

$$\begin{aligned} p^* &= \frac{X^{\beta}}{2}(m - \alpha \pm \sqrt{(\alpha - m)^2 - 4CX^{-\beta}}), \\ \nu_1 &= \left((-S_X + \lambda_1 \kappa)\kappa\gamma \mp \Omega + \lambda_1 \kappa\gamma \left(\delta + \frac{1}{X^{\beta}C - \bar{p}^2} \left(\frac{\dot{p}}{\bar{p}} (X^{\beta}C - \bar{p}^2) + \bar{p}^2 \beta\kappa \right) \right) \right) \Omega^{-1}, \end{aligned}$$

where $\Omega = \sqrt{(\alpha - m)^2 - 4CX^{-\beta}}$ and $\dot{p} = \pm \frac{2\beta\bar{p}^2\kappa(\gamma\bar{p} - X)}{X^{\beta+1}\Omega}$.

The costate equations are

$$\begin{aligned} \dot{\lambda}_1 &= (r + \mu + \kappa)\lambda_1 - \mu S_X(X) - \lambda_2 \frac{\beta p^2}{X^{\beta+1}} - \pi_1 \frac{p\beta}{X^{\beta+1}}, \\ \dot{\lambda}_2 &= (r + \mu - \delta)\lambda_2 - \mu - \nu_1. \end{aligned}$$

It might not pay off or even be possible to survive the recession with certainty. In that case, the firm would go bankrupt if at time T^*, which is determined by the end time conditions,

$$\mathcal{H}(T^*) = 0,$$

and $Y(T^*) = 0$ as well as $Y = p(m - \alpha - p/X^\beta) - C \leq 0$. This leads to a minimum brand image $X_{\min}$ so that no immediate bankruptcy occurs, where

$$X_{\min} = \left(\frac{4C}{(m-\alpha)^2}\right)^{\frac{1}{\beta}}.$$

For the free end time problem the transversality condition

$$\lambda_i(T^*) = 0,$$

for $i = 1, 2$ has to hold.

4 Results

4.1 Parameter Values for Numerical Calculations

Since not all results can be derived analytically, some numerical calculations have to be conducted. We adopt the values used and motivated in Caulkins et al. (2011):

The following parameter values were used:

r	γ	β	C	κ	m	δ	μ	α
0.04	5	0.5	7.5	2	3	0.1	0.5	[0, 1]

(2)

And in addition we model different gradations of recession severity by considering different values of the parameter α.

4.2 Severe Recession

In a recession a firm goes bankrupt if it does not have enough cash and/or reputation. If the recession is severe the reputation and/or cash required to survive with certainty is particularly large. So if the reputation and/or cash are low, the firm faces a positive probability of bankruptcy.

As Fig. 1 shows there is a weak Skiba curve, which corresponds to the one-dimensional stable manifold of a saddle point. Even though the curve might appear rather flat at the images, for X_0 large enough, one would never go bankrupt even with $Y_0 = 0$. If the firm is able to survive with certainty, the cash state diverges to infinity in the first stage. Note, however, that the probability that the recession has not yet ended goes to zero faster than the cash balance accumulates, so the expected cash on hand at the end of the recession remains finite. Below the Skiba curve the bankruptcy probability is always greater than zero. If the reputation is low a firm would try to charge a price as high as possible in order to improve its reputation, subject to the demand remaining non-negative. If, however, the reputation is high enough, one would try to lower the price in order to strengthen demand. Of course such measures have negative effects on the reputation. While the available cash has no big impact on the pricing strategy itself, the initial amount of cash is crucial for determining whether a firm is able to survive recession or not.

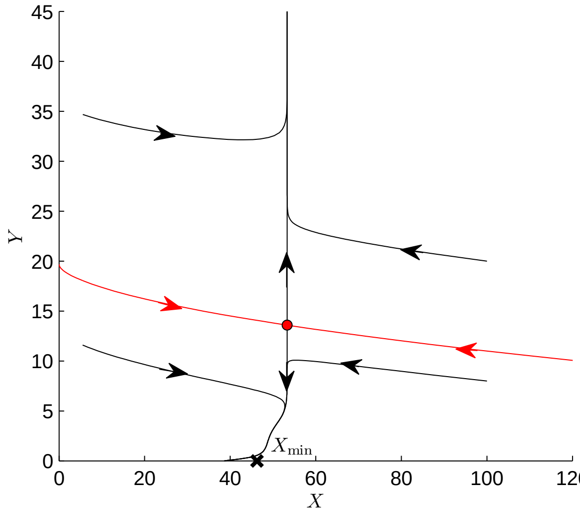

Figure 1: Phase portrait for $\alpha = 0.9$

4.3 Mild recession

The level $X_{\min}$ is smaller in a mild than in a severe recession, reflecting that in a mild recession it is easier to avoid bankruptcy.

Fig. 2 reveals that there is one weak Skiba curve in the case of a mild recession. The big differences relative to the severe cases are that for any $X > X_{\min}$ a firm can avoid bankruptcy with certainty and also that the cash needed for survival is much lower. There is no admissible steady state anymore which is responsible for the Skiba curve. What happens here is that the second costate diverges at the solution path leading to $X_{\min}$.

The strategy is the same as before. If X is very small one would raise the price as high as possible in order to gain reputation. However, if the reputation is already big, one would lower the price in recession times in order to strengthen short-term demand.

4.4 Intermediate Recession Case 1 ($\alpha = 0.836$)

With an intermediate recession we can find two weak Skiba curves, as can be seen in Fig. 3. If the initial values of X and Y are very small, then one would go bankrupt with positive probability. On the lower weak Skiba curve one approaches a steady state which is actually unstable, i.e. the Jacobian evaluated at this point only has positive eigenvalues. The reason why this is possible is that on the steady state the constraint $Y = 0$ is active and on the solution path where one approaches the steady state, it is not.

If the initial state values X_0 and/or Y_0 are rather large, but still such that one would go bankrupt with positive probability, i.e. one starts below the weak Skiba curve, the following can occur (see besides Fig. 3, the “zoom in” in Fig. 4): If the initial reputation is small (and much cash is available) one would increase the price in order to gain reputation. If X becomes large,

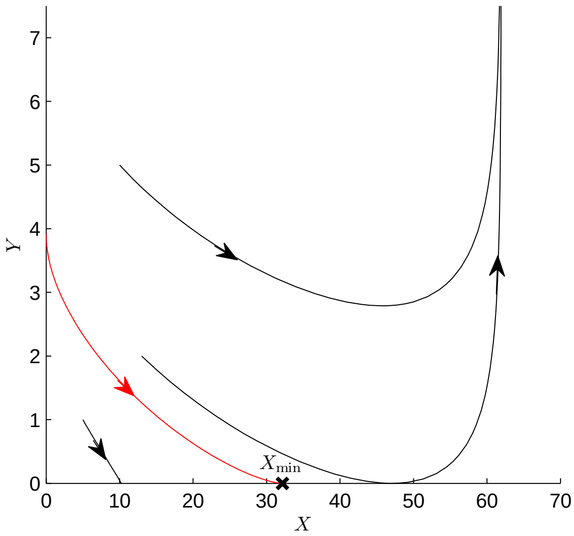

Figure 2: Phase portrait for $\alpha = 0.7$

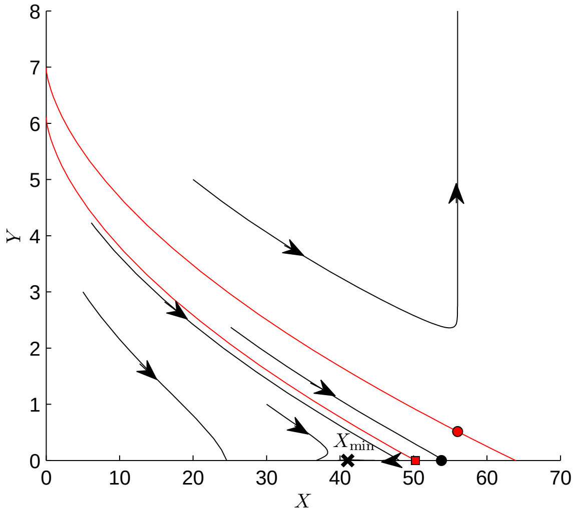

Figure 3: Phase portrait for $\alpha = 0.836$

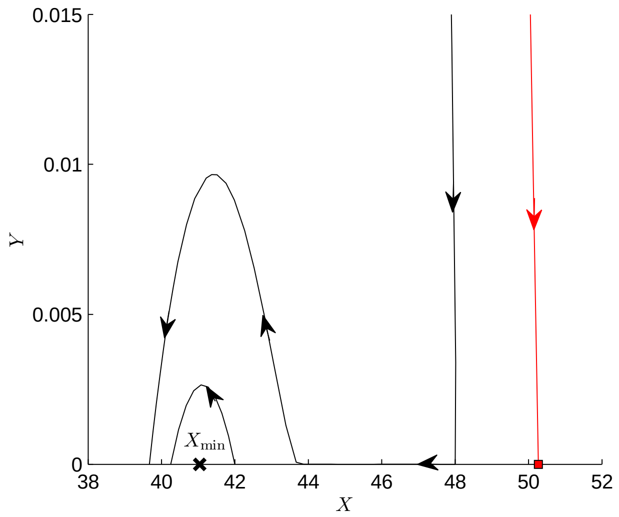

Figure 4: Phase portrait for $\alpha = 0.836$

one would start to decrease the price in order to increase revenues. Yet these increases are not large enough due to the low reputation, so the available cash decreases. After a certain time no cash is left and one would proceed by lowering the price. Then, after some additional time, price is decreased even further, which has the effect that revenue through the strengthened demand becomes so big that it exceeds the costs and the cash can grow again. Note that the negative effect of this additional price reduction is that reputation, and thus future demand suffers. Still the positive effect on cash generates a delay of (possible) bankruptcy time. After a while the price is so low that the costs exceed the revenues and cash decreases again. The decision maker is then not able to escape bankruptcy any longer.

Between the two Skiba curves, one would approach a steady state where the cash is also zero. This steady state is a saddle point. Note that one is able to survive bankruptcy then with certainty but without cash. The optimal strategy would be to increase the price to gain reputation until one would reach or exceed the steady state and reduce the efforts to strengthen demand when reputation is large enough. The second weak Skiba curve is, like in the severe case, the 1-dimensional stable manifold of a steady state. Above this curve, one would be able to survive with certainty. The cash again diverges if the initial cash and/or reputation is large enough.

4.5 Intermediate recession Case 2 ($\alpha = 0.8275$)

For a slightly lower α, the steady state responsible for the upper Skiba curve becomes inadmissible, thus there is only one weak Skiba curve as depicted in Fig. 5. Also, the steady state (the saddle point) on the boundary disappears. The only remaining steady state is the one responsible for the lower Skiba curve (the unstable steady state). As in the previous case, if the initial reputation is greater than $X_{\min}$, but not large enough so that one would still go bankrupt, it can make sense to choose a strategy where the cash first becomes zero but then increases again due to the strengthened demand caused by the lower price, see Fig. 6.

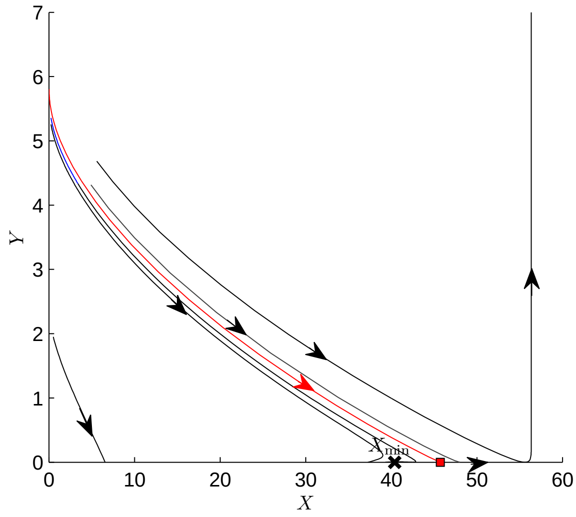

Figure 5: Phase portrait for $\alpha = 0.8275$

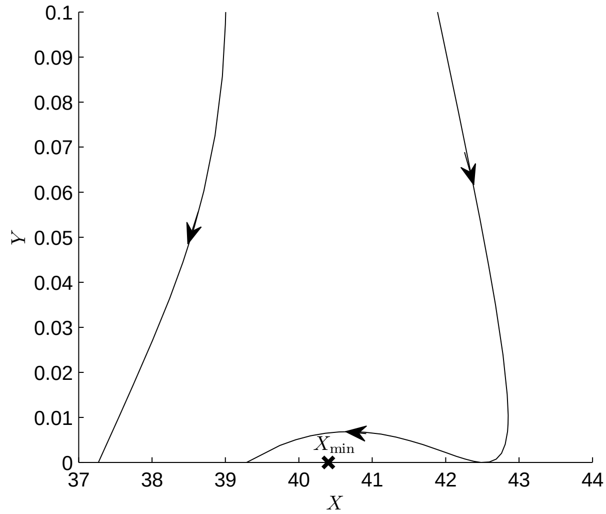

Figure 6: Phase portrait for $\alpha = 0.8275$

4.6 Isobankruptcy curves

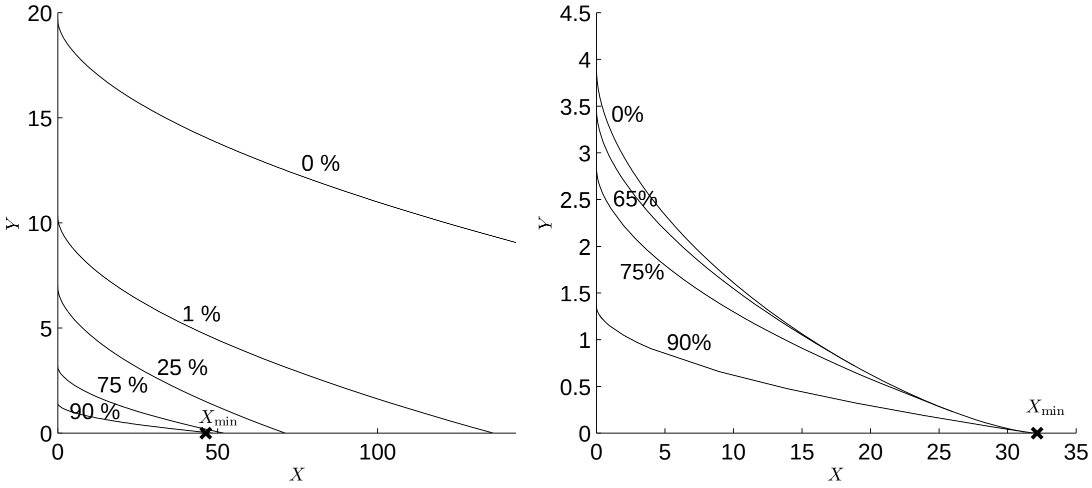

Figure 7: Isobankruptcy curves for the severe recession in the left panel ($\alpha = 0.9$) and for the mild recession in the right panel ($\alpha = 0.7$)

Fig. 7 shows the probability of going bankrupt for certain state values when the recession is either severe (with alpha=0.9) or mild (with alpha=0.7). Obviously, when the recession is severe, one needs a better reputation and/or greater amount of cash to be able to survive with certainty. While the chance of going bankrupt is very small in the severe case when starting closely below the Skiba curve, i.e. the 0%-curve, in the mild case it is still very big. The reason for this is that the time needed to reach bankruptcy is much longer in the severe case (as the danger of bankruptcy is higher for a larger range of initial state values in a severe recession even if reputation and cash are available). Further, for a mild recession one would never go bankrupt if the initial reputation is above a certain level ($X_{\min}$), even if there is no cash available. In a severe recession this is not the case, but there we can also see that the chance of not going bankrupt increases with the initial reputation.

5 Conclusion

Textbook descriptions imply that executives manage companies to maximize shareholder value, perhaps modeled as a discounted integral of future operating profits. But those who have actually owned and run businesses also speak of crises when only last minute efforts and lucky breaks allow the firm to avoid insolvency. Such moments reflect (near) loss of liquidity, not necessarily negative economic worth. If capital markets were truly perfect, this would be a distinction without a difference, but in reality the inability to borrow against an asset is not altogether uncommon.

In the language of firm-level dynamic optimization models, academic models often think in terms of a single generic capital stock, and there is no liquidity constraint. If the firm wants to invest in capital it always can; there is no notion of having to generate investment dollars from current operating surpluses. Implicitly, if the firm did not have the cash on hand, it is assumed that it could always borrow or sell equity to produce such cash in whatever quantity is necessary. The firm feels the full cost (investment spending counts negatively in the objective), but it does not worry whether current bank balances are high enough to cover the check.

In contrast, crisis-induced war stories reflect times when the decision maker is temporarily in a situation of managing two or more capital stocks that are not fungible in the short run, even though at some later economic stage they may be. Going negative with respect to one (the cash account) triggers bankruptcy, even if the total value of the firm's assets is positive. Indeed, it is precisely because the firm still owns some valuable assets that the creditors want a bankruptcy proceeding, so they have an organized way of divvying up those assets or restructuring the firm so it can use those assets to generate future profits.

Sometimes decision makers know exactly when an asset will be able to be used to generate cash. For example, an IPO might already have been scheduled. More often the duration of the first stage of the problem is uncertain. That makes the problem more challenging. If one guesses the duration will be brief and so does not steward cash wisely, one might go bankrupt, but if the cash constraint is relaxed sooner than expected, then in retrospect one may wish one had invested more aggressively throughout.

This paper looked at a particular version of this two-stage optimization problem, a version inspired by articles about how firms sought to manage prestigious brands during the recent global recession, when even firms with healthy fundamentals had trouble raising cash.

The results, in brief, are as follows. When the recession is severe, the firm responds to the lower demand by steering reputation to a lower level than in normal times. If it happened to have an enormous war chest at the start of the recession, the firm could wait out the recession regardless of its length, basically covering negative net operating revenues with interest earned on that war chest. But that is an unrealistic situation; it would require cash on hand to be a very large multiple of annual revenues during normal times. So the usual situation would be to manage a balanced erosion of both brand capital and cash on hand, hoping the recession ends soon enough to avoid bankruptcy. Initially the firm would draw down cash relatively quickly to minimize erosion of brand integrity, but as cash balances dwindled it would alter priorities, giving up brand capital relatively more quickly. (Trajectories always move southwest, but initially move south by southwest before shifting to west by southwest.

Results are similar when the recession is mild, except that there is positive net operating revenue in the region of the favored level of reputation during the recession. So there is no chance of bankruptcy; the trajectories move north by north east, not south by southwest. The exception would be firms that began a mild-to-moderate recession in an already weakened state, with brand reputation far below steady-state no-recession levels and without a large war chest. This might apply to start-up firms trying to break into the prestige market. Their best strategy is to continue to price high enough to raise the reputational capital, even though that burns through cash, hoping that the recession will end soon enough.

Intermediate recessions are, reasonably enough, something of a blend of these situations. Firms that start very strong can manage to a position with positive net operating revenues. Firms that start weak should keep prices so high that they drive toward insolvency, just gambling the recession will end soon enough. There are, though, quite interesting and complicated trajectories for firms of intermediate strength facing an intermediate recession. In particular, their trajectories may not be monotonic in either state variable, varying over time between heading southeast, due west on the boundary with zero cash on hand, then northwest, and finally southwest before crashing into insolvency. In terms of boundaries, this means traveling through the interior to a boundary of the feasible region, following the boundary for a time, going back into the interior, and then finally crashing into the boundary in a way that forces liquidation and the end of the problem. Absent this analysis, such meandering might have been maligned as sure evidence of indecisive leadership, but it can in fact be optimal to reverse course multiple times as one's solvency clock is running down.

Recall that we introduced this analysis as an example of a two-stage problem in which regions of state-space are forbidden in Stage 1 (but not in Stage 2) in the sense that entering such a region terminates the problem. Since terminating the problem means forfeiting the salvage value computed based on Stage 2 and the payoff in Stage 2 can be entirely different than a simple extension of the current payoff being accrued in Stage 1, the looming possibility of a sudden removal of the forbidden region can apparently induce quite unusual behavior in Stage 1 optimal trajectories. From a policy perspective, this is a way of thinking about the origins of the social costs of a market failure and, hence, as a potential justification for government intervention in those markets. And we suspect this character of two-stage problems will prove to be a fruitful topic for further research of a more general character.

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