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Abstract

For certain goods or services, the quality of the product can be assessed by customers only after consumption. Assuming that the demand potential of a profit-maximizing firm is influenced both by this experience quality as well as by advertising, we are able to determine the optimal time paths for pricing, advertising and quality. In particular, it is shown that there may exist two optimal trajectories separated by an indifference threshold in which the firm has the same utility of converging to either of the two long-run steady states. Or, to put it in a nutshell: it is illustrated how the optimal mix of marketing instruments may lead to history-dependence. One interpretation is that there may be a market failure such that a government subsidy could help reach the steady state that is better for the Economy in the sense of having greater sales and a higher quality product.

Keywords: marketing; optimal control; history-dependence; Skiba point

1 Introduction

In a path-breaking paper on information and consumer behavior, Nelson (1970) distinguishes between qualities of a brand that consumers can determine by inspection *prior* to purchase, denoted as search qualities (sometimes also called 'design quality'; Nelson (1974) exemplifies it by the style of a dress), and qualities which can be evaluated only *after* the purchase, so-called *experience qualities*. Examples of these sorts of goods or services are medical services, new vacation resorts, restaurants, repairs, or simply a bottle of wine. Typically, the situation is characterized by asymmetric information where firms produce goods or services of a certain quality that is not known to customers prior to purchase.

Kotowitz and Mathewson (1979a,b) consider a firm faced with rational, albeit ignorant, consumers. These consumers hold prior perceptions about aspects of quality, which determine their purchase behavior. Differences between expected and experienced quality lead to reevaluation of expectations. The monopolistic firm affects these perceptions by advertising and quality variations. For a discussion of the socially optimal marketing instruments in the Kotowitz-Mathewson model, see the excellent survey of Sethi (1977). Fruchter (2009) considers a model where a high price as well as advertising raises the consumers' perceived quality, whereas the latter in turn has a positive effect on sales.

Assuming that the quality, expected by the consumer on the basis of goodwill is known to the producer, Conrad (1985) considers the following dynamics of the goodwill: if goodwill has promised a lower (higher) quality than the consumer experiences after the purchase, the goodwill will increase (decrease). Using optimal control theory, the author is able to derive the optimal paths of quality and advertising. Another paper analyzing the optimal mix of advertising and experience quality is Ringbeck (1985). He considers a profit-maximizing monopolist whose stock of customers is increased by advertising, but decays by bad quality. Differential game approaches are considered in El Ouardighi and Pasin (2006); El Ouardighi et al. (2008), and El Ouardighi et al. (2013).

The ignorance of the customers about quality can be ameliorated by communicating with others who have already used the service or bought the product. Spremann (1985) (a brief description of Spremann's approach to quality is given in Feichtinger et al., 1994) assumes the seller's reputation as a second signal depending on two factors: the quality-price ratio as perceived by former customers and quantity already sold. Spremann's model of signaling quality by reputation leads to two optimal equilibria: the producer is either cheap or expensive. Both equilibria are remarkable departures from price-corresponds-to-quality mode, which turns out to be sub-optimal if reputation is being generated. Feichtinger et al. (1988) provide an extension of Spremann's model. It is a particular feature of the last two models that the accumulation of reputation depends on the quality-price ratio obtained by former customers' judgements. Caulkins et al. (2006) consider a two-state optimal control model dealing with experience quality. One of their state variables, denoted as reputation, reflects the expected quality of a good. Numerical analysis with parameters based on the U.S. cocaine market show that there might exist multiple equilibria and even persistent cycles.

Kostami and Rajagopalan (2014) study the impact of the quality of a product on the market demand potential. There is an inherent trade-off between the quality and the speed of a service. Working faster will result in more sales and less waiting time but may also result in lower quality and dissatisfied customers. By using queueing theory, they are able to find the optimal balance between price and speed for the customers revenue and the congestion costs due to quality.

The present paper extends Kostami and Rajagopalan (2014) by considering advertising as another means to increase demand potential, along with experience quality. We find that there are history dependent optimal trajectories in the sense that it takes an initially large demand potential to keep demand potential large in the long run. The intuition is that investment in quality pays off especially when a large number of customers benefits from it. When the initial demand potential is not large enough the firm chooses a time path ending at a steady state where demand potential is relatively low. Before reaching this steady state the firm invests less in quality, while it first increases and then decreases price on the trajectory where demand potential is decreasing and approaches the steady state with low demand potential. The price increase is done to reduce the number of customers so that not too many of them suffer from the quality decrease, which prevents a too fast reduction of the demand potential. The larger steady state may be better from the point of view of the whole economy, because sales are larger and the product has higher quality. We sketch how the government can stimulate the firm to choose the path towards this better steady state by subsidizing quality investments.

The paper is organized as follows. Section 2 presents the model in an optimal control setting. Section 3 discusses the optimal policies regarding pricing, quality investment and advertising. In particular, the existence of multiple long-run steady states is shown.

In that context, the indifference point separating the basins of attraction is discussed. Section 4 briefly considers a model variant where quality costs are linearly dependent on sales, where it is shown that history dependence survives this model change. The final Section 5 deals with the conclusions and possible extensions.

2 The model

Denote by p the price of a good, a the advertising rate, q the quality of the product, and K is the maximum volume obtained when prices are cut to zero; hereinafter referred to as “demand potential”. Moreover, the quantity sold (sales) is $S = S(p, K)$ with $S_p < 0$ and $S_K \geq 0$. As in Kostami and Rajagopalan (2014) we assume

$$S = K - \alpha p, \quad (1)$$

where α is a positive constant, and the price is restricted to $p \in [0, K/\alpha]$. The demand potential is dynamically influenced by advertising and price as follows:

$$\dot{K} = a/K^\theta - \delta (K - \alpha p) (\bar{q} - q) - \beta K. \quad (2)$$

Here \bar{q} is the perfect quality (perfect in the sense of perfectly conforming to the customer’s highest ideals; this is for instance the case when the product is completely free of defects), and β and δ are two positive parameters, while $\theta \in [0, 1]$. The effect of advertising on the demand potential is not independent of K . It makes sense to assume that the impact of a decreases with K . If the demand potential is large, most consumers who are in principle interested in the good have already been reached, so it will be increasingly difficult to make it even larger. This justifies the term $aK^{-\theta}$, $0 \leq \theta \leq 1$.¹ The second term of \dot{K} reflects that consumers get upset by quality deficits, i.e. when q falls below the perfect quality level \bar{q} . If they experience lack of quality, they do not want to buy the product anymore, implying that demand potential is reduced. This effect is particularly large if there are lots of consumers, so when sales, $K - \alpha p$, are large. The third term, $-\beta K$, has to do with increasing competition over time due to the fact that new products are invented to which consumers can switch, thereby reducing demand potential of the current product.

The firm is a profit maximizer, implying that the objective is as follows:

$$\max_{p, q, a} \int_0^\infty e^{-rt} (p(K - \alpha p) - c(q) - k(a)) dt. \quad (3)$$

Note that in the objective functional the quality costs, $c(q)$, are independent of the production. One could think of a situation that once one has achieved a certain quality level one could produce as many products as wanted with this quality. It is like inventing

¹Alternatively, we could impose that the impact of a on the rate of change of K increases first with K , but then decreases, where the decrease captures the saturation effect as in our main model. The increase when K is small is due to word-of-mouth: when the number of customers is still low, demand potential increases because customers tell each other about the product. This of course happens more when there are more consumers, i.e. when demand potential increases. A logistic function would satisfy this assumption. By assuming that, the system dynamics (2) would be replaced by the following state equation

$$\dot{K} = aK (\bar{K} - K) - \delta (K - \alpha p) (\bar{q} - q) - \beta K.$$

a more clever design, either of the product itself or of the production process. Think for instance of intangible goods such as software: once the computer program is finished, no extra costs are needed to sell 1, 2, 3, or 1000 products. Section 4 briefly considers the case where quality costs are linearly increasing in production.

The last term in the objective functional (3), $k(a)$, reflects the advertising costs. As in Kotowitz and Mathewson (1979a) and Gould (1970) we assume convex costs of quality and advertising, respectively. This corresponds to the marginally decreasing impact both of quality as well as advertising (see also Jørgensen and Zaccour, 2004). Hence, we have

$$c(q) = \frac{1}{2}\gamma q^2, \quad k(a) = \frac{1}{2}\kappa a^2. \quad (4)$$

The following constraints have to be taken into consideration:

$$K/\alpha \geq p \geq 0, \quad \bar{q} - q \geq 0, \quad K \geq 0. \quad (5)$$

Next, we establish necessary optimality conditions. To do so we set up the Hamiltonian of our optimal control problem:

$$H = p(K - \alpha p) - \frac{1}{2}\gamma q^2 - \frac{1}{2}\kappa a^2 + \lambda(a/K^\theta - \delta(K - \alpha p)(\bar{q} - q) - \beta K),$$

where the costate variable is denoted by λ . The Lagrangian is

$$L = H + \mu_1(\bar{q} - q) + \mu_2(K - \alpha p).$$

Nonnegativity constraints on price and demand potential are disregarded at first. We check afterwards whether the optimal solution satisfies these conditions.

The first order condition for price setting is

$$L_p = K - 2\alpha p + \alpha\delta\lambda(\bar{q} - q) - \alpha\mu_2 = 0. \quad (6)$$

Hence, when there is no quality deficit, i.e. $\bar{q} = q$, price equals $K/2\alpha$, which maximizes instantaneous revenue. For lower quality levels price is higher. Consumers finding out that quality is lower get disappointed by the product, which reduces demand potential. To diminish this effect the firm charges a higher price, which reduces demand, so that fewer consumers experience this lower quality.

The firm sets quality such that it satisfies

$$L_q = -\gamma q + \lambda\delta(K - \alpha p) - \mu_1 = 0 \quad \Rightarrow \quad q = \frac{\lambda\delta}{\gamma}(K - \alpha p). \quad (7)$$

The firm especially invests in quality when sales, $K - \alpha p$, are large so that many customers realize quality is high. Otherwise, demand potential would decline a lot because of the many customers experiencing the low quality.

Inserting (7) into (6) we find

$$p = \frac{\gamma}{\alpha(\alpha\delta^2\lambda^2 - 2\gamma)} \left(K \left(1 - \frac{\alpha\delta^2\lambda^2}{\gamma} \right) + \alpha\delta\lambda\bar{q} - \alpha\mu_2 \right).$$

Concerning advertising we have

$$L_a = -\kappa a + \lambda/K^\theta = 0 \quad \Rightarrow \quad a = \frac{\lambda}{\kappa K^\theta}.$$

Interpretation of this condition is straightforward. The firm advertises a lot when unit advertising costs are low, when current demand potential is low so that there is still a lot to gain, and when the firm assigns a high value to additional demand potential, i.e. the costate value λ is large.

The Legendre Clebsch condition is fulfilled if

$$\mathcal{H} = \begin{pmatrix} -2\alpha & -\alpha\delta\lambda & 0 \\ -\alpha\delta\lambda & -\gamma & 0 \\ 0 & 0 & -\kappa \end{pmatrix}$$

is negative definite. This guarantees that the previously calculated control values are maximizers of the Hamiltonian.

The costate equation is

$$\dot{\lambda} = r\lambda - L_K = (r + \beta)\lambda - p + \frac{\lambda\theta a}{K^{\theta+1}} + \lambda\delta(\bar{q} - q) - \mu_2,$$

The complementary slackness conditions

$$\begin{aligned} \mu_1(\bar{q} - q) &= 0, \\ \mu_2(K - \alpha p) &= 0, \end{aligned}$$

have to be fulfilled.

In case the control constraint $\bar{q} - q \geq 0$ is active we have

$$p = \frac{K}{2\alpha}, \quad q = \bar{q}, \quad \mu_1 = -\gamma\bar{q} + \frac{\lambda\delta K}{2}, \quad \mu_2 = 0.$$

In case the constraint $K - \alpha p \geq 0$ is active we have

$$p = \frac{K}{\alpha}, \quad q = 0, \quad \mu_1 = 0, \quad \mu_2 = \frac{K}{\alpha} + \delta\lambda\bar{q}.$$

3 Numerical results

We numerically determine the optimal solution, using the toolbox OCMat; see http://orcos.tuwien.ac.at/research/ocmat_software/. The underlying boundary value approach is described in Grass et al. (2008) and Grass (2012).

The parameters used are

$$\delta = 0.05, \alpha = 0.5, \kappa = 2, \bar{q} = 100, \gamma = 0.05, r = 0.03, \beta = 0.01, \theta = 1.$$

The (K, λ) -phase portrait (Figure 1) shows that there are two stable steady states, K_1 and K_2 . Note that the Legendre Clebsch condition is fulfilled on the optimal solution paths. Regarding the optimal trajectory approaching the larger steady state K_2 , this figure shows that the costate variable λ increases with the demand potential. Thus,

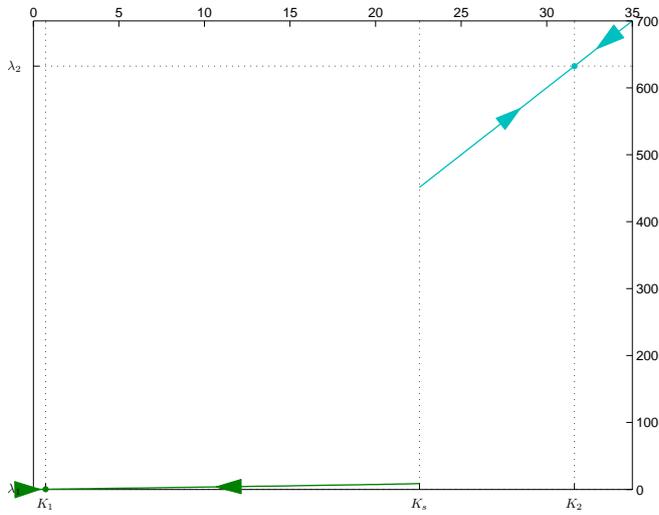


Figure 1: State-costate phase portrait for the base case, where the Skiba point K_s separates the basins of attraction of the two equilibria with demand potential levels K_1 and K_2

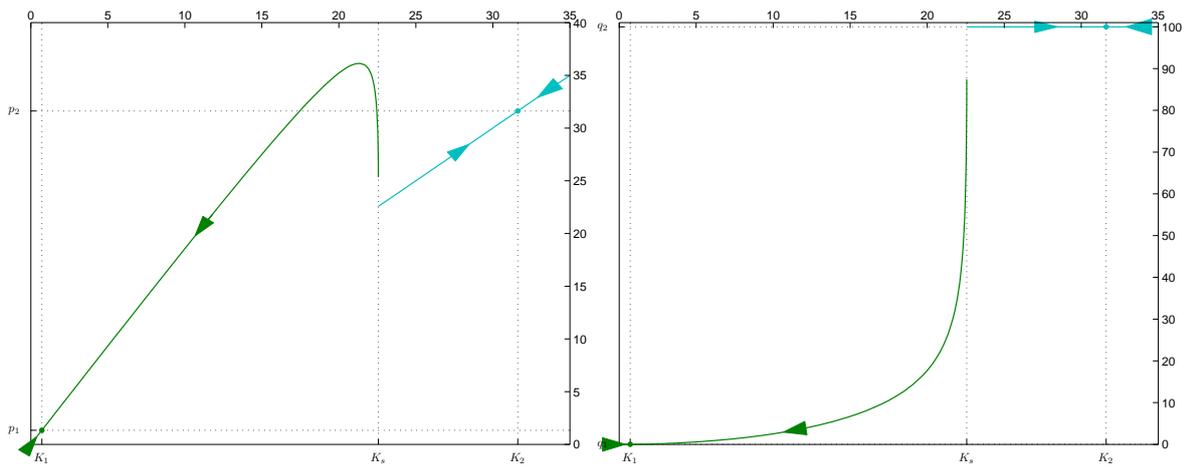


Figure 2: (K, p) -phase portrait (left panel) and (K, q) -phase portrait (right panel) for the base case. K_1 is an equilibrium where price and product quality is low. At K_2 the product is expensive and has high quality.

the usual scarcity argument (the shadow price of a good increases if the good becomes more scarce) does not hold. After plugging in $p = K/2\alpha$, which holds on the trajectory approaching the larger steady state, the objective becomes convexly increasing in K , so that also marginal revenue is increasing in K .² Therefore, the shadow value of K increases with K , implying that the costate goes up with K .

The reason that $p = K/2\alpha$ holds on the trajectory approaching the larger steady state is as follows. On this trajectory demand potential K is relatively large, which attracts lots of customers. Therefore, it pays off for the firm to put q at the upperbound \bar{q} (see the right panel of Figure 2), and this implies via (6) that price is put at the level that maximizes instantaneous revenue. This explains why the $p - K$ trajectory is on the 45 degree line ($\alpha = 0.5$) when approaching the larger steady state, see the left panel of Figure 2. The right panel of Figure 2 reveals that quality is monotonically increasing in K , i.e. a firm invests especially in quality once demand potential is large, and consequently the number of customers is high. At such a moment a huge quality deficit would mean that many customers get disappointed by the product so this needs to be prevented by keeping quality high.

When approaching the lower steady state the firm has less incentive to invest in quality. Hence, quality falls below its upperbound \bar{q} , making the term $-\delta(K - \alpha p)(\bar{q} - q)$ nonzero. Then the firm has an incentive to charge a higher price than $K/2\alpha$ in order to reduce sales, since this in turn reduces the negative effect of the quality deficit on demand potential. This explains the initial increase in price (see the left panel of Figure 2) on the trajectory starting at a demand potential just below the indifference level K_S . In particular, the explanation for the price spike when approaching the small steady state is as follows: At the beginning of the trajectory there is not a large quality deficit. Then the firm keeps quantity large and price low. Along the trajectory the quality deficit increases. Then the firm wants to limit the resulting negative effect on demand potential by reducing quantity and thus increasing price. On the other hand, demand potential falls along the trajectory. This lowers demand which has a negative effect on price. When demand potential has reduced sufficiently, this negative price effect starts to dominate and price starts to decrease.

Figure 1 also shows an indifference point, or Skiba point (see, e.g., Grass et al., 2008), K_S , in between steady states with high and with low levels of potential demand K . This is caused by the control state interaction term $-\delta(K - \alpha p)(\bar{q} - q)$. The aim of investing in quality is to increase demand potential, or at least to diminish its rate of reduction. This especially pays off when K is large. Therefore, when K is small there is less investment in quality and K converges to the small steady state. When K is large the firm invests in quality and K converges to the larger steady state. This is confirmed by Figure 2. We

²To see that consider the model without quality, i.e.

$$\max_{a,p} \int_0^{\infty} e^{-rt} (p(K - \alpha p) - \kappa a^2/2) dt, \quad (8)$$

s.t.

$$K = a/K^\theta - \beta K. \quad (9)$$

Maximization w.r.t. p yields $p = K/2\alpha$. Substituting this back into (8) gives

$$\max_{a,p} \int_0^{\infty} e^{-rt} (K^2/4 - \kappa a^2/2) dt.$$

This feature remains valid when the quality q is included.

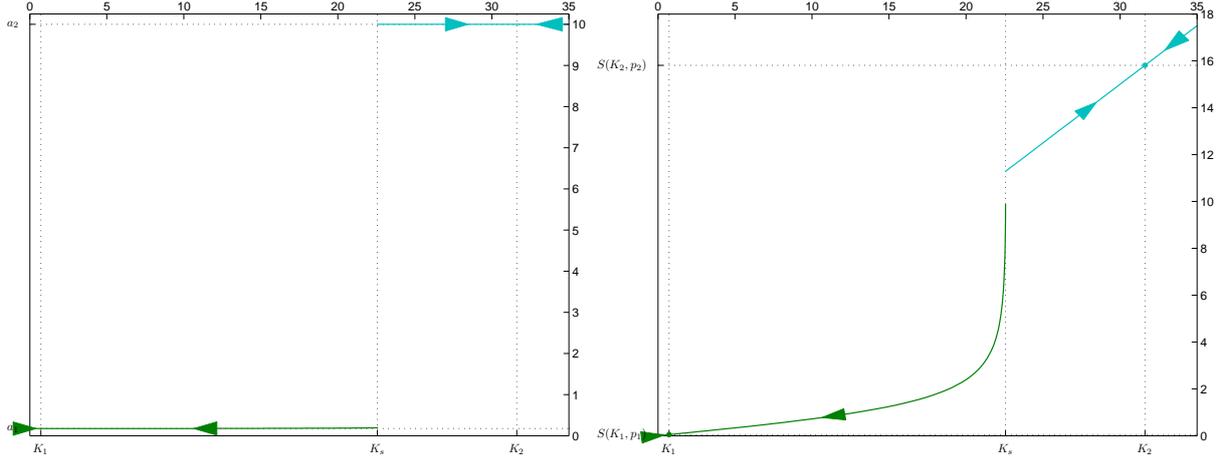


Figure 3: (K, a) -phase portrait for the base case (left panel) and phase portrait for the demand potential K and the sales $S = K - \alpha p$ (right panel). At the K_1 equilibrium both advertising and sales are negligible, whereas the equilibrium with high demand potential, K_2 , is accompanied by high advertising and large sales.

conclude that the optimal solution is history dependent in the sense that it requires a large initial demand potential to keep demand potential large in the long run. Otherwise the firm keeps a small demand potential forever, where it ends up in the steady state K_1 .

If K is small so quantity, $K - \alpha p$, is small too, the firm does not gain much by investing in a high q , because the large $(q - \bar{q})$ is squandered by being multiplied by a small amount of sales. In other words, the quality investment only affects a small output. In this situation we have a sort of no-name non-brand mode in which one competes just on price, with low a (see Figure 3) and low q . And then there is a high q , high a , high K (and then high price) equilibrium, which might look like the typical (over-priced) national brand, one that convinces people to buy despite the high price by having both a good product and a lot of advertising.

Once K is large, demand is high and it is worthwhile to keep K large by advertising, see Figure 3. If K is small, the firm keeps K small and the advertising policy is adjusted to that by keeping it at a very low level. Note that the latter takes place despite the fact that the term a/K^θ makes advertising especially effective when K is small. This advertising term does not favor the appearance of an indifference point, but apparently it is not influential enough to prevent that result.

The upper steady state would seem to be better (higher quality, so more attractive product, large consumption) for the economy than the lower one. Therefore, in a situation where currently demand potential is small, it may be optimal for a government to intervene by temporarily subsidizing quality investments so that demand potential becomes so large that convergence to this larger steady state is optimal for the self-interested firm to pursue of its own accord. Subsidizing quality investments implies reducing γ . To illustrate the effect of this, when γ is zero, q will equal its upperbound \bar{q} . Then quality disappears out of the model, price is such that $p = K/2\alpha$ all the time, and in the resulting solution there is only one stable steady state, which is the larger steady state in our present model. So, when quality investments are sufficiently subsidized, the firm, starting out with small K , will follow the trajectory towards the large steady state. Once the trajectory passes the Skiba threshold K_S , the firm will continue to grow towards the larger steady state on its own, i.e. without needing the subsidy to make this behavior

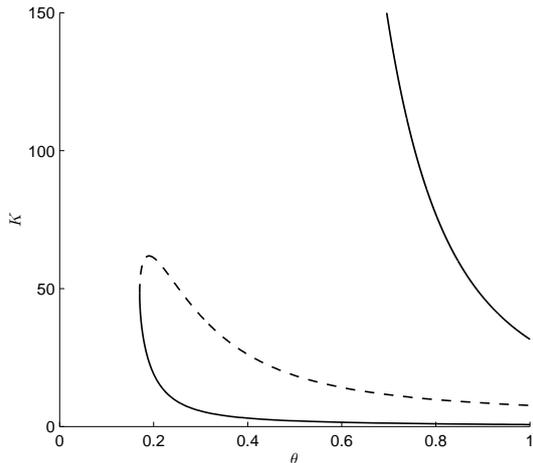


Figure 4: The two potential long-run equilibria as a function of θ , where θ measures the reduction in the effect of advertising due to an increased current demand potential

optimal. (Note: this argument is somewhat ad hoc. We have not considered strategic efforts by the firm to anticipate or influence government policy, e.g., with political donations. So the argument might apply with respect to a small firm or a broad policy, not one in which this particular firm is the primary stakeholder for that policy intervention.)

Figure 4 shows how the steady states change if parameter θ changes. Both steady states are bigger for smaller θ ; however, while the change for the lower steady state is not particularly large, it is large for the upper one. There is an intermediate steady state which is not optimal. This steady state is relevant for the location of the Skiba point. Below a certain threshold the lower and the intermediate steady states disappear. Then it is optimal to approach the upper steady state, where the long-run value of K is huge. For $\theta = 0$, no optimal solution exists. If θ is large it implies that advertising has less effect on the demand potential, especially when K gets larger. Therefore, the firm will advertise less and demand potential will be lower in the long run.

Figure 5 shows the steady states dependency on β . A higher value of β implies that the firm has to deal with more competition over time because new products are invented to which consumers can switch. This reduces profitability and therefore the firm spends less money on its own product, resulting in less advertising and lower long run demand potential. The observed solution remains qualitatively similar as the two long run equilibria keep on existing for the various beta levels.

4 Quality Costs Linearly Increasing in Production

In another variant $c(q)$ denotes the costs for producing one unit of the good with quality q :

$$\max_{p,q,a} \int_0^{\infty} e^{-rt} ((p - c(q)) (K - \alpha p) - k(a)) dt. \quad (10)$$

The rest of the model stays as it is. Hence, instead of quality coming from some more clever design that needs to be developed just once and then can be copied at zero marginal

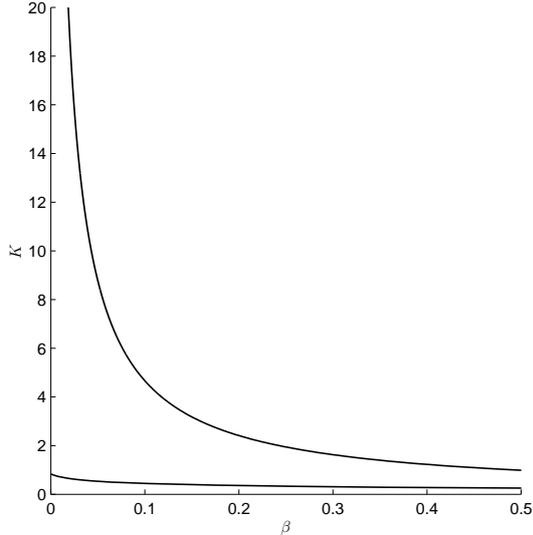


Figure 5: The two potential long-run equilibria as a function of β , where β is a measure of the increase of competition over time.

costs like in the model of Section 2, here producing quality goods requires the firm to spend more on better materials, devote more labor hours per unit produced, etc. Broadly speaking, the earlier model may be more applicable to invention and to intellectual goods, whereas this model may be more applicable to quality for traditional physical goods.

Keeping the same parameter values as in the previous section, except that we replace $\alpha = 0.5$ by $\alpha = 0.1$, the different phase portraits are depicted in Figures 6–8. We conclude that most of the qualitative features of the previous section are preserved here, including the history dependence property, i.e. it requires a large initial value of demand potential ($K > K_s$) to converge to the steady state with higher quality products and more sales. What is different is that now price is monotonic everywhere, because the price spike has disappeared. The fact that quality costs are now multiplied by sales provides another incentive to reduce sales by having a high price. This explains why it is optimal that price is high initially on the path starting at K_s^- and converging to the lower steady state. Price declines along with demand potential, because, as before, demand declines with the reduction of demand potential and this has a negative effect on price.

5 Conclusions

In this paper we assumed that the demand potential for a product depends on advertising, price and the quality of the good or the service. Demand is directly influenced by price and demand potential, where the latter is in turn positively influenced by advertising and negatively by the gap between the experience quality and the consumers' notion of 'perfect' quality. If this gap is large the firm has an incentive to increase price to reduce this negative influence. This together with the direct effect of price on demand results in a non-monotonic pricing behavior.

We find history dependent behavior, where a so-called Skiba point defines a threshold demand potential level that forms the boundary between initial demand potential levels leading to a profit-maximizing firm to pursue two quite different strategies. When that

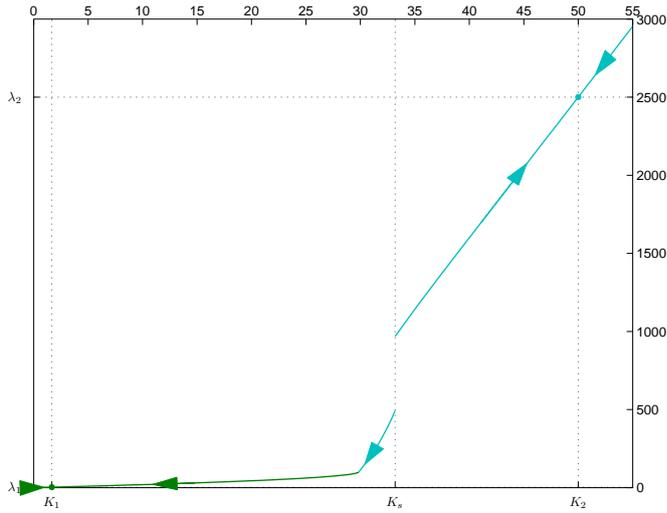


Figure 6: State-costate phase portrait if quality costs linearly increase in production

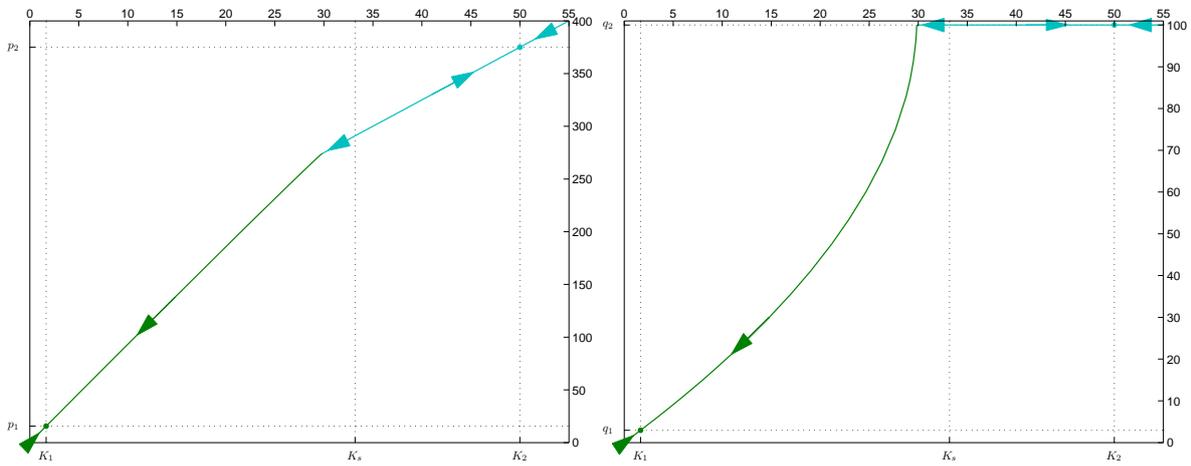


Figure 7: (K, p) -phase portrait (left panel) and (K, q) -phase portrait (right panel) if quality costs linearly increase in production

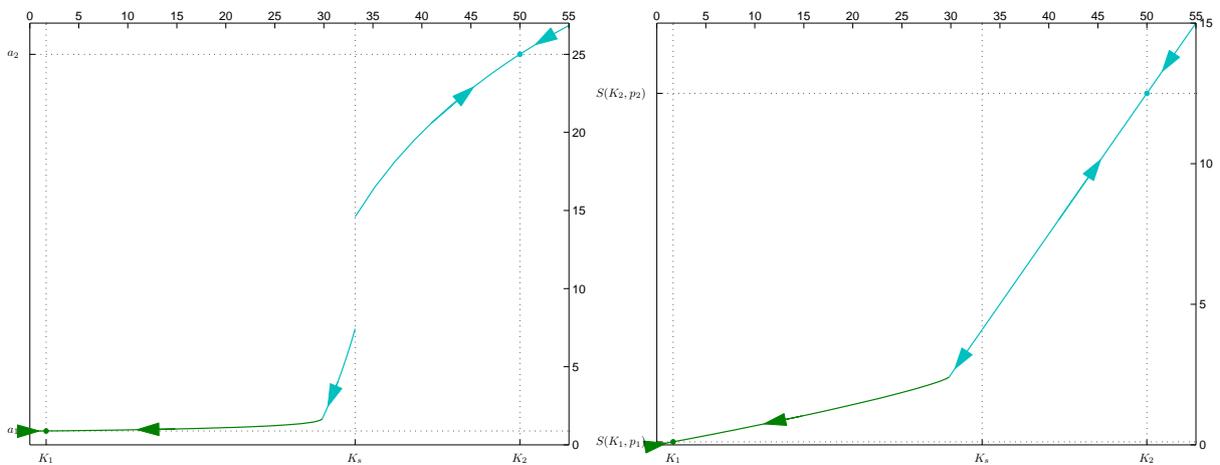


Figure 8: (K, a) -phase portrait (left panel) and phase portrait for the demand potential K and the sales $S = K - \alpha p$ (right panel) if quality costs linearly increase in production

initial level is low, the demand potential remains low, hardly any advertising occurs and the firm's quality investments are also low. When that initial potential is high, the firm invests more in advertising and quality and ends up in a steady state with high advertising, a large product quality, many sales and still a high product price. The latter solution may be judged to be preferable for society, so the government may have an incentive to subsidize quality investments to assure the firm will choose this latter solution path. In further work it would be interesting to set up a (differential) game between government and firm (or industry) to investigate this issue when the firm is large enough and the policy is sufficiently narrowly targeted at that one firm, to make it worthwhile for the firm to try to anticipate or influence the government action.

In our approach, the market demand potential can be interpreted as a sort of capital Stock that measures the reputation of a firm. Acting as the state variable of the underlying optimal control model, it is not only influenced by advertising but also by the experience quality. A more complicated model would allow the firm to have an additional capital stock which is influenced by price. For the interesting consequences of such a price reputation interaction we refer to the work of Caulkins et al. (2011).

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