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CONTRACTS AND INFORMATION STRUCTURE IN A SUPPLY CHAIN WITH OPERATIONS AND MARKETING INTERACTION

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Abstract. The objective of the paper is to study how wholesale price and revenue sharing contracts affect operations and marketing decisions in a supply chain under different dynamic informational structures. We suggest a differential game model of a supply chain consisting of a manufacturer and a single retailer that agree on the contract parameters at the outset of the game. The model includes key operational and marketing activities related to a single product in the supply chain. The manufacturer sets a production rate and the rate of advertising efforts while the retailer chooses a purchase rate and the consumer price. The state of the game is summarized in the firms' backlogs and the manufacturer's advertising goodwill. Depending on whether the supply chain members have and share state information, they may either make decisions contingent on the current state of the game (feedback Nash strategy), or precommit to a plan of action during the whole game (open-loop Nash strategy). Given a contract type, the impact of the availability of information regarding the state of the game on the firms' decisions and payoffs is investigated. It is shown that double marginalization can be better mitigated if the supply chain members adopt a contingent strategy under a wholesale price contract and a commitment strategy under a revenue sharing contract.

Keywords. Supply chain management, Wholesale price contract, Revenue sharing contract, Information structure, Operations, Marketing.

1. Introduction

Operations management and marketing are critical functions for not only individual manufacturing firms but also, with increased global competition, entire supply chains. Also, the two functions are strategically interlinked, since marketing's role is to create and manage demand, and operations is responsible for efficiently meeting the demand. Conflicts between marketing and operations management arise, since the two functions have differing objectives, marketing to enhance demand, and operations to minimize manufacturing and inventory/backlog costs, so management of the interface between the areas is critical. The importance of effective management of the interface has been recognized for some time (Malholtra and Sharma, 2002), and a review of models of the interface is provided by Tang (2010).

An important research area of operations management and marketing literature dealing with supply chains/marketing channels is the study of inefficiencies caused by lack of coordination. A classic example is the double marginalization problem that occurs when a retailer pays a supplier a fixed transfer price per unit ordered. The mark-ups applied by both chain members lead to lower ordering quantities, higher consumer prices, and smaller overall profits than if decisions were centralized (Spengler, 1950). To avoid this inefficiency, decision making in the supply chain must be coordinated. This often entails using an appropriate contract to regulate the flow of payments between members of the supply chain.

The coordination problem has been addressed in a sizable body of literature in operations management as well as marketing. This literature has, however, tended to disregard the interactions between the two functional areas: *"Operations management has a wealth of literature that deals with the inventory aspects of the supply chain, but ignores marketing expenses… At the same time, the marketing literature tends to deal with customer acquisition costs in the form of advertising […], but ignores operational issues"* (Simchi-Levi *et al*., 2004, p. 612).

The current research focuses on two types of contracts, the wholesale price contract (WPC) and the revenue sharing contract (RSC). A RSC lowers the transfer price to the retailer's benefit. The retailer then pays the supplier a part of its revenue. The relative merits of the two contracts in static settings are well known (e.g., Cachon and Lariviere, 2005). Mortimer (2008) provides an empirical study of WPC and RSC in the video rental industry. Revenue sharing contracts have been used in other industries, for example, in telecommunication services (Qin, 2008; Chakravarty and Werner, 2011) and cell phone manufacturing (Linh and Hong, 2009).

An operations management approach to coordination with WPC and RSC has often employed a static setup (see, e.g., Lariviere, 1999; Cachon, 2003). Less is known, however, about how the two contracts work in a supply chain involving operational and marketing functional areas with repeated interaction. Thus, a primary aim of the paper is to study the relative merits of the two contracts in a game setting where a retailer and its supplier first agree on a contractual scheme and then make operations and marketing decisions over time in the supply chain.

In this setting, firms' strategy depends on the extent to which chain members have, and share, state information (Başar and Olsder, 1999; Dockner *et al*., 2000). If a chain member has no such information, it cannot condition its actions on the state vector, and the firm's actions can be based only on time. Although decisions are dynamically optimized, strategic interaction takes place at the initial instant of time only; each player makes a commitment to execute a predetermined plan of action. This is known as an open-loop strategy. Such strategy applies in inventory management when there exists information delays (Bensoussan *et al*., 2007), inaccurate records (DeHoratius and Raman, 2008), or hidden information. A good example of the latter is when a retailer is unwilling to reveal consumer sales data to its supplier.

Conversely, if chain members have, and share, state information, a firm's actions can also be based on the current state. This is known as a feedback strategy. Strategic interaction takes place throughout the game because players make decisions that are contingent upon the current state and time. In a supply chain, this situation is plausible whenever mutual trust is prevalent.

Both open-loop and feedback strategies have been applied in the supply chain (e.g., Gaimon, 1998; Kogan and Tapiero, 2007) and marketing channel (e.g., Jørgensen and Zaccour, 2004) literature dealing with intertemporal decisions.

A major aim of the paper is to compare the performance of different informational structures under a wholesale price and a revenue sharing contract, respectively. For example, given a contract, a comparison will reveal the relative merits of commitment and contingent strategies and quantify how availability of information on the current state of key operational and marketing variables affects supply chain members' decisions and payoffs.

The paper focuses on production and sales of a single product/brand and suggests a dynamic model that includes key operational and marketing activities in the supply chain. The operations-marketing model that we use in the current research is a variant of the model in El Ouardighi *et al*. (2008) and Jørgensen (2011) (see also Jørgensen, 1986; Eliashberg and Steinberg, 1987) and an advertising goodwill model; see, e.g., Jørgensen and Zaccour (2004). None of these papers, however, was concerned with the availability of state information in a supply chain.

The setup is one in which a single manufacturer sets her production rate and the national advertising rate for her product. Manufacturer advertising supposedly builds up a stock of consumer goodwill. There is a single retailer who sets the purchase rate and the consumer price. The state of the system is represented by the stock of consumer goodwill as well as the manufacturer's and retailer's respective backlogs of the product.

The following research issues are addressed:

- How are operations and marketing decisions in the supply chain, under a WPC and an RSC, respectively, affected by the availability of state information?
- Depending on the availability of state information, how do the two kinds of contracts compare in the long run?
- How do the two kinds of strategies mitigate channel inefficiency (double marginalization) under each contract?
- How are supply chain members' payoffs affected by the type of strategy and contract?

The main contribution of the current research lies in the fact that we look simultaneously at two important aspects of supply chain management: the choice of a contract that specifies how payments flow in the supply chain, and the role of information for the design of operations and marketing strategies in the supply chain. We show that with a wholesale price contract, feedback strategies mitigate the double marginalization problem because an increase in price does not deter growth of demand. Conversely, with a revenue sharing contract, open-loop strategies provide results that are closest to those of a cooperative strategy.

The paper proceeds as follows. Section 2 develops a differential game model where a manufacturer and a retailer agree on a supply chain contract at the outset. Sections 3 and 4 study the operations and marketing decisions in the supply chain under the two contracts in the context of open-loop and feedback Nash equilibria. Section 5 compares the contracts and the strategies. Section 6 concludes the paper.

2. Differential game model

A manufacturer's product is ordered by an exclusive retailer who resells the product to final consumers. Each chain member controls two decisions: one in marketing and one in operations. The manufacturer determines its production rate and investment in advertising goodwill, whereas the retailer controls its procurement rate and the consumer price.

Time t is continuous and the game starts at time zero. State variables are the manufacturer's and retailer's backlogs as well as the manufacturer's advertising goodwill. Backlogging means that there is a delay in the delivery of some of the product quantity the retailer ordered from the manufacturer, as well as from retailer to customers, with associated costs to the manufacturer and retailer. Backlogging reflects an on-time fulfillment rate of less than 100%, which is not unusual in practice. Feichtinger and Hartl (1985), and Erickson (2011, 2012) show that backlogging may be profitable in the long run because production and purchase costs can be deferred. Sapra *et al*. (2010) argue that backlogging can be optimal because it adds to the allure and sense of exclusivity of a product and stimulate its demand.

The manufacturer's backlog at time *t* is denoted by $X(t)$ and evolves over time according to

$$\dot{X}(t) = v(t) - u(t), \qquad X(0) = X_0 \geq 0 \tag{1}$$

where $u(t) \geq 0$ is the manufacturer's production rate and $v(t) \geq 0$ is the retailer's purchase rate. We assume that the manufacturer has a safety stock that can be used to fill backlogged products if there is backlogging (see, e.g., Maimon *et al*., 1998). The reason is that whenever the production rate is lower than the purchasing rate, the difference has to be filled, though with delay, thanks to the safety stock.

The retailer's backlog is $Y(t)$ and evolves according to

$$\dot{Y}(t) = S(t) - v(t), \qquad Y(0) = Y_0 \geq 0 \tag{2}$$

where $S(t)$ is the consumer demand rate.

Consumer demand is affected negatively by the retail price $p(t)$ and positively by advertising goodwill $G(t)$ in a simple linear fashion

$$S(t) = \alpha - \beta p(t) + G(t) \tag{3}$$

where $\alpha, \beta > 0$ and constant. The market potential is time-dependent and equals $\alpha + G(t)$.

The manufacturer's goodwill evolves according to the Nerlove and Arrow (1962) model

$$\dot{G}(t) = w(t) - \delta G(t), \qquad G(0) = G_0 \geq 0 \tag{4}$$

where $w(t) \geq 0$ is the manufacturer's national advertising effort and $\delta > 0$ a constant decay rate.

If advertising goodwill is omitted in the demand function, equations (1) - (4) reduce to a variant of the model developed by Jørgensen (1986), Eliashberg and Steinberg (1987), Desai (1992, 1996), Kogan (2012) and others. If supply chain members adopt a zero-stock policy, they produce and order to meet retailer and consumer demand, respectively, at any time and equations (1)-(4) reduce to the model studied in Jørgensen and Zaccour (2004).

Our next task is to define a payoff functional for each firm. Operations management literature usually assumes short planning horizons to evaluate the performance of operational strategies (Kogan and Tapiero, 2007). Marketing literature has often adopted an infinite planning horizon, partly to disclose long-run effects of marketing strategies, and partly for mathematical convenience (Erickson, 2003; Jørgensen and Zaccour, 2004). This paper assumes an infinite planning horizon, that is $t \in [0, +\infty[$, which enables us to study the long-run stability of game equilibria. The use of an infinite horizon is also motivated by the inclusion of advertising goodwill in the model. Building a stock of goodwill takes time and assuming a finite (and possibly short) horizon might leave out interesting aspects of goodwill evolution.

We assume that an agreement on (*i*) the type of contract and (*ii*) its parameter(s) has been reached before playing the game. Here supply chain members can choose among a WPC or an RSC. The WPC has one parameter, the transfer price ω, while the RSC has two parameters, the transfer price and the share, ϕ, of the retailer's revenue that the manufacturer gets from the retailer. The parameters ω and ϕ are exogenously determined and remain constant over time.

- The transfer price $\omega \geq 0$ is paid by the retailer to the manufacturer for each unit purchased and we denote by $\omega|_{WPC}$ and $\omega|_{RSC}$ the transfer prices that apply under a WPC and an

RSC, respectively. We require a transfer price to be positive and $\omega_{|RSC}$ be lower than $\omega_{|WPC}$, that is, the retailer would agree to an RSC only if the manufacturer lowers the transfer price that applies in the WPC: *quid pro quo*.

- The second parameter is the manufacturer's share ϕ of the retailer's revenue and we require this share to be nonnegative and less than one. The WPC is a special case of an RSC with $\phi = 0$ and a larger transfer price. The rationale for an RSC is to decrease the retailer's unit procurement cost in order to induce the retailer to buy more units.

It remains to formulate the objective functions of the two firms. As to the manufacturer's costs, we suppose that the manufacturing cost increases with the production rate according to the quadratic function $au(t)^2/2$, $a > 0$ and constant. The manufacturer incurs a cost of advertising effort, expressed by the quadratic function $bw(t)^2/2$, $b > 0$ and constant. Finally, the manufacturer's cost of backlogging is $cX(t)^2/2$, $c > 0$ and constant. Note that if the backlogging turns to inventory, i.e., $X < 0$, the manufacturer then incurs an inventory cost. Finally, if the manufacturer uses a safety stock, its cost is sunk and be disregarded.

Assume that both firms employ a constant discounting rate, denoted by $r > 0$. Then the manufacturer's objective is

$$\underset{u(t),w(t)\geq 0}{\text{Max}} \Pi^M = \int_0^\infty e^{-rt}\left[\phi p(t)S(t) + \omega v(t) - au(t)^2/2 - bw(t)^2/2 - cX(t)^2/2\right]\mathrm{d}t \tag{5}$$

The retailer's gross revenue is $p(t)S(t)$ under a WPC, and $\bar{\phi} p(t)S(t) - \omega_{|RSC} v(t)$, $\bar{\phi} \equiv 1-\phi$, under an RSC. The retailer's ordering/processing cost is increasing and is convex in the purchase rate: $dv(t)^2/2$, $d > 0$ and constant. Finally, the retailer's backlogging cost is $eY(t)^2/2$, $e > 0$ and constant.

The retailer's objective is

$$\underset{v(t),p(t)\geq 0}{\text{Max}} \Pi^R = \int_0^\infty e^{-rt}\left[\bar{\phi} p(t)S(t) - \omega v(t) - dv(t)^2/2 - eY(t)^2/2\right]\mathrm{d}t \tag{6}$$

We note that all costs for the manufacturer and retailer, production, advertising, retailer ordering, and backlog, are modeled as being quadratic. This is to be consistent with existing literatures, which typically assume strictly convex costs for advertising (Jørgensen and Zaccour, 2004; Erickson, 2003), production (Jørgensen *et al.*, 1999), ordering (Jørgensen, 1986), and backlog (Feichtinger and Hartl, 1985; Erickson, 2011).

To avoid being entangled in mathematical subtleties we confine our interest to equilibrium outcomes for which the objective integrals in (5) and (6) converge for all admissible states, controls, and parameter values.

3. Open-loop Nash equilibrium strategies

This section identifies an open-loop Nash equilibrium (OLNE). The assumption here is that information on the backlogs as well as the goodwill stock is not available and the firms' strategies depend on time only.

Omitting from now on the time argument when no confusion can arise, the manufacturer's (current-value) Hamiltonian is

$$H^M = \phi p(\alpha - \beta p + G) + \omega v - au^2/2 - bw^2/2 - cX^2/2$$
$$+\lambda_1(v-u) + \lambda_2(\alpha - \beta p + G - v) + \lambda_3(w - \delta G) \quad (7)$$

where $\lambda_1(t)$, $\lambda_2(t)$, and $\lambda_3(t)$ are the manufacturer's (current-value) costate variables, associated with state variables, X, Y, and G, respectively.

The costate equations are given by

$$\dot{\lambda}_1 = r\lambda_1 + cX \quad (8)$$

$$\dot{\lambda}_2 = r\lambda_2 \quad (9)$$

$$\dot{\lambda}_3 = (r+\delta)\lambda_3 - \lambda_2 - \phi p \quad . \quad (10)$$

The retailer's (current-value) Hamiltonian is

$$H^R = \bar{\phi} p(\alpha - \beta p + G) - \omega v - dv^2/2 - eY^2/2 + \mu_1(v-u) + \mu_2(\alpha - \beta p + G - v) + \mu_3(w - \delta G) \quad (11)$$

where $\mu_1(t)$, $\mu_2(t)$, and $\mu_3(t)$ are (current-value) costates.

The costate equations are

$$\dot{\mu}_1 = r\mu_1 \quad (12)$$

$$\dot{\mu}_2 = r\mu_2 + eY \quad (13)$$

$$\dot{\mu}_3 = (r+\delta)\mu_3 - \bar{\phi} p - \mu_2 \quad (14)$$

The state equations are as in (1), (2), and (4). For the case where optimal controls are positive, necessary optimality conditions are (1), (2), (4), (8)-(10), (12)-(14), and

$$H_u^M = 0 \Rightarrow u^{ol} = -\lambda_1/a \quad (15)$$

$$H_w^M = 0 \Rightarrow w^{ol} = \lambda_3/b \quad (16)$$

for the manufacturer, and

$$H_v^R = 0 \Rightarrow v^{ol} = -(\omega + \mu_2)/d \tag{17}$$

$$H_p^R = 0 \Rightarrow p^{ol} = \left[\left(\alpha + G^{ol}\right)\Big/\beta + \left(dv^{ol} + \omega\right)\Big/\bar{\phi}\right]\Big/2 \tag{18}$$

for the retailer. The superscript "*ol*" refers to "open-loop''. It is readily shown that the Hamiltonians are strictly concave in the decision variables, which guarantees unique maxima. Analysis of the necessary conditions provides the following results.

Lemma 1. For the case where optimal controls are positive, the manufacturer's OLNE production rate and advertising effort rate satisfy

$$\dot{u}^{ol} = ru^{ol} - cX^{ol}/a, \qquad u_\infty^{ol} = cX_\infty^{ol}/ra \tag{19}$$

$$\dot{w}^{ol} = (r+\delta)w^{ol} - \phi p^{ol}/b, \qquad w_\infty^{ol} = \phi p_\infty^{ol}/b(r+\delta) \tag{20}$$

where the subscript ∞ indicates that controls are computed in steady state.

Proof. See A1**.**

The second equation in (19) shows - as expected - that the larger the backlog, the larger the production rate in steady state. The second equation in (20) shows that, in steady state, the larger the consumer price, the larger the manufacturer's advertising effort. This is because price and goodwill have opposite effects on demand. The second equation in (20) also shows that a larger discounting rate implies a smaller steady state advertising effort. The intuition is that shortsighted manufacturers should not invest very much in long-run advertising goodwill.

Proposition 1. Under a WPC, the manufacturer's advertising effort rate is always zero. Under an RSC, the effort rate is always positive.

Proof. See A2**.**

The first part of Proposition 1 tells a manufacturer operating under a WPC that it is not worthwhile to invest in advertising (to increase consumer demand) because the manufacturer's gross revenue is independent of retail sales. Consequently, brand goodwill decreases steadily over time. With an RSC, the manufacturer has an incentive to raise brand goodwill - and hence consumer demand - because the manufacturer receives a positive share of the retailer's revenue.

Lemma 2. The retailer's OLNE purchase rate and consumer price satisfy

$$\dot{v}^{ol} = rv^{ol} - \left(eY^{ol} - r\omega\right)\Big/d, \qquad v_\infty^{ol} = \left(eY_\infty^{ol} - r\omega\right)\Big/rd \tag{21}$$

$$\dot{p}^{ol} = \frac{1}{2}\left(\dot{G}^{ol}/\beta + d\dot{v}^{ol}/\bar{\phi}\right), \qquad p_\infty^{ol} = \left[\left(\alpha + G_\infty^{ol}\right)\Big/\beta + eY_\infty^{ol}/r\bar{\phi}\right]\Big/2 \tag{22}$$

Proof. See A3**.**

Eq. (21) shows that in steady state, the larger the retailer's backlog the higher the purchase rate. Given that the transfer price is higher under a WPC than an RSC, the retailer has a lower steady state purchase rate under a WPC. This is intuitive. Using (22) shows that (in steady state) the larger the retailer's backlog, the higher the consumer price. Also this is intuitive: if the retailer is understocked, she should decrease demand by raising the consumer price.

Consider an RSC in steady state and use (3), (20), and (22). A higher consumer price leads (by (3)) to lower consumer demand but it also implies a higher level of manufacturer advertising (by (20)). More advertising implies a higher stock of goodwill and (by (3)) greater consumer demand. By (22), a higher goodwill stock implies a higher consumer price. The managerial implication of these effects is that should – for any reason - the retailer increase the consumer price, the resulting decrease in demand/sales will be mitigated by the manufacturer, who responds by increasing advertising to make the goodwill stock larger and thereby stimulate consumer demand.

The next proposition shows that an equilibrium under a WPC exhibits nice structural properties.

Proposition 2. Under a WPC, the steady state is unique and the equilibrium path converging to the steady state is monotonic.

Proof. See A4.

Under a WPC, steady state values for production and backlogging, as well as procurement and backlogging, are all strictly positive if the transfer price is not larger than the steady state consumer price, i.e., $p^{ol}_{\infty|WPC} \geq \omega_{|WPC}$ This requirement is likely to be satisfied in practice; one would not expect a retailer to have a negative gross margin.

Technically, long-run feasibility and local stability of the equilibrium under a WPC did not require additional assumptions. We conclude that in steady state, backlogs are constant and consumer demand equals production, which in turn equals the purchase rate. Advertising effort as well as the goodwill stock are zero.

Proposition 3. Under an RSC, the supply chain has a unique steady state. This steady state is feasible and is a saddle point in the control-state space if the following conditions are satisfied

$$r < 1/\beta\delta b - \delta \text{ and } \phi < \tilde{\phi}$$

where $\tilde{\phi}$ is an upper threshold of ϕ that is strictly lower than 1. Under the two conditions, the equilibrium path converging to the steady state is monotonic.

Proof. See A5**.**

According to Proposition 3, the equilibrium path to the steady state under an RSC is monotonic if the sharing parameter is sufficiently small, $\phi \le \tilde{\phi}$. The parameter ϕ may be seen as a proxy for the bargaining position for the manufacturer. If ϕ is higher than $\tilde{\phi}$, the steady state cannot be reached from some or all initial states (Engwerda, 1998). The implication is that the RSC is not superior to a WPC as a long-term contract in an OLNE because it does not ensure stability while WPC does. This interesting feature of the RSC was not envisioned in the literature on supply chain contracting and coordination in static or short-term setups (Cachon, 2003; Cachon and Lariviere, 2005; El Ouardighi *et al*., 2008; El Ouardighi and Kim, 2010). Long-run feasibility and stability of an equilibrium under an RSC can be ensured only by contracts that have a sharing parameter being lower than the threshold $\tilde{\phi}$. In addition, the inequality $r < 1/\beta\delta b - \delta$ must hold. The interpretation of this inequality is that the discount rate must not be "too large", that is, supply chain members must be relatively farsighted.

In the case where the threshold for the sharing parameter is close to 1, the manufacturer's production rate and the retailer's purchase and sales rates are close to zero. This situation is sustainable, however, because the retailer is able to compensate the high manufacturer share in the sales revenue by charging a high consumer price, which is backed up by a substantial manufacturer advertising effort. (Technically, the local monotonicity of the equilibrium path can still be assured, though globally the paths may be non-monotonic).

It can be shown that whatever the compensation scheme, the manufacturer's steady state backlog rate, relative to the retailer's steady state purchase rate, i.e., $X_\infty^{ol}/v_\infty^{ol}$, equals ra/c (see A.5). However, the retailer's steady state backlog rate, relative to consumer sales, i.e., $Y_\infty^{ol}/S_\infty^{ol}$, is lower under an RSC than a WPC if $\phi \le \left(\omega_{|WPC} - \omega_{|RSC}\right)/\omega_{|WPC}$. If this inequality is satisfied, an RSC is more effective than a WPC in meeting demand in the sense that relative understocking at the retailer's outlet is reduced. This result confirms what happens in practice when an RSC is introduced. The main idea of an RSC is to enable the retailer to buy more units by lowering the transfer price, compared to a WPC. This reduces the likelihood of the retailer being understocked.

The condition $\phi \le \left(\omega_{|WPC} - \omega_{|RSC}\right)/\omega_{|WPC}$ can be illustrated by the data from the Blockbuster case in Mortimer (2008). We rewrite the condition as follows: $\bar{\phi} \ge \omega_{|RSC}/\omega_{|WPC}$. The left-hand side of this inequality is the fraction of retail revenue that the retailer keeps for herself while the right-hand side is a positive number, strictly less than 1. In the Blockbuster example, $\bar{\phi}$ is approximately 60% while the fraction on the right-hand side is \$8/\$65 = 0.12. Hence the condition is satisfied.

4. Feedback Nash equilibrium strategies

If information on backlogs and advertising goodwill is available to both firms throughout the game, they may condition their current actions on the current values of these stocks, as well as on time. In this case, one can look for a feedback Nash equilibrium (FBNE). To identify an FBNE, we derive the Hamilton-Jacobi-Bellman (HJB) equations of the firms and then characterize the equilibrium strategies. The model we have formulated has an infinite horizon and parameters are constant. In such a case, one can look for strategies that are stationary in the sense that they depend on the state but not on time (explicitly).

Let $V^M(X,Y,G)$ be the manufacturer's value function, which represents the optimal profit of the manufacturer in a game that starts out at time t in state $(X(t),Y(t),G(t))$. The manufacturer's HJB equation then is

$$rV^M = \underset{u,w}{\text{Max}}\Big\{\phi p(\alpha-\beta p+G)+\omega v-au^2/2-bw^2/2-cX^2/2 \\ +V_X^M(v-u)+V_Y^M(\alpha-\beta p+G-v)+V_G^M(w-\delta G)\Big\} \quad (23)$$

where subscripts on V^M denote partial differentiation. Necessary conditions for a maximum on the right-hand side of (23) are, if production and advertising rates are positive,

$$u^{fb} = -V_X^M/a \quad (24)$$

$$w^{fb} = V_G^M/b \quad (25)$$

where the superscript "*fb*" refers to ''feedback''.

The retailer's HJB equation is

$$rV^R = \underset{v,p}{\text{Max}}\Big\{\bar{\phi}p(\alpha-\beta p+G)-\omega v-dv^2/2-eY^2/2+V_X^R(v-u)+V_Y^R(\alpha-\beta p+G-v)+V_G^R(w-\delta G)\Big\} \quad (26)$$

where $V^R(X,Y,G)$ is the retailer's value function. Necessary conditions for a maximum are, if the purchase rate and the consumer price are positive,

$$v^{fb} = \left(V_X^R - V_Y^R - \omega\right)/d \quad (27)$$

$$p^{fb} = \left[(\alpha+G)/\beta - V_Y^R/\bar{\phi}\right]/2 \quad (28)$$

Proposition 4. Whenever all decision variables are positive, FBNE strategies for production, advertising effort, procurement, and consumer price are as follows

$$u^{fb} = -\left(B^M + 2C^M X + I^M Y + J^M G\right)\Big/a \tag{29}$$

$$w^{fb} = \left(F^M + J^M X + K^M Y + 2H^M G\right)\Big/b \tag{30}$$

$$v^{fb} = \left[B^R - D^R - \omega + \left(2C^R - I^R\right)X + \left(I^R - 2E^R\right)Y + \left(J^R - K^R\right)G\right]\Big/d \tag{31}$$

$$p^{fb} = \frac{1}{2}\left[\frac{\alpha}{\beta} - \frac{D^R}{\bar{\phi}} - \frac{I^R}{\bar{\phi}}X - \frac{2E^R}{\bar{\phi}}Y + \left(\frac{1}{\beta} - \frac{K^R}{\bar{\phi}}\right)G\right] \tag{32}$$

in which $B^M, C^M, F^M, H^M, I^M, J^M, K^M, B^R, C^R, D^R, E^R, I^R, J^R, K^R$ are time-independent parameters of the two value functions

$$V^M(X,Y,G) = A^M + B^M X + C^M X^2 + D^M Y + E^M Y^2 + F^M G + H^M G^2 + I^M XY + J^M XG + K^M YG \tag{33}$$

$$V^R(X,Y,G) = A^R + B^R X + C^R X^2 + D^R Y + E^R Y^2 + F^R G + H^R G^2 + I^R XY + J^R XG + K^R YG \tag{34}$$

Proof. See A6**.**

The FBNE strategies are determined analytically by (29)-32) as linear functions of the three state variables. To determine the value function coefficients one has to solve a system of 14 nonlinear algebraic equations which does not seem to be possible. Therefore we use a numerical procedure. From the set of numerical solutions we select those that ensure a globally asymptotically stable steady state solution (if it exists).

Technical Remark. See A7.

For the numerical solutions we use the following baseline parameters.

Parameter	α	β	a	b	c	d	e	δ	r	ω	ϕ
Value	1 000	10	0.1	2	0.05	0.01	0.1	0.1	0.1	$\omega_{\mid \mathrm{WPC}} = 100$, $\omega_{\mid \mathrm{RSC}} = 0$	$\phi_{\mid \mathrm{WPC}} = 0$, $\phi_{\mid \mathrm{RSC}} = 0.8$

Table 1. Baseline parameters

In the supply chain management literature, it is usual to set the value of *r* between 0.05 (e.g., El Ouardighi and Erickson, 2015) and 0.1 (e.g., El Ouardighi, 2014). Here, we choose the upper value *r* = 0.1 to emphasize the relative importance of short time operational performances. Note that the wholesale price is set equal to zero in the RSC. In sensitivity analyses, a broad range of values is used for all parameters except ω and ϕ. For each parameter, the solutions are calculated for values deviating 25% and 50% above and below the baseline value. Neither the feasibility nor the qualitative pattern of the solutions generated was affected by variations up to 50% from the baseline values.

Technical Remark. See A8**.**

We use the optimal values of the contract parameters in an FBNE, that are derived in the next section (see 5.1). For an optimal WPC with transfer price $\omega = 73.98$, the eigenvalues of the Jacobian in (A8) are all real and negative, which ensures monotonic convergence to the stable steady state. The coefficients of the value functions for this solution are given in Table 2.

A^M	B^M	C^M	D^M	E^M	F^M	H^M	I^M	J^M	K^M
112 829.5367	-9.1361	-0.0327	59.244	-0.0215	127.5785	-0.0302	-0.0502	-0.034	-0.032
A^R	B^R	C^R	D^R	E^R	F^R	H^R	I^R	J^R	K^R
74405.9114	-2.199571	0.00003	-75.1883	-0.0151	88.6002	0.0651	0.0007	-0.00249	-0.0068

Table 2. Value function coefficients in a WPC

Using (29)-(32) and the values in Table 2, FBNE strategies with the optimal WPC are

$$u_{|WPC}^{fb} = 91.36123 + 0.32733X + 0.050286Y + 0.34015G \tag{35}$$

$$w_{|WPC}^{fb} = 63.78926 - 0.0170075X - 0.016004Y - 0.030274G \tag{36}$$

$$v_{|WPC}^{fb} = -99.1623 - 0.0019X + 3.0371Y + 0.4402G \tag{37}$$

$$p_{|WPC}^{fb} = 87.59417 - 0.0000395X + 0.015146Y + 0.053449G \tag{38}$$

According to (35), the production rate increases if the manufacturer or retailer backlog grows. This is to avoid understocking and is expected. A higher level of goodwill increases consumer demand which motivates increasing production.

Eq. (36) says that the manufacturer should use less advertising effort to promote consumer demand when there are backlogs or when the goodwill stock is already large. These results confirm intuition.

Using (37) shows that a larger manufacturer backlog (i.e., poorer availability) deters retailer procurement, as expected. The retailer buys more if her own backlog increases. Realizing that a higher level of goodwill increases consumer demand induces the retailer to purchase more.

According to (38), the retailer sets a lower consumer price when there is a backlog at the manufacturer level (due to insufficient supply). This is consistent with the manufacturer's decreasing advertising effort in such situations. In contrast, the retailer sets a higher price to lower demand when there is a backlog at the retail level. Finally, a higher level of goodwill increases consumer demand and justifies an increase in the consumer price.

For an optimal RSC with parameters $\omega = 0, \phi = 0.7$ (see Section 5.1 below), the eigenvalues associated with the stable steady state are real and negative and hence convergence to the steady state is monotonic. The values of the coefficients of the value functions under an RSC are given in Table 3.

A^M	B^M	C^M	D^M	E^M	F^M	H^M	I^M	J^M	K^M
112 599.0816	-36.9961	-0.0327	33.6214	-0.0201	101.9747	0.0321	-0.0458	-0.0327	-0.0295
A^R	B^R	C^R	D^R	E^R	F^R	H^R	I^R	J^R	K^R
113 011.2873	-2.1122	0.00001	-6.1348	-0.0144	93.9565	0.0275	0.0003	-0.001	-0.005

Table 3. Value function coefficients in an RSC

Using (29)-(32) and the values in Table 3, the strategies in an optimal RSC are

$$u^{fb}_{|RSC} = 369.96161 + 0.6551X + 0.45885Y + 0.32778G \tag{39}$$

$$w^{fb}_{|RSC} = 50.98738 - 0.016389X - 0.014757Y + 0.032101G \tag{40}$$

$$v^{fb}_{|RSC} = 402.262 - 0.0013X + 2.8889Y + 0.3985G \tag{41}$$

$$p^{fb}_{|RSC} = 60.22474 - 0.000065X + 0.048083Y + 0.058467G \tag{42}$$

Comparing the signs of the value function coefficients under a WPC in (35)-(38) with those of an RSC in (39)-(42) shows no differences, except for manufacturer advertising effort which depend negatively on goodwill under a WPC, but positively under an RSC. There is no straightforward explanation to this, noting that goodwill enters in a complicated manner in the value functions given by (33)-(34). With this exception we conclude that there are no qualitative differences between the way in which the state variables influence the operations and marketing decisions under the two contracts.

5. Comparisons of contracts and strategies

We assume, as would be the case in practice, that the choice of a contract precedes that of strategy type. We wish to assess the "coordination ability" of the two contracts, under open-loop and feedback strategies, respectively, and characterize the evolution of operations and marketing variables over time. For this purpose we use the analytical solutions from Sections 3 and 4. To see how efficient a given contract is in coordinating the supply chain, we need a benchmark which is taken to be optimal cooperative solution which is defined as the solution which maximizes the overall profit of the supply chain.

Payoffs are investigated when contract parameters and strategies vary. For each strategy, an optimal contract is first defined. Then, contracts and strategies are compared with the cooperative solution in terms of their payoffs and policies.

To determine the cumulative payoffs and the operations and marketing policies under each contract and each strategy, we use the baseline parameters of Table 1. Solution paths are calculated for initial state values $\left(X_0, Y_0, G_0\right) = \left(0, 0, 50\right)$. Thus the assumption is that firms start out with zero backlogs. For the numerical solution, a boundary value approach and a continuation algorithm, described in Grass *et al*. (2008) and Grass (2012) are used.

5.1. Optimal contracts and strategies

The reader should be aware that now we abandon the assumption that contract parameters have fixed values that are exogenously given. In this section we wish to determine an optimal contract for each type of strategy (open-loop and feedback). The meaning of ‘’optimal’’ will be made clear below.

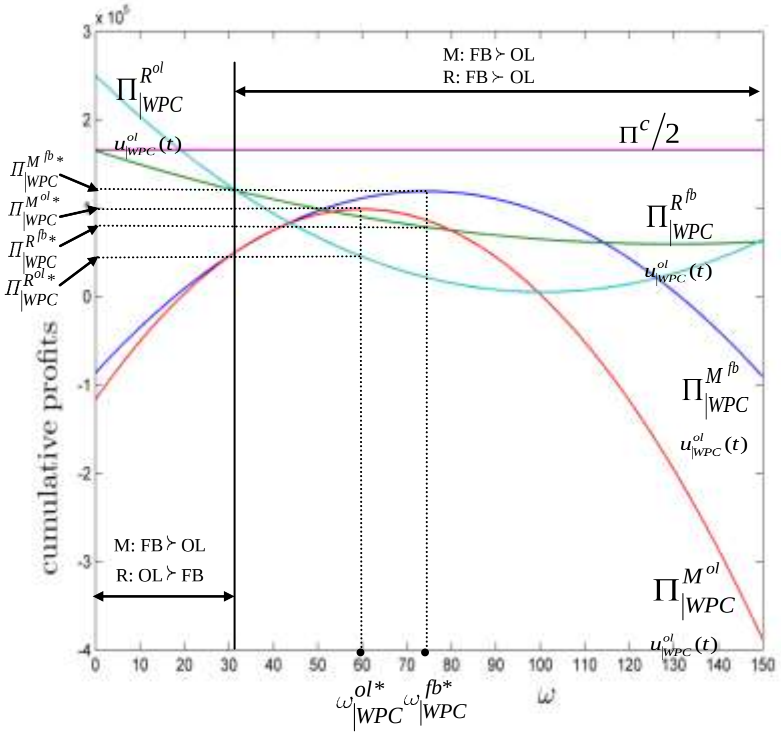


Figure 1. OLNE and FBNE payoffs in a WPC, as functions of the transfer price

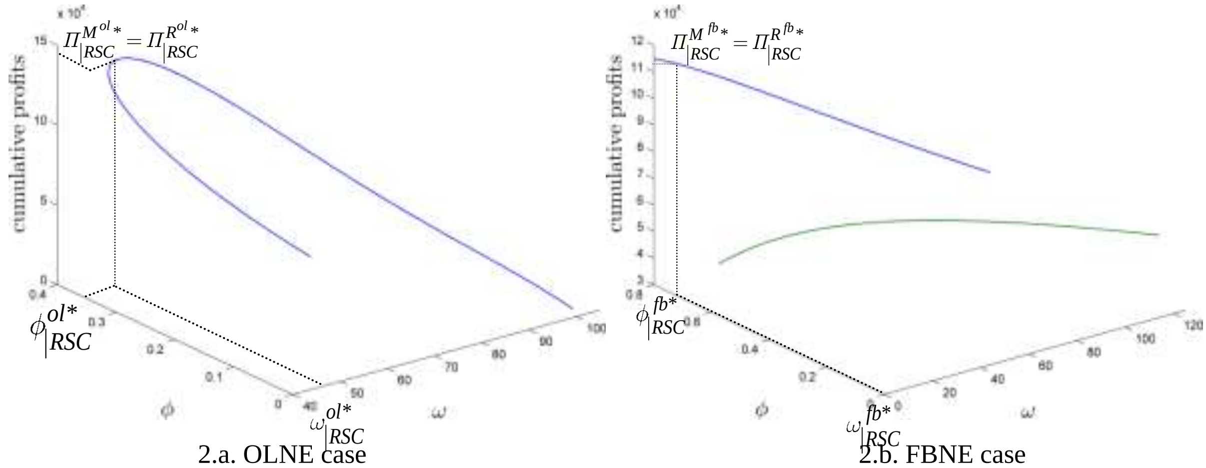
Starting with a WPC (that is, given the sharing parameter ϕ is zero) we vary the transfer price ω to find out what happens to manufacturer and retailer profits in an OLNE and a FBNE, respectively. Then, considering an RSC, we vary both the transfer price ω and the sharing parameter ϕ to answer the same question. As in El Ouardighi (2014), the manufacturer determines an optimal WPC by choosing a transfer price that maximizes her individual profit. The retailer can accept or reject the contract. Given that the retailer accepts the contract, Figure 1 shows the firms’ cumulative profits under WPC and both strategy types. Figure 1 depicts the manufacturer and retailer overall payoffs in a WPC and an OLNE, $\Pi^{M^{ol}}_{|WPC}$ and $\Pi^{R^{ol}}_{|WPC}$, respectively, as well as $\Pi^{M^{fb}}_{|WPC}$ and $\Pi^{R^{fb}}_{|WPC}$ in a WPC and a FBNE, respectively, as functions of the transfer price ω. For comparison, an uniform division of the cooperative profits, $\Pi^c/2$, is also depicted.

An optimal WPC is characterized both by a higher transfer price ($\omega^{fb*}_{|WPC} = 73.98 > \omega^{ol*}_{|WPC} = 60.15$) and payoff in a FBNE than in an OLNE, for both manufacturer, $\Pi^{M\,fb*}_{|WPC} = 1.19125 \times 10^5 > \Pi^{M\,ol*}_{|WPC} = 9.8865 \times 10^4$ and retailer, $\Pi^{R\,fb*}_{|WPC} = 7.8993 \times 10^4 > \Pi^{R\,ol*}_{|WPC} = 4.4626 \times 10^4$. Therefore, if the supply chain members are able to have and share state information, both firms prefer an optimal WPC and FBNE to an optimal WPC and OLNE because they get higher profits although it creates a greater double marginalization effect. The reason is that a higher transfer price in a FBNE leads to a higher consumer price, but - as a countermeasure to stimulate consumer demand - also induces the manufacturer to develop goodwill through increased advertising effort. A higher transfer price is desirable in a FBNE because it increases supply chain members' payoffs due to greater sales. This suggests that advertising in a FBNE could mitigate the effects of double marginalization on profits by stimulating the consumer demand. If the supply chain members do not share state information, or if the cost of getting and sharing state information is too large, that is, greater than $\Pi^{M\,fb*}_{|WPC} - \Pi^{M\,ol*}_{|WPC}$ for the manufacturer and $\Pi^{R\,fb*}_{|WPC} - \Pi^{R\,ol*}_{|WPC}$ for the retailer, an optimal WPC in an OLNE is the only option for both firms. Although the transfer price in this case is smaller than in a FBNE, double marginalization is stronger because the manufacturer cannot use advertising to mitigate its effects on sales and profits. The rationale behind this result is suggested by equation (10), which represents the rate of change of the manufacturer's marginal incentive to invest in goodwill advertising. Under WPC and OLNE, the manufacturer has no incentive to invest in advertising because the retailer's backlog and marginal sales revenue are both payoff-irrelevant for the manufacturer (i.e., both λ_2 and ϕ are zero). In an OLNE, the retailer's marginal sales revenue is payoff-relevant for the manufacturer if ϕ is positive, that is, under a RSC. Under a WPC, the retailer's backlog is payoff-relevant for the manufacturer if λ_2 is non-zero. In particular, if the retailer's backlog is implicitly beneficial for the manufacturer, it will increase sales through greater goodwill advertising. In return, greater sales will induce the retailer to increase its purchase rate from the manufacturer to incur lower backlog costs. Overall, the manufacturer invests in advertising under WPC if it has an interest in raising the retailer's backlog cost. Conversely, if the manufacturer cannot observe the retailer's backlog, advertising efforts are not beneficial.

The search for a mutually beneficial alternative to an optimal WPC should aim at minimizing the difference between the supply chain's cooperative and the sum of non-cooperative payoffs. The reason is that an optimal RSC is acceptable by both firms if it is profit Pareto-improving, that is, no firm gets a lower profit under an optimal RSC than under an optimal WPC. The joint cooperative profits are the upper bound on firms' profits and the Nash bargaining scheme (Dockner *et al.*, 2000) is used to determine the RSC parameters. The retailer computes and selects the value of the sharing parameter that minimizes the difference between the cooperative profit and the sum of non-cooperative payoffs, that is, $\Pi^C - \left(\Pi^M_{|RSC} + \Pi^R_{|RSC}\right)$.

To motivate the above we note that if the retailer sets the sharing parameter to maximize her own profit we can end up in a solution which is not Pareto-improving. That is, for a fixed transfer price being lower than that under a WPC, the best non-cooperative solution for the retailer might be to share as little revenue as possible with the manufacturer. For the manufacturer to agree to charge a transfer price which is lower than that under a WPC, the RSC should be profit Pareto-improving. Given that the manufacturer charges such a transfer price, the retailer has to choose a sharing parameter between 0 and 1 as neither 0 nor 1 are Pareto-improving, but there is an interval of values of the sharing parameter that is profit Pareto-improving. We know the upper bounds on the firms' profits and to get the best Pareto-improving RSC (given the value of the transfer price) we must find the value of the sharing parameter that makes individual profits as close as possible to their respective upper bounds. The adoption of a RSC is not to maximize the retailer's profits but to coordinate the SC. This is the way in which we compute the curves in Figure 2.

Under an RSC, one contract parameter needs to be exogenously specified (Cachon and Lariviere, 2005; Jørgensen, 2011). We assume that the manufacturer sets a more advantageous transfer price to the retailer than that under an optimal WPC. The retailer, in turn, computes the value of the sharing parameter that minimizes the difference between the supply chain's cooperative and the sum of non-cooperative payoffs. To identify an optimal RSC, we compute for each strategy the intersection points between the firms' payoffs curves that are at the shortest distance from the cooperative payoffs curve. Figure 2 represents the curves of these intersection points as functions of transfer price and sharing parameter in an OLNE (2.a) and an FBNE (2.b).



2.a. OLNE case 2.b. FBNE case

Figure 2. Optimal RSC in OLNE and FBNE

As expected, an optimal RSC is characterized by a strictly lower transfer price than an optimal WPC in OLNE or in FBNE. However, while the transfer price should be strictly positive in an OLNE ($\omega^{ol*}_{|RSC} = 45.5$), it is zero in an FBNE ($\omega^{fb*}_{|RSC} = 0$). The optimal sharing parameter is considerably larger in an FNBE than in an OLNE, $\phi^{fb*}_{|RSC} = 0.7 > \phi^{ol*}_{|RSC} = 0.35$. The firms' payoffs are significantly greater in an OLNE than in an FBNE, $\Pi^{M\,ol*}_{|RSC} = \Pi^{R\,ol*}_{|RSC} = 1.40689 \times 10^5 > \Pi^{M\,fb*}_{|RSC} = \Pi^{R\,fb*}_{|RSC} = 1.18 \times 10^5$. Both firms prefer an optimal RSC in an OLNE to an optimal RSC in an FBNE. Sensitivity analysis shows that this holds also true for smaller discounting rates and greater demand price-sensitivity. This suggests that farsightedness under an RSC increases the value of the commitment which is inherent in the OLNE. However, because $\Pi^{M\,fb*}_{|WPC} > \Pi^{M\,fb*}_{|RSC}$, an RSC in an FBNE is not profit Pareto-improving because the manufacturer prefers a WPC in an FBNE. Actually, an RSC in an FBNE is a dominated option for both firms. Under an RSC and OLNE, the state information is disregarded which makes it not only more profitable but also less constraining in terms of state information collecting and sharing than RSC and FBNE for both firms. Because both firms' profits are strictly greater under optimal RSC in an OLNE than under optimal WPC in an FBNE, $\Pi^{M\,ol*}_{|RSC} > \Pi^{M\,fb*}_{|WPC}$ and $\Pi^{R\,ol*}_{|RSC} > \Pi^{R\,fb*}_{|WPC}$, both firms are willing to adopt an RSC in an OLNE. Finally, if supply chain members are unable to condition their decisions upon the current state, they have a strong incentive to select an RSC because the increase in profits entails no additional constraints in terms of collecting and sharing state information. This result is notable, although it may be counterintuitive. Therefore, a switch from a WPC to an RSC may require a change in the strategy type, from a contingent to a commitment strategy.

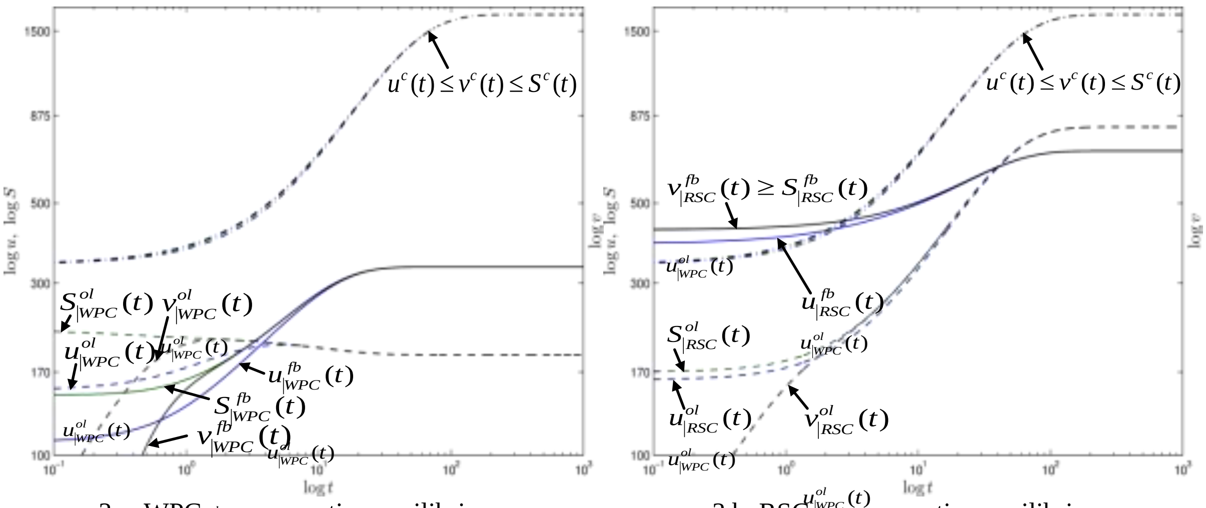
Each contract type enables the supply chain members to effectively "align their interests" so that they both prefer the same type of equilibrium—either a commitment or a contingent strategy—depending on the state information availability under a WPC, and a commitment strategy under an RSC.

5.2. Optimal operations and marketing policies

Supply chain members play, for any given contract, the Nash strategies identified above. To analyze the transient behavior of operations and marketing policies, we generate the time paths of the control variables under each contract (WPC and RSC) and strategy (open-loop and feedback).

In Figs. 3.a-3.b, the time paths of the manufacturer's production rate, the retailer's purchase rate and the sales rate are graphed on a logarithmic scale for the cooperative and non-cooperative equilibria. The time axis is also logarithmically scaled to show the paths in more detail on an initial interval of time where the significant dynamic changes take place.

In the cooperative setting (Figs. 3.a-3.b), operations decision variables and sales have S-shaped time paths. The production and the purchase rates are initially low and increase steadily over time until the steady state is reached in finite time. Though not quite visible, the transient sales are greater than the purchase rate, which in turn exceeds the production rate. This makes the backlogs of both firms increase. Except during an initial phase, the cooperative production, purchase and sales rates are greater than in the non-cooperative case.



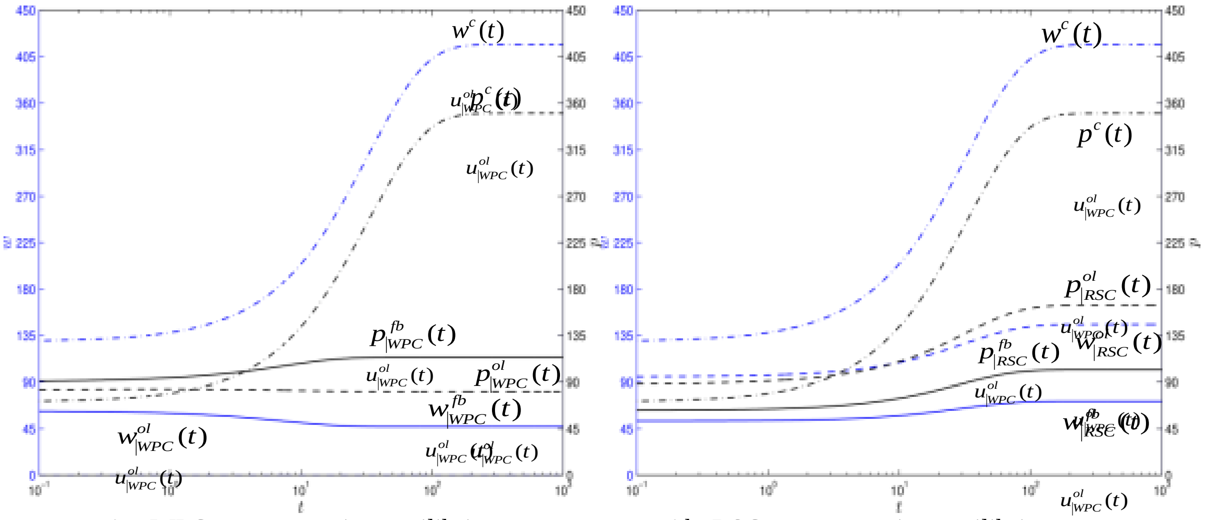
3.a. WPC *vs.* cooperative equilibrium 3.b. RSC *vs.* cooperative equilibrium

Figure 3. Operations policies and sales under optimal WPC and RSC in OLNE and FBNE *vs.* cooperative case

Under a WPC and OLNE (Fig. 3.a), operations policies and sales have quite different patterns than in the cooperative setting. Sales are (slightly) monotonically decreasing while the production and purchase rates first increase and then (slightly) decrease. Initially, the manufacturer holds a positive inventory because the production rate is greater than the retailer's purchase rate. Later on, the purchase rate surpasses the production rate for a finite time interval until they both equalize at the steady state. However, the purchase rate starts at a lower level than sales until the steady state. The production, purchase and sales rates in an OLNE are significantly lower than in FBNE during an initial period. With a WPC and FBNE (Fig. 3.a), convergence of these three variables is similar than to that of an OLNE, except that all time paths are increasing. The dynamic adjustment of the purchase and sales rates toward the steady state is similar to that in the cooperative solution.

Under an RSC (Fig. 3.b), operations policies in OLNE and FBNE, by and large, follow the same pattern as the cooperative solution. Yet they differ in that the OLNE strategy displays (inferior) parallel paths to the cooperative paths, while the FBNE paths are flatter with greater initial values than the cooperative equilibrium and lower steady state values than the OLNE.

Figs. 4.a-b show the time paths of the manufacturer's advertising effort and the consumer price for the cooperative solution and the non-cooperative equilibria. In the cooperative equilibrium, advertising effort and consumer price also have S-shaped time paths with increasing values until the steady state is reached. The marketing instruments are used in a way such that an increase in the consumer price (which decreases demand) goes along with an increase in advertising effort (which increases demand).



4.a. WPC *vs*. cooperative equilibrium 4.b. RSC *vs*. cooperative equilibrium

Figure 4. Marketing policies under optimal WPC and RSC in OLNE and FBNE *vs*. cooperative equilibrium

Under a WPC in an OLNE (Fig. 4.a), the advertising effort is zero while the consumer price decreases (slightly) over time. According to Eq. (22), the decreasing time path of the consumer price results from the decrease of both the manufacturer's goodwill and the retailer's purchase rate (see Fig. 3.a). The decreasing time path of sales (Fig. 3.a) is due to the fact that goodwill decreases faster than the consumer price. Under a WPC and FBNE, marketing instruments affect demand in the same way: An increase in the consumer price goes along with a decrease in advertising effort (both decrease demand).

Marketing instruments are strategic complements under an RSC in both OLNE and FBNE, as in the cooperative solution. For a given strategy type, the time paths of marketing strategies are affected by the contract type. With an RSC and OLNE, the marketing policy prescribes under-advertising and successively over-pricing over a (brief) initial time interval and under-pricing thereafter. Cooperation is the more effective way of increasing demand because cooperative advertising effort is greater.

A WPC and OLNE or FBNE will in general lead to lower consumer prices than an RSC and OLNE. In a static setting, Cachon and Lariviere (2005) obtain that the consumer price is lower with an RSC than a WPC. In a dynamic setting, the retailer can set a higher consumer price under an RSC by rewarding the manufacturer for her efforts to develop the market potential through increased goodwill. The strategic interaction between the two marketing instruments eliminates the need to reduce the consumer price, a standard argument for the use of an RSC. Finally, the strategy type with the highest consumer price is more profitable.

In Fig. 5, the phase diagram of the manufacturer's backlog and advertising goodwill and the retailer's backlog is depicted for the cooperative solution and the non-cooperative equilibria.

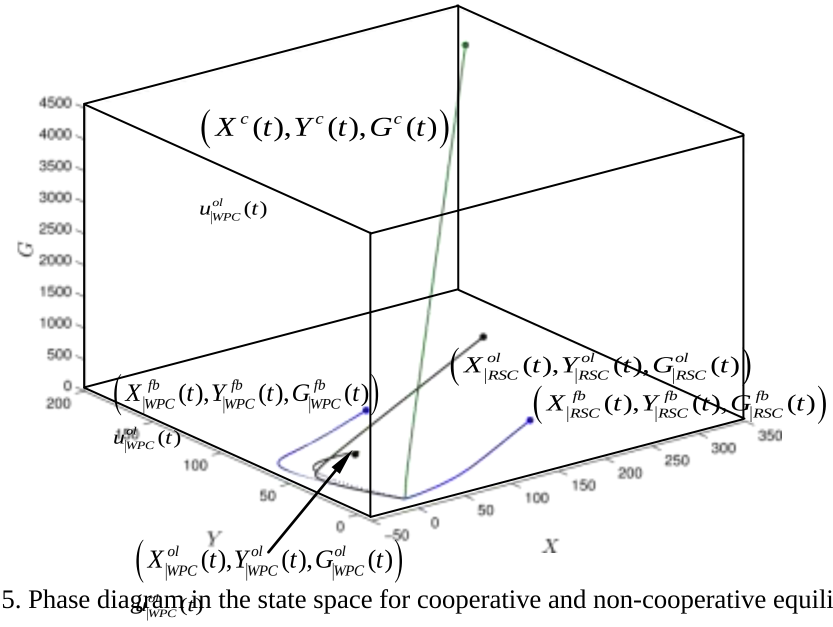
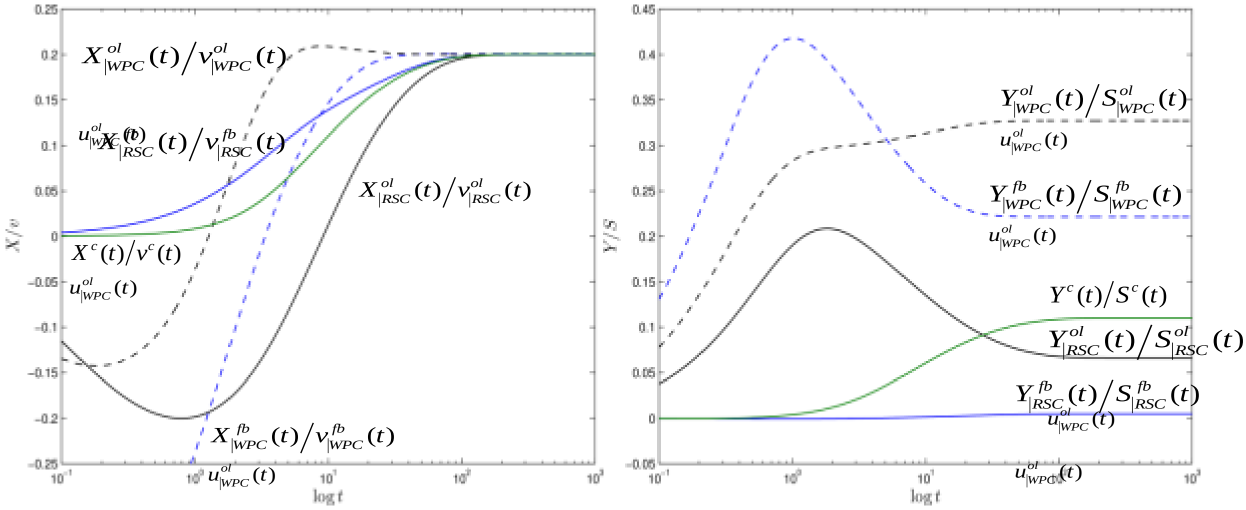


Figure 5. Phase diagram in the state space for cooperative and non-cooperative equilibria

Although cooperation is more effective than non-cooperation in developing demand (owing to higher advertising goodwill), it also leads to larger backlogs for the manufacturer and retailer. In the non-cooperative game, the case of a WPC and OLNE can be described in a way opposite to that of the cooperative solution because it leads to a lower sales rate, due to absence of advertising effort, as well as smaller backlogs for both firms. An RSC and OLNE has more similarities with the cooperative solution than a WPC and OLNE. Sensitivity analysis verifies this for low values of the production and advertising cost coefficients and high price-sensitivity of demand. An RSC and FBNE differs from a WPC and FBNE because the former results in a lower retailer backlog and greater manufacturer backlog and goodwill. This holds for low values of the production cost coefficient and the retailer's backlogging cost, high values of the advertising cost coefficient, and high price-sensitivity of demand.

Due to the differences between the cooperative and non-cooperative values of the retailer's purchase rate and sales volume, it is appropriate to assess understocking at the manufacturing and retailing levels in relative instead of absolute terms. To do so, we determine the transient paths of the relative backlog for each firm, that is, $X(t)/v(t)$ for the retailer, defined as the percentage of backlogged retailer's order at the manufacturer's plant, and $Y(t)/S(t)$ for the manufacturer, representing the percentage of backlogged consumer demand by the retailer. These time paths are illustrated in Figs. 6.a-6.b.



6.a. Manufacturer's relative backlog rate
6.b. Retailer's relative backlog rate

Figure 6. Manufacturer's and retailer's relative backlog rates in cooperative and non-cooperative equilibria

As suggested in Section 3, in an OLNE the contract type does not affect the manufacturer's steady state relative backlog. Fig. 6.a suggests that the manufacturer's steady state relative backlog, i.e., $X_\infty/v_\infty = ra/c$, is also independent of the strategy type in both the cooperative solution and the non-cooperative equilibria. However, the manufacturer's transient relative backlog shows significant differences between cooperation and non-cooperation. In a cooperative solution and under an RSC and FBNE, the manufacturer has no initial inventory and the convergence of the relative unavailability of the product to the steady state is slower in the cooperative case. In contrast, in an OLNE under both contracts and in an FBNE and an RSC, the manufacturer holds a positive inventory during an initial time interval. The inventory gradually turns into backlogging as time passes.

In Fig. 6.b, the retailer's steady state relative backlog is affected by both the compensation scheme and the type of game equilibrium. As suggested in Section 3, in an OLNE, the retailer's steady state relative backlog is greater under a WPC than an RSC if the manufacturer's relative share of the retailer's revenue does not exceed the relative reduction of the transfer price that the retailer obtain from the manufacturer under an RSC versus a WPC, i.e., if $\phi \leq \left(\omega_{|WPC} - \omega_{|RSC}\right) / \omega_{|WPC}$; This requirement is fulfilled in our case both for OLNE and FBNE. Fig. 6.b suggests that the retailer's relative backlog also is greater under a WPC and FBNE than an RSC and FBNE. A WPC and OLNE is the least effective in meeting demand, followed by a WPC and FBNE. At the steady state, relative backlogging at the retailer is even lower under an RSC and both strategy types than in the cooperative equilibrium. The lowest (relative) unavailability level at the retailer is observed for an RSC and FBNE. Overall, the relative availability is better under an RSC than under a WPC for both strategy types. However, the retailer's transient relative backlog evolves conversely to the manufacturer's rate under RSC and OLNE, as under WPC with both strategy types.

		Compensation scheme	
		Wholesale price contract	Revenue sharing contract
Decision rule	Open-loop strategy	Unadvertised, lower priced product with poorer marketing and operational performance	Higher advertised and priced product with greater marketing performance and lower operational performance
	Feedback strategy	Advertised and higher-priced product with greater marketing and operational performance	Lower advertised and priced product with lower marketing performance and greater operational performance

Tab. 4. Supply chain outcomes under WPC and RSC with open-loop and feedback strategies

The key results of our analysis are summarized in Table 4. The cooperative outcome reflects a vertically integrated supply chain that sells a heavily advertised, high-priced product where (relative) availability is of high concern. In contrast, under a WPC and OLNE, the supply chain sells an unadvertised, lower-priced product with poorer availability. As for a WPC and FBNE, the RSC and OLNE leads to higher-priced and advertised products. Marketing is important in both cases, and availability has a significantly higher priority under RSC and OLNE than under WPC and FBNE. Finally, an RSC and FBNE has the most effective operational performance in terms of relative availability.

6. Conclusions

In this paper, we analyze how the performances of wholesale price and revenue sharing contracts are affected by the information structure and the resulting decision rules in a supply chain in terms of operational and marketing decisions, and payoffs.

We point out an important, yet undisclosed merit of the wholesale price contract, that it is always a stable contract in the long run, regardless of whether state information is available or not in the supply chain. In contrast, when state information is unavailable in the supply chain, employing a revenue sharing contract is not optimal in the long term if the manufacturer's bargaining power is excessively large. Our results also suggest that the unavailability of information on the current state of key operational and marketing variables is more detrimental under wholesale price contract than under revenue sharing contract. An important implication of this is that it serves supply chain members better to share information on their respective inventories under a wholesale price than a revenue sharing contract.

The analysis of the operations/marketing game has provided a series of observations that should be useful for operations, marketing, and supply chain managers:

- Integrating marketing and operations decisions is strongly believed to be beneficial to the supply chain. We have demonstrated that using advertising to create goodwill leads to higher consumer prices and stimulates sales while maintaining high product availability.
- Noncooperative behavior leads to under-investment in goodwill and a lower consumer price. If supply chain firms play a noncooperative game, the type of strategy that better enhances individual profits depends on the type of contract chosen. With a wholesale price contract, feedback strategies provide better outcomes than open-loop strategies. Under a revenue sharing contract, open-loop strategies provide preferable results for all firms.
- With a wholesale price contract, feedback strategies mitigates the double marginalization problem because an increase in price does not deter growth of demand. While such strategies are effective in expanding sales, their effectiveness in meeting demand is limited. In such a situation, supply chain members should try to improve, throughout the game, the mutual availability of information on the current state of key operational and marketing variables of the supply chain.
- With a revenue sharing contract, open-loop strategies provide results that are closest to those of a cooperative strategy. Such strategies are effective both in expanding and meeting demand. Supply chain members could profitably precommit to a plan of action during the entire game if the manufacturer's bargaining power is not excessively large and supply chain members are farsighted.

The current research contributes to the understanding of the role of contracts, the use of information in a supply chain, and the implications for supply chain members' operations and marketing strategies. The importance of the availability of information and the strategies used by supply chain members under dynamic conditions has received little attention in the literature. Moreover, the literature seems to have followed two disparate streams, one in marketing and another in operations management. This paper has demonstrated the importance of the informational basis on which marketing and operations strategies are designed and the role of the contact under which decisions are made. The research also contributes to the stream of literature which views marketing and operational decision-making from an integrated point of view.

Appendix

A1. First, note that the manufacturer's backlog (for both positive and negative values) has a negative influence on its objective function because backlogging is costly and hence we expect that its costate λ_1 is negative. Next, because the manufacturer's goodwill has no (positive) influence on its objective under a WPC (RSC), λ_3 is expected to be zero (positive). For λ_2, equation (9) can be explicitly solved and has the solutions $\lambda_2 = 0$ and $\lambda_2 = Ce^{rt}$, where C is an arbitrary, nonzero constant of integration. Equations (15) and (16) show the following. A positive value of the manufacturer's production rate requires a negative value of the costate λ_1. A positive value of the manufacturer's advertising effort requires a positive value of the costate variable λ_3. The manufacturer's optimal decisions in (15)-(16) do not depend on λ_2 and henceforth we choose the zero solution for λ_2. Differentiating (15) wrt time and using (8) and (15) gives (19). Differentiating (16) wrt time and using (10) and (16) yields (20). □

A2. Regarding the first part of the proposition, note that WPC implies $\phi = 0$ and, in (20), $\dot{w}^{ol} = (r+\delta)w^{ol}$ and $w_{\infty}^{ol} = 0$. The equation $\dot{w}^{ol} = (r+\delta)w^{ol}$ has the general solution $w^{ol}(t) = De^{(r+\delta)t}$, where D is a constant of integration. For $D = 0$, the solution is $w^{ol} = 0$, while $D > 0$ makes $w^{ol} \mapsto \infty$ when $t \mapsto \infty$. Since $w^{ol}(0) \geq 0$, $D < 0$ is not possible, which proves that $w^{ol} = 0$. For the second part of the proposition, we note that, in RSC the term ϕp in (10) is positive when we exclude the possibility that $p^{ol} = 0$. We know that λ_3 non-negative is necessary for $w^{ol} \geq 0$. Suppose $\lambda_3 = 0$. Then the optimality condition (10) cannot hold. Hence λ_3 must be positive and the second part of the proposition follows from (16). □

A3. A similar argument as the one used for costate λ_2 leads to choose the solution $\mu_1 = 0$ in (12). The retailer's backlog has a negative influence on its objective, which suggests that the costate μ_2 should be negative. Finally, because goodwill positively influences the retailer's objective, whatever the compensation scheme, μ_3 is expected to be positive. According to (17), a positive value of the retailer's purchase rate requires $-\mu_2 \geq \omega$, which means that the retailer's unit purchase cost is lower than the imputed cost of backlogging one unit at the retail level. (The consumer price stated in (18) is clearly positive). Differentiating (17) wrt time and using (13) and (17) gives (21). Differentiating (18) wrt time and using (4), (20) and (21) yields (22). □

A4. To compute the steady state under WPC, we form the associated canonical system in the state-control space. To do so, we plug the RHS of (18) for p in the RHS of (20) and (2) to eliminate the equation of p. We obtain the following system of 6 linear equations

$$\begin{bmatrix} \dot{X}^{ol}_{|WPC} \\ \dot{Y}^{ol}_{|WPC} \\ \dot{G}^{ol}_{|WPC} \\ \dot{u}^{ol}_{|WPC} \\ \dot{w}^{ol}_{|WPC} \\ \dot{v}^{ol}_{|WPC} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & -1-\frac{\beta d}{2} \\ 0 & 0 & -\delta & 0 & 1 & 0 \\ -\frac{c}{a} & 0 & 0 & r & 0 & 0 \\ 0 & 0 & 0 & 0 & r+\delta & 0 \\ 0 & -\frac{e}{d} & 0 & 0 & 0 & r \end{bmatrix} \begin{bmatrix} X^{ol}_{|WPC} \\ Y^{ol}_{|WPC} \\ G^{ol}_{|WPC} \\ u^{ol}_{|WPC} \\ w^{ol}_{|WPC} \\ v^{ol}_{|WPC} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\alpha+\beta\omega}{2} \\ 0 \\ 0 \\ 0 \\ \frac{r\omega}{d} \end{bmatrix} \tag{A4.1}$$

The resolution of (A4.1) along with (18) gives

$$\begin{bmatrix} u^{ol}_{\infty|WPC} & w^{ol}_{\infty|WPC} & v^{ol}_{\infty|WPC} & p^{ol}_{\infty|WPC} & X^{ol}_{\infty|WPC} & Y^{ol}_{\infty|WPC} & G^{ol}_{\infty|WPC} \end{bmatrix}^T =$$
$$\begin{bmatrix} \frac{\alpha-\beta\omega_{|WPC}}{2+\beta d} & 0 & \frac{\alpha-\beta\omega_{|WPC}}{2+\beta d} & \frac{(1+\beta d)\alpha+\beta\omega_{|WPC}}{\beta(2+\beta d)} & \frac{ra(\alpha-\beta\omega_{|WPC})}{c(2+\beta d)} & \frac{r(d\alpha+2\omega_{|WPC})}{e(2+\beta d)} & 0 \end{bmatrix}^T \tag{A.4.2}$$

To check the stability of the system, we find the roots of the characteristic equation associated with the Jacobian in (A4.1), that is (Grass *et al.*, 2008),

$$\xi^{ol}_{1|WPC} = -\delta,\ \xi^{ol}_{2|WPC} = r+\delta,\ \xi^{ol}_{3,4|WPC} = r/2 \pm \sqrt{r^2/4 + e(\beta d+2)/2d},\ \xi^{ol}_{5,6|WPC} = r/2 \pm \sqrt{r^2/4 + c/a} \tag{A1.3}$$

All the eigenvalues are real, 3 having a positive sign and 3 having a negative sign. The saddle-point property of the steady state is thus granted and the path converging to it is monotonic. □

A5. Under RSC, the canonical system in the state-control space is

$$\begin{bmatrix} \dot{X}^{ol}_{|RSC} \\ \dot{Y}^{ol}_{|RSC} \\ \dot{G}^{ol}_{|RSC} \\ \dot{u}^{ol}_{|RSC} \\ \dot{w}^{ol}_{|RSC} \\ \dot{v}^{ol}_{|RSC} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & -1-\frac{\beta d}{2\bar{\phi}} \\ 0 & 0 & -\delta & 0 & 1 & 0 \\ -\frac{c}{a} & 0 & 0 & r & 0 & 0 \\ 0 & 0 & -\frac{\phi}{2\beta b} & 0 & r+\delta & -\frac{\phi d}{2\bar{\phi} b} \\ 0 & -\frac{e}{d} & 0 & 0 & 0 & r \end{bmatrix} \begin{bmatrix} X^{ol}_{|RSC} \\ Y^{ol}_{|RSC} \\ G^{ol}_{|RSC} \\ u^{ol}_{|RSC} \\ w^{ol}_{|RSC} \\ v^{ol}_{|RSC} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{2}\left(\alpha + \frac{\beta\omega_{|RSC}}{\bar{\phi}}\right) \\ 0 \\ 0 \\ -\frac{\phi}{2b}\left(\frac{\alpha}{\beta} - \frac{\omega_{|RSC}}{\bar{\phi}}\right) \\ \frac{r\omega_{|RSC}}{d} \end{bmatrix} \tag{A5.1}$$

The solution of the system, along with (18), gives

$$u^{ol}_{\infty|RSC} = \frac{\beta\delta b(r+\delta)\bar{\phi}\alpha + [\phi - \beta\delta b(r+\delta)]\beta\omega_{|RSC}}{\beta\delta b(r+\delta)(2\bar{\phi}+\beta d) - \phi(\bar{\phi}+\beta d)} \quad \text{(A5.2)}$$

$$w^{ol}_{\infty|RSC} = \frac{\phi\delta\left[(\bar{\phi}+\beta d)\alpha + \beta\omega_{|RSC}\right]}{\beta\delta b(2\bar{\phi}+\beta d)(r+\delta) - \phi(\bar{\phi}+\beta d)} \quad \text{(A5.3)}$$

$$v^{ol}_{\infty|RSC} = \frac{\beta\delta b(r+\delta)\bar{\phi}\alpha + [\phi - \beta\delta b(r+\delta)]\beta\omega_{|RSC}}{\beta\delta b(r+\delta)(2\bar{\phi}+\beta d) - \phi(\bar{\phi}+\beta d)} \quad \text{(A5.4)}$$

$$p^{ol}_{\infty|RSC} = \frac{\delta b(r+\delta)\left[(\bar{\phi}+\beta d)\alpha + \beta\omega_{|RSC}\right]}{\beta\delta b(r+\delta)(2\bar{\phi}+\beta d) - \phi(\bar{\phi}+\beta d)} \quad \text{(A5.5)}$$

$$X^{ol}_{\infty|RSC} = \frac{ra\left\{\beta\delta b(r+\delta)\bar{\phi}\alpha + [\phi - \beta\delta b(r+\delta)]\beta\omega_{|RSC}\right\}}{c\left[\beta\delta b(r+\delta)(2\bar{\phi}+\beta d) - \phi(\bar{\phi}+\beta d)\right]} \quad \text{(A5.6)}$$

$$Y^{ol}_{\infty|RSC} = \frac{r\bar{\phi}\left\{\beta\delta b(r+\delta)d\alpha + [2\beta\delta b(r+\delta) - \phi]\omega_{|RSC}\right\}}{e\left[\beta\delta b(r+\delta)(2\bar{\phi}+\beta d) - \phi(\bar{\phi}+\beta d)\right]} \quad \text{(A5.7)}$$

$$G^{ol}_{\infty|RSC} = \frac{\phi\left[(\bar{\phi}+\beta d)\alpha + \beta\omega_{|RSC}\right]}{\beta\delta b(r+\delta)(2\bar{\phi}+\beta d) - \phi(\bar{\phi}+\beta d)} \quad \text{(A5.8)}$$

We check the local stability of the steady state. Because the number of negative eigenvalues is at most three, a change in signs implies that the number of negative eigenvalues is reduced and therefore the number of stable eigendirections of the steady state declines, making the locally stable steady state unstable. Because the determinant of the Jacobian is given as the product of the eigenvalues, and three negative eigenvalues exist for $\phi = 0$ (see **A4**), the determinant becomes zero if one of the eigenvalues becomes zero for $\phi > 0$. Due to the sparsity of the Jacobian in (A5.1), the determinant is given as

$$\left|J_{|RSC}\right| = ce[\phi(\bar{\phi}+\beta d) - \beta\delta b(2\bar{\phi}+\beta d)(r+\delta)]/2\bar{\phi}\beta abd \quad \text{(A5.9)}$$

It then suffices to find the zero values of (A5.9). For ϕ, this leads us to find the roots of

$$-\phi^2 + \phi[1+\beta d + 2\beta\delta b(r+\delta)] - \beta\delta b(2+\beta d)(r+\delta) = 0 \quad \text{(A5.10)}$$

that is,

$$\phi_{1,2} = \beta\delta b(r+\delta) + (1+\beta d)/2 \pm \sqrt{\beta\delta b(r+\delta)[\beta\delta b(r+\delta)-1] + (1+\beta d)^2/4} \quad \text{(A5.11)}$$

Five scenarios are then possible, depending on the value of the discriminant $D \equiv \beta\delta b(r+\delta)[\beta\delta b(r+\delta)-1] + (1+\beta d)^2/4$ in (A5.11), that is,

- $D < 0$, which implies that the number of negative and positive eigenvalues does not change;
- $D > 0$ so that either $0 < \phi_1 < \phi_2 < 1$ or $\phi_1 < 0 < \phi_2 < 1$ exists;
- $D > 0$ so that either $\phi_1 < 0 < 1 < \phi_2$ or $0 < \phi_1 < 1 < \phi_2$ exists.

If $\beta\delta b(r+\delta)\geq 1$, it is obvious that $D>0$. Conversely, if $\beta\delta b(r+\delta)<1$, we have: $\beta\delta b(r+\delta)[\beta\delta b(r+\delta)-1]\geq -1/4$, which also implies that $D>0$. Therefore, the first scenario is invalid. Regarding the second scenario, it can be shown that $0<\phi_1<1$ if $\beta\delta b(r+\delta)<1$, which invalidates the third and fourth scenarios, that is, $\phi_1<0<\phi_2<1$ and $\phi_1<0<1<\phi_2$. However, if $\beta\delta b(r+\delta)\geq 1$, we get $\phi_2>1$. Conversely, if $\beta\delta b(r+\delta)<1$, we have

$$\phi_2 \geq \beta\delta b(r+\delta)+1/2+\sqrt{1/4-\beta\delta b(r+\delta)[1-\beta\delta b(r+\delta)]}$$
$$=\beta\delta b(r+\delta)+1/2+\sqrt{(1/2-\beta\delta b(r+\delta))^2}=\beta\delta b(r+\delta)+1/2+\sqrt{(\beta\delta b(r+\delta)-1/2)^2} \quad \text{(A5.12)}$$

If $\beta\delta b(r+\delta)>1/2$, then we get

$$\phi_2 \geq \beta\delta b(r+\delta)+1/2+\beta\delta b(r+\delta)-1/2=2\beta\delta b(r+\delta)>1 \quad \text{(A5.13)}$$

Conversely, if $\beta\delta b(r+\delta)\leq 1/2$, we obtain

$$\phi_2 \geq \beta\delta b(r+\delta)+1/2+1/2-\beta\delta b(r+\delta)=1 \quad \text{(A5.14)}$$

Therefore, the second scenario according to which $0<\phi_1<\phi_2<1$ is also invalid. Finally, only the smallest root in (A5.11), denoted as $\tilde{\phi}\equiv\phi_1$, where $\phi_1<1$ requires that $\beta\delta b(r+\delta)<1$, should be considered as a bifurcation threshold, which validates the fifth scenario whereby $0<\phi_1<1<\phi_2$, so that for $0<\phi<\tilde{\phi}$, the number of negative eigenvalues cannot change. Next, requiring non-negativity of the manufacturer's production and the retailer's purchase rate under RSC, we get $\phi\leq\dfrac{\delta b(r+\delta)(\alpha-\beta\omega_{|RSC})}{\delta b(r+\delta)\alpha-\omega_{|RSC}}<1$. It can be shown that for $\dfrac{\delta b(r+\delta)(\alpha-\beta\omega_{|RSC})}{\delta b(r+\delta)\alpha-\omega_{|RSC}}<\tilde{\phi}$ to hold, it is necessary that $\beta\delta b(r+\delta)>1$. This contradicts the condition that $\beta\delta b(r+\delta)<1$, and proves that $\phi<\tilde{\phi}<1$ is a necessary and sufficient condition for the feasibility of the steady state. □

A6. Substituting from (24)-(25) and (27)-(28) into (23) and (26) yields:

$$rV^M=-\frac{cX^2}{2}+\frac{\phi G^2}{4\beta}+\left(\frac{\phi\alpha}{2\beta}+\frac{V_Y^M}{2}-\delta V_G^M\right)G+\frac{V_X^{M2}}{2a}+\frac{V_G^{M2}}{2b}-\frac{\phi\beta V_Y^{R2}}{4\bar{\phi}^2}+\frac{V_X^MV_X^R}{d}-\frac{V_X^MV_Y^R}{d}$$
$$-\frac{V_Y^MV_X^R}{d}+\left(\frac{1}{d}+\frac{\beta}{2\bar{\phi}}\right)V_Y^MV_Y^R-\frac{\omega V_X^M}{d}+\left(\frac{\alpha}{2}+\frac{\omega}{d}\right)V_Y^M+\frac{\omega V_X^R}{d}-\frac{\omega V_Y^R}{d}+\frac{\phi\alpha^2}{4\beta}-\frac{\omega^2}{d} \quad \text{(A6.1)}$$

$$rV^R=-\frac{eY^2}{2}+\frac{\bar{\phi}G^2}{4\beta}+\left(\frac{\bar{\phi}\alpha}{2\beta}+\frac{V_Y^R}{2}-\delta V_G^R\right)G+\frac{V_X^{R2}}{2d}+\left(\frac{\beta}{4\bar{\phi}}+\frac{1}{2d}\right)V_Y^{R2}+\frac{V_X^MV_X^R}{a}-\frac{V_X^RV_Y^R}{d}$$
$$+\frac{V_G^RV_G^M}{b}-\frac{\omega V_X^R}{d}+\left(\frac{\alpha}{2}+\frac{\omega}{d}\right)V_Y^R+\frac{\bar{\phi}\alpha^2}{4\beta}+\frac{\omega^2}{2d} \quad \text{(A6.2)}$$

We need to establish the existence of bounded and continuously differentiable value functions V^M and V^S that solve the HJB equations as well as unique and nonnegative solutions *X*, *Y*, and *G* to the state equations. To do so, we make the following conjectures

$$V^M = A^M + B^M X + C^M X^2 + D^M Y + E^M Y^2 + F^M G + H^M G^2 + I^M XY + J^M XG + K^M YG \quad \text{(A6.3)}$$

$$V^R = A^R + B^R X + C^R X^2 + D^R Y + E^R Y^2 + F^R G + H^R G^2 + I^R XY + J^R XG + K^R YG \quad \text{(A6.4)}$$

from which we obtain

$$V_X^M = B^M + 2C^M X + I^M Y + J^M G \quad \text{(A6.5)}$$

$$V_Y^M = D^M + 2E^M Y + I^M X + K^M G \quad \text{(A6.6)}$$

$$V_G^M = F^M + 2H^M G + J^M X + K^M Y \quad \text{(A6.7)}$$

$$V_X^R = B^R + 2C^R X + I^R Y + J^R G \quad \text{(A6.8)}$$

$$V_Y^R = D^R + 2E^R Y + I^R X + K^R G \quad \text{(A6.9)}$$

$$V_G^R = F^R + 2H^R G + J^R X + K^R Y \quad \text{(A6.10)}$$

Plugging the RHS of (A6.5) and (A6.7) in (24) and (25), respectively, gives (29) and (30). Plugging the RHS of (A6.8) and (A6.9) in (27), and the RHS of (A6.9) in (28) and rearranging, respectively, yields (31) and (32). Inserting the RHS of (A6.5)-(A6.10) into (A6.1)-(A6.2), and equating coefficients, we develop a system of 20 connected and quadratic equations in $A^M,...,K^M, A^R,...,K^R$, which can be solved numerically with given values for the parameters. This leads to the following coupled quadratic equations

$$rA^M = B^{M2}/2a + F^{M2}/2b - \phi\beta D^{R2}/4\bar{\phi}^2 - \left[\omega\left(\omega + B^M - B^R + D^R\right) + B^M D^R + D^M B^R - B^M B^R\right]/d$$
$$+ (1/d + \beta/2\bar{\phi})\, D^M D^R + (\alpha/2 + \omega/d)\, D^M + \phi\alpha^2/4\beta \quad \text{(A6.11)}$$

$$rB^M = \left[\omega\left(2C^R - 2C^M - I^R\right) + 2\left(B^M C^R + C^M B^R\right) - B^M I^R - 2C^M D^R - 2D^M C^R - I^M B^R\right]/d$$
$$+ F^M J^M/b + (1/d + \beta/2\bar{\phi})\left(D^M I^R + I^M D^R\right) + 2B^M C^M/a - \phi\beta D^R I^R/2\bar{\phi}^2 + (\alpha/2 + \omega/d)\, I^M \quad \text{(A6.12)}$$

$$rC^M = 2C^{M2}/a + J^{M2}/2b - \phi\beta I^{R2}/4\bar{\phi}^2 + (1/d + \beta/2\bar{\phi})\, I^M I^R - \left[2\left(C^M I^R + I^M C^R\right) - 4C^M C^R\right]/d - c/2 \quad \text{(A6.13)}$$

$$rD^M = \left[\omega\left(I^R - I^M - 2E^R\right) + \left(B^M I^R + I^M B^R\right) - 2B^M E^R - D^R I^M - D^M I^R - 2E^M B^R\right]/d$$
$$+ F^M K^M/b + 2(1/d + \beta/2\bar{\phi})\left(E^M D^R + D^M E^R\right) + B^M I^M/a - \phi\beta D^R E^R/\bar{\phi}^2 + 2(\alpha/2 + \omega/d)\, E^M \quad \text{(A6.14)}$$

$$rE^M = I^{M2}/2a + K^{M2}/2b - \phi\beta E^{R2}/\bar{\phi}^2 + (4/d + 2\beta/\bar{\phi})\, E^M E^R - \left[2\left(I^M E^R + E^M I^R\right) - I^M I^R\right]/d \quad \text{(A6.15)}$$

$$rF^M = \left[\omega\left(J^R - J^M - K^R\right) + \left(B^M J^R + J^M B^R\right) - B^M K^R - D^R J^M - D^M J^R - K^M B^R\right]/d + \left(2H^M/b - \delta\right)F^M$$

$$+\left(1/d+\beta/2\bar{\phi}\right)\left(D^M K^R+K^M D^R\right)+B^M J^M/a-\phi\beta D^R K^R/2\bar{\phi}^2+D^M/2+\left(\alpha/2+\omega/d\right)K^M+\phi\alpha/2\beta \tag{A6.16}$$

$$rH^M=2H^{M2}/b+J^{M2}/2a-\phi\beta K^{R2}/4\bar{\phi}^2+\left(1/d+\beta/2\bar{\phi}\right)K^M K^R-\left(K^R J^M+K^M J^R-J^M J^R\right)/d$$
$$+K^M/2-2\delta H^M+\phi/4\beta \tag{A6.17}$$

$$rI^M=2C^M I^M/a+J^M K^M/b-\phi\beta E^R I^R/\bar{\phi}^2-2\left(2C^M E^R+2E^M C^R+I^M I^R-C^M I^R-I^M C^R\right)/d$$
$$+\left(2/d+\beta/\bar{\phi}\right)\left(E^M I^R+I^M E^R\right) \tag{A6.18}$$

$$rJ^M=2C^M J^M/a+\left(1/d+\beta/2\bar{\phi}\right)\left(I^M K^R+K^M I^R\right)-\phi\beta I^R K^R/2\bar{\phi}^2+I^M/2+\left(2H^M/b-\delta\right)J^M$$
$$-\left[I^R J^M+I^M J^R-2\left(C^M J^R+J^M C^R\right)+2\left(C^M K^R+2K^M C^R\right)\right]/d \tag{A6.19}$$

$$rK^M=I^M J^M/a+2\left(1/d+\beta/2\bar{\phi}\right)\left(E^M K^R+K^M E^R\right)-\phi\beta E^R K^R/\bar{\phi}^2+E^M+\left(2H^M/b-\delta\right)K^M$$
$$-\left[K^R I^M+K^M I^R+2\left(E^R J^M+E^M J^R\right)-\left(I^M J^R+J^M I^R\right)\right]/d \tag{A6.20}$$

$$rA^R=\bar{\phi}\alpha^2/4\beta+\omega^2/2d+\left(B^R\right)^2/2d-\omega B^R/d+\left(\omega/d+\alpha/2\right)D^R+\left(2\bar{\phi}+\beta d\right)\left(D^R\right)^2/4\bar{\phi}d+B^M B^R/a+F^M F^R/b-B^R D^R/d \tag{A6.21}$$

$$rB^R=-2\omega C^R/d+\left(\omega/d+\alpha/2\right)I^R+\left(2\bar{\phi}+\beta d\right)D^R I^R/2\bar{\phi}d+2\left(B^M C^R+C^M B^R\right)/a+\left(F^M J^R+J^M F^R\right)/b$$
$$+\left[2B^R C^R-B^R I^R-2C^R D^R\right]/d \tag{A6.22}$$

$$rC^R=2\left(C^R\right)^2/d+\left(2\bar{\phi}+\beta d\right)\left(I^R\right)^2/4\bar{\phi}d+4C^M C^R/a+J^M J^R/b-2C^R I^R/d \tag{A6.23}$$

$$rD^R=-\omega I^R/d+2\left(\omega/d+\alpha/2\right)E^R+\left(2\bar{\phi}+\beta d\right)D^R E^R/\bar{\phi}d+\left(B^M I^R+I^M B^R\right)/a+\left(F^M K^R+K^M F^R\right)/b$$
$$+\left[B^R I^R-2B^R E^R-D^R I^R\right]/d \tag{A6.24}$$

$$rE^R=-e/2+\left(I^R\right)^2/2d+\left(2\bar{\phi}+\beta d\right)\left(E^R\right)^2/\bar{\phi}d+I^M I^R/a+K^M K^R/b-2E^R I^R/d \tag{A6.25}$$

$$rF^R=\bar{\phi}\alpha/2\beta-\omega J^R/d+\left(\omega/d+\alpha/2\right)K^R+D^R/2-\delta F^R+\left(2\bar{\phi}+\beta d\right)D^R K^R/2\bar{\phi}d+\left(B^M J^R+J^M B^R\right)/a$$
$$+2\left(F^M H^R+H^M F^R\right)/b+\left(B^R J^R-B^R K^R-D^R J^R\right)/d \tag{A6.26}$$

$$rH^R=\bar{\phi}/4\beta+\left(J^R\right)^2/2d+K^R/2-2\delta H^R+\left(2\bar{\phi}+\beta d\right)\left(K^R\right)^2/4\bar{\phi}d+J^M J^R/a+K^M K^R/b-J^R K^R/d \tag{A6.27}$$

$$rI^R=-\left(I^R\right)^2/d+2\left(C^M I^R+I^M C^R\right)/a+\left(J^M K^R+K^M J^R\right)/b+2\left(C^R I^R-2C^R E^R\right)/d \tag{A6.28}$$

$$rJ^R=I^R/2-\delta J^R+\left(2\bar{\phi}+\beta d\right)I^R K^R/2\bar{\phi}d+2\left(C^M J^R+J^M C^R\right)/a+2\left(J^M H^R+H^M J^R\right)/b$$
$$+\left(2C^R J^R-2C^R K^R-I^R J^R\right)/d \tag{A6.29}$$

$$rK^R=E^R-\delta K^R+\left(2\bar{\phi}+\beta d\right)E^R K^R/\bar{\phi}d+\left(I^M J^R+J^M I^R\right)/a+2\left(K^M H^R+H^M K^R\right)/b$$
$$+\left(I^R J^R-2E^R J^R-I^R K^R\right)/d \tag{A6.30}$$

□

A7. Plugging the right-hand side of (32) into (3) gives the equilibrium sales rate

$$S = \alpha/2 + \left[\beta\left(D^R + I^R X + 2E^R Y\right) + \left(\bar{\phi} + \beta K^R\right)G \right] \Big/ 2\bar{\phi} \tag{A7.1}$$

Plugging the RHS of (29)-(30), (31), and (35) into (1)-(2), and (4), respectively, the system is

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{G} \end{bmatrix} = \begin{bmatrix} \dfrac{2C^R - I^R}{d} + \dfrac{2C^M}{a} & \dfrac{I^R - 2E^R}{d} + \dfrac{I^M}{a} & \dfrac{J^R - K^R}{d} + \dfrac{J^M}{a} \\ \dfrac{I^R - 2C^R}{d} + \dfrac{\beta I^R}{2\bar{\phi}} & \dfrac{2E^R - I^R}{d} + \dfrac{\beta E^R}{\bar{\phi}} & \dfrac{K^R - J^R}{d} + \dfrac{1}{2} + \dfrac{\beta K^R}{2\bar{\phi}} \\ \dfrac{J^M}{b} & \dfrac{K^M}{b} & \dfrac{2H^M}{b} - \delta \end{bmatrix} \begin{bmatrix} X \\ Y \\ G \end{bmatrix} + \begin{bmatrix} \dfrac{B^M}{a} - \dfrac{\omega - B^R + D^R}{d} \\ \dfrac{\beta D^R}{2\bar{\phi}} + \dfrac{\omega - B^R + D^R}{d} + \dfrac{\alpha}{2} \\ \dfrac{F^M}{b} \end{bmatrix} \tag{A7.2}$$

The FBNE is globally asymptotically stable if the eigenvalues of the Jacobian matrix associated with the dynamic system in (A7.2) all are negative. □

A8. The roots of the system of 20 non-linear equations in **A6** are computed with *Matlab* 7.5.0.342 (R2007b). To ensure that all equilibria are identified, a two-step approach is necessary. First, we randomly search for equilibria and we use an algorithm (*Matcont*) to continue these equilibria by changing a parameter value of the model. If the branch of any equilibrium undergoes a limit point bifurcation, this is detected by the algorithm and further equilibria can be detected. For both contracts, 10,000 initial vectors were chosen randomly for the coefficients and used to solve the equation system. Only a single steady state satisfying the criterion of global asymptotic stability was found for each contract. *Matcont* was used to detect any other equilibria due to bifurcation. However, even after this step, a single and stable steady state remained in all cases. □

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