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Institutional change, education and population growth: lessons from dynamic modelling

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Abstract

This is one of the first papers that links population growth, education and institutional change within a dynamic optimization. The basic premise is, following interesting papers of Boucekkine with different coauthors dealing with the Arab Spring, that elites manage first the ruling and then the transition to a 'democratic' government. We are less optimistic concerning the economic efficiency of domestic resource use and, more important, extend this framework by accounting for (endogenous) population growth. These extensions render the survival of any elite much less feasible than without population growth. Only a (cynical) elite worrying about the size of the population allows for a longrun and interior outcome. Therefore, the rulers and the elite have to account for a second phase in which they lose control over a country's (financial) resources. If the elite lacks sufficient stakes in the second phase, it will 'take the money and run', i.e., no investment, at least close to the (endogenous) terminal time.

Keywords: Elites, two stage optimization, population growth, lack of longrun interior solutions.

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1 Introduction

This is one of the first papers that links population growth, education (one of the key factors affecting population growth, in particular, the education of girls) and institutional change (which is more specific than conflicts) within dynamic optimization models. It is motivated by political events like the Arab Spring and by papers (and presentations) of Raouf Boucekkine with different coauthors, in particular, Boucekkine et al (2016) and Boucekkine et al (2019a). Although the proposed models are far from trivial involving two (and more) states and in some cases two stages, we are able to characterize the intertemporal policies. Therefore, our theoretical analysis is complementary to recent empirical papers like Acemoglu et al. (2020) on population growth and conflict and Boucekkine et al (2019b) on education and illiberalism.

Of the different explanations of an uprising, we mention here two complementary ones. Kuran's theory of preference falsification, e.g., in Kuran (1989), addresses the decisions of individuals: Individuals hide their true preference for the opposition if this is individually opportune and any uprising or revolution requires that the support for the opposition surpasses a critical threshold. This theory can explain that revolutions and the subsequent changes of political leaders are often an event that surprises even the experts. Examples abound, from the French, to the Russian and the Islamic (in Iran) revolution, the collapse of socialism in 1989 and more recently the Arab Spring in 2011 of which some (e.g., Fahmi (2019)) predict a second coming, e.g., with reference to the ongoing protests in Algeria. Although this is a very important and plausible explanation, we depart from a second and complementary approach that addresses the issue of revolutions from the perspective of a ruling elite and its possible voluntary power hand-over with theoretical reference to the American sociologist Lipset and a recent book of Albertus and Menaldo (2018). This second route has been proposed in Boucekkine et al (2016) and in Boucekkine et al (2019a). They assume that an elite has access to a rent from selling abroad a resource and also at home by providing a vital input for domestic production in order to explain events like the Arab Spring that started in Tunisia in 2011 and is continuing until today in Algeria. Other potential applications are to past regime changes in South Africa, Chile (of Pinochet), maybe Russia, and the recent changes that Muhammad bin Salman, clearly a member of the ruling elite, initiated in Saudi Arabia.

The papers, Boucekkine et al (2016) and Boucekkine et al (2019a), are our starting point. Both consider an economy with constant capital stock and population ruled by an elite. The elite has access to the revenues from selling a resource, at home and abroad. Gross domestic product is the output of two variable inputs (aside from constant capital and labor): the domestic resource use (a flow) and human capital (a stock build up by investment into education). Although the elite's concern is its own level of consumption, it invests into human capital, which expands output, raises wages and also the population's demand for its share of the resource rent. A central assumption is that the elite can determine the transition and can retain some stakes under the future (democratic?) regime. Arithmetically, the elite solves an optimal control problem, possibly involving two stages before and after the hand-over. Although we follow this approach too, the role of elites is seen differently by different authors. It is not entirely different from Acemoglu and Robinson (2006) who argue that "democracy consolidates when elites do not have strong incentive to overthrow it." Recently, Guriev and Treisman (2019) argue that an informed elite is in opposition and constrains the actions of recently emerging autocrats within formal democracies. Similarly, Boucekkine et al (2019b) observe that rising educational levels are incompatible with illiberalism and thus increase the probability of a regime change.

Our first extension is to address how population growth affects and constrains an elite's policies. This extension is crucial from an applied perspective, because population growth is a major force and even a threat to any elite. The reason is that many young people, in particular, too many 3rd and 4th sons without any economic and social perspective are prone to revolt. Acemoglu et al. (in press) is an empirical confirmation how the drop in morbidity since 1940 (due to breakthroughs in medicine, hygiene and fights against malaria, which are all exogenous for developing countries) led to population growth, which in turn had sizeable effect on civic conflicts. While population declines in the industrialized countries, it grows dramatically in most Sub-Saharan (doubling within a few decades) and Arab countries of which Jordan, Oman, Kuwait, Bahrein, United Arab Emirates and Qatar have the world's highest population growth for 2010 over 2005 (of course, the huge annual growth rates of 12 and 13% per annum for the latter two includes migration). Indeed, the ignorance of population growth is, according to Lianos (2020), "the elephant in the living room" in (in his case in environmental but we think much beyond that) economics. The second is the modification of the optimistic assumption that the resource is used as a productive input for domestic production. Instead its use is by and large in the form of cheap, actually absurdly cheap, petrol, which will lead to enormous political problems when prices must be raised eventually as the recent revolts in Ecuador (following increasing petrol prices to still moderate levels $ 2.39 per gallon, see *The Economist* (2019b)) and in Iran (but from absurdly low levels below the price of bottled water) document. Even accounting for refining, petrochemicals and the many airlines in the Gulf we doubt that these industries deliver a profit based on a genuine comparative advantage because of the need of hiring foreign personal (engineers from the West, workers from the Indian subcontinent, cabin crews and pilots from all over the world, etc.). Instead they just benefit from the cheap input (natural gas, oil and kerosene) which were presumably better sold abroad, compare, Ghoddusi et al (2020).

The paper consists of a sequence of models, some of them only briefly sketched. The upshot of our analysis is that the elite's intertemporal optimization problems do not have a longrun solution (i.e., the canonical equations implied by the, also sufficient, first order optimality conditions do not lead to a saddlepoint stable steady state) in most cases, including variants close to the ones suggested in Boucekkine, et al (2016) and Boucekkine et al (2019a). The extension considered in this paper, population growth, renders (optimal) control by the elite in the longrun even less possible unless the elite heavily penalizes population. Therefore, it is implicitly necessary to account for a second stage after the elite hands the power over to the 'people' unless one subscribes to the cynical description of the elite just in the above sentence. The elite can choose the time when to hand over to a future government and/or leaves the country. Given the impossibility that any kind of ruling, by elites or kings, can survive under the assumed constraints, it is crucial what are their stakes in a future government or in exile? Therefore, an elite facing the constraints addressed in this paper will only provide for a handover if it has an 'after life' after the handover. However, it is questionable nowadays whether an elite can accumulate resources outside the country for the exile. For example, *The Economist* (Africa's money launderers, Oct.12th, 2019, p38) reports that anti corruption campaigners make the stashing of illicit wealth, such as exercised by Sani Abacha of Nigeria and Bokossa of Central Africa, much harder nowadays. This can have unintended consequences as dictators and elites have only the option to 'take the money and run' as Adelman hypothesized about OPEC rulers. Or formulated positively, the elite chooses to behave well if that allows it to retain some benefits after a negotiated hand-over. According to Guriev and Treisman (2019) describing how 'human' autocrats became in the recent decade (political killings and the number of political prisoners declined substantially) it is the elite (the informed ones, say in Russia) that opposes the autocrats and limits their actions. However, given the formal set up of an intertemporal optimization problem, our models are, at least to some extent, also applicable to a ruler facing an informed elite.

The model departing from Boucekkine et al (2016) and Boucekkine et al (2019a) and its extensions and variants are introduced and analyzed in Section 2. The importance and even the necessity of a second stage arises due to the lack of (saddlepoint) stable longrun solutions. Therefore, Section 3 analyzes a two stage problem: The elite rules during the first phase, uses a resource rent for consumption, investment (into education) and the build up of foreign assets. In the second phase, the elite lives in exile from the assets transferred abroad during the period of ruling.

2 Models and implications

2.1 No population growth

The papers Boucekkine et al (2016) and Boucekkine, et al (2019a) consider an elite having access to the revenues from resource sale. The elite uses the revenues from sales (export, domestic) for own consumption, for financing education, for subsidizing the domestic use of the resource, and directly for transfers to the population (e.g., including little or no taxes). Gross domestic product (Y) is the output of the inputs of physical and of human capital and of the domestic resource use. Our first modification is a simplification that ignores the contribution of domestic resource use for economic activity, because driving large SUVs through the desert provides little value added. Therefore, total output (Y) is given by the production function F with only the two inputs of physical (K) and human (H) capital,

$$Y = F(K, H) = Pf(k, h).$$

Introducing per capita terms, P denoting the population (capital letters refer to aggregate and small letters to per capita values),

$$k := \frac{K}{P}, h := \frac{H}{P}; \tag{1}$$

and fixing per capita capital k (as implicitly in Boucekkine et al (2016) and Boucekkine et al (2019a)) yields:

$$y = \frac{Y}{P} = f(k, h) = a\varphi(h) = ah^{\alpha},\ a := Ak^{1-\alpha}.$$

Assuming, $\alpha = 1$ (i.e., ah-technology in analogy to the AK-framework, see Rebelo (1991)), the equilibrium wage is,

$$w = y. \tag{2}$$

El-Mattrawy and Semmler (2006) find a large Solow residual for Egypt and find that a substantial (but not the entire part) can be explained by human capital.

Elite and rulers own a resource that gives the elite access to the revenues R, e.g., oil revenues. However, also aid from governments and NGOs could be the source of the elite's rent, which broadens the applicability of the model. Indeed many rulers and their elite turned into kleptocrats pocketing aid money instead of using it for development, and e.g., Acemoglu et al (2004) mention Democratic Republic of the Congo (Zaire) under Mobutu Sese Seko, the Dominican Republic under Rafael Trujillo, Haiti under the Duvaliers, Nicaragua under the Somozas, Uganda under Idi Amin, Liberia under Charles Taylor, and the Philippines under Ferdinand Marcos, not to forget Jean-Bedel Bokassa of Central Africa who "would slip his guest diamonds to thank him for France's support" according to *The Economist* (2019a).

The elite spends the rent R for own consumption C, the maximization of which is its objective (using the constant discount rate, $\rho > 0$, and for concreteness logarithmic utility),

$$\max \int_0^\infty \exp(-\rho t) \ln C(t)\, dt, \tag{3}$$

for transfers to the workers Θ and $\theta := \Theta/P$ per capita, and for education E , $e := E/P$ per capita,

$$R = C + \Theta + E. \tag{4}$$

Human capital (H) follows the standard capital accumulation rule (also used in Boucekkine et al (2016) and Boucekkine, et al (2019a)),

$$\dot{H} = \beta E - \delta H,\ H(0) = H_0 \text{ given},\ H \geq 0. \tag{5}$$

because workers (= domestic population except for the elite) cannot invest in education. However, they are not entirely passive, because they will revolt if their income, consisting of their wage (w) plus the handout θ, is considered to be too low. Therefore, in order to deter a revolt, the following inequality,

$$w + \theta \geq z + \tau(h),\ \tau' > 0, \tag{6}$$

must be satisfied; z denotes the subsistence level and $\tau(h)$ a threshold, which is increasing with respect to education because it fosters awareness (as also Guriev and Treisman (2019) and Boucekkine et al (2019b) stress). Therefore, the inequality (6) must hold as long as the elite is in charge and the rent R must be sufficiently large, at least $R > zP$, to allow for positive consumption at all, $C > 0$. In the most simple version of a linear endogenous threshold level (again as in Boucekkine et al (2016) and Boucekkine et al (2019a)),

$$\tau(h) = bh,$$

and assuming an ah-technology, the constraint (6) becomes in terms of the minimal per capita handout,

$$\theta \geq z + (b-a)h, \ b > a. \tag{7}$$

This above assumed inequality states that the elite has to surrender some of its rent and that this amount per capita is increasing with human capital.

Remark 1 *If $b < a$, the elite could extract taxes after investing into education in order to finance its consumption, hence the assumption, $b > a$. This possibility that the elite can tax its people, i.e., $\theta < 0$, arises also for a concave $\tau(h)$ if investments into education render human capital sufficiently productive and at the same time the population relatively docile, $h > \hat{h} : z + \tau(h) - ah = 0$, a situation that may apply to China's elite today. In fact, we wanted to rule this possibility out a priori, because we do not think that China's unique experience (e.g., decades of a one-child policy) applies to the countries we have in mind. Furthermore, if taxation became feasible, the elite would prefer a higher population.*

Remark 2 *As mentioned in the introduction and due to our point of departure from Boucekkine et al (2016) and Boucekkine et al (2019a), we refer to the elite as the decision maker although it could be a much smaller set of an autocrat (to use the term of Guriev and Treisman (2019)) and his inner circle.*

Proposition 1 *Considering this slightly simplified version of Boucekkine et al (2016) and Boucekkine, et al (2019a), and still a constant population, P is constant, no longrun interior solution exists for the elite's intertemporal optimization problem (3) - (7), i.e., the canonical equations implied by the (also sufficient) first order conditions do not converge to an interior steady state.*

The economic intuition explaining this lack of a sustainable rule by the elite is that the longrun objective, i.e., substituting the steady state value of human capital, $H = \beta E/\delta$, and accounting for the budget constraint, i.e., (7), yields a strictly concave objective,

$$\max_E U(E) := \ln\left(R - (b-a)\frac{\beta}{\delta}E - E - zP\right),$$

but one, which has no generic interior solution since the corresponding first order condition (foc), $U' = 0$, cannot be met since $U' < 0$ at $E = 0$ and U' is declining.

2.2 Population growth

The first extension is the account for exogenous population growth (at the constant rate g). Given a finite rent and a growing population, it is a no-brainer that this process cannot go on forever and that the elite can at best enjoy its rent for a limited time. Indeed, *the elite's optimal policy is to not invest into education and to offer only the transfers (as long as feasible), $R > \Theta = zP + (b-a)H$, that just avoid and thereby delay the revolution*. There are examples, e.g., 'Tsarist Russia' and the elites in some developing countries enjoy a luxury life but spend little for education, e.g., Mobuto Sese Seko and some of the others mentioned in the introduction.

The second addition is that education lowers population growth (g denotes the population growth rate),

$$\dot{P} = g(h)P, \; P(0) = P_0, \; g' < 0, \; P \geq 0, \tag{8}$$

$$g(h) = \gamma - \pi h, \; \gamma > 0, \; \bar{h} := \frac{\gamma}{\pi}. \tag{9}$$

This link between population growth and education is not only an assumption but an empirical regularity, in particular, educating girls lowers fertility rates, even drastically in some countries. The assumption of a linear relation is chosen for reasons of simplicity. It implies a stationary population at a unique level of individual education denoted by $\bar{h}$ in (9). As a consequence, the elite has to pass between Scilla (an uneducated and growing population which cannot be fed from some future point onwards from the constant rent R) and Charybdis (educating the population sufficiently in order to achieve a stationary population but which will demand larger individual handouts).

Given the exogenous rent R, then $\bar{P}$ is the maximum level of population that can be educated, which is further diminished to $\hat{P}$ if accounting for the need of transfers according to (7),

$$\hat{P} := \frac{\beta\pi R}{\beta\pi z + \gamma(\beta(b-a) + \delta)} < \bar{P} := \frac{\beta\pi R}{\delta\gamma}.$$

Of course, only initial conditions below $\hat{P}$ make sense.

One possibility is to add a soft constraint: the elite values the subjective welfare of its people (their income minus their education dependent demands), or respectively, accounts for the costs from demonstrations, uprisings, and the risk of a revolution of an unhappy population. However, we bypass this (anyway also not allowing for an interior longrun solution) and replace the constraint in (7) about avoiding uprisings or revolutions with equality. This yields the objective,

$$\max_{e(t)\geq 0} \int_0^\infty \exp(-\rho t) \ln(R - (b-a)H - eP - zP) dt, \tag{10}$$

with the single control, per capita expenditures for education (e), and the two states H and P. Even this problem does not allow for an optimal longrun rule by the elite:

Proposition 2 *No steady state exists for the optimal longrun interior solution, $e > 0$, for the optimal control problem with the objective (10) and the state differential equations (5) and (8).*

Proof. Setting up the

$$\mathcal{H} = \ln(R - (b-a)H - eP - zP) + \lambda(\beta eP - \delta H) + \mu(\gamma P - \pi H) \tag{11}$$

the first order conditions are:

$$\mathcal{H}_e = \beta\lambda P - \frac{P}{R - (b-a)H - eP - zP} \Longrightarrow$$

$$e^* = \max\left\{\frac{R - (b-a)H - zP}{P} - \frac{1}{\beta\lambda P}, 0\right\}, \tag{12}$$

$$\dot{\lambda} = (\rho + \delta)\lambda + \pi\mu + \frac{b-a}{R - (b-a)H - eP - zP}, \tag{13}$$

$$\dot{\mu} = (\rho - \gamma)\mu + \frac{z+e}{R - (b-a)H - eP - zP} - e\beta\lambda. \tag{14}$$

Note that $\mathcal{H}_{ee} < 0$ for both solutions in (12). Assuming (indirectly) an interior solution of e^* and substituting it into the state and costate equations yields the canonical equations system,

$$\dot{H} = \beta(R - (b-a)H - zP) - \frac{1}{\lambda} - \delta H, \tag{15}$$

$$\dot{P} = \gamma P - \pi H, \tag{16}$$

$$\dot{\lambda} = (\rho + \delta + \beta(b-a))\lambda + \pi\mu, \tag{17}$$

$$\dot{\mu} = (\rho - \gamma)\mu + \beta z\lambda. \tag{18}$$

Solving first for steady states of the costates yields for the linear equation system (17) and (18),

$$\lambda = \mu = 0, \tag{19}$$

as the unique solution. This rules out the interior solution and thus implies $e^* = 0$ in the longrun. ■

Remark 3 *This finding extends to a concave threshold, e.g.,* $\tau(h) = \tilde{b}\sqrt{h}$*, as well as to a convex specification of* τ*. The reasons, at least the arithmetical ones, are: (i)* τ *appears only as a negative element in consumption and (ii) utility is logarithmic. Therefore, the specification of* τ *affects the above interior solution of* e^* *only indirectly (a different term is subtracted from consumption), but plays no role in the adjoint equations after substituting the optimal control. Thus it allows again only for the trivial solution (19) for the steady states of the costate differential equations system and thus implies the boundary solution,* $e^* = 0$ *in the longrun.*

Why is it uneconomical (from the elite's perspective) to sustain education in the longrun given this more or less straightforward optimal control model with a concave objective? The reason is that the implied static/stationary objective (replacing H by its steady state value conditional on P, however, the same holds for carrying out the analysis with respect to H),

$$\max_P U(P) := \ln\left(R - (b-a)\frac{\gamma}{\pi}P - \frac{\gamma\delta}{\beta\pi}P - zP\right), \tag{20}$$

does not allow for an interior solution with the choice of population $P \geq 0$, as instrument, because

$$U' = -\frac{\beta((b-a)\gamma + z\pi) + \delta\gamma}{\beta\pi R - \beta((b-a)\gamma + z\pi)P - \gamma\delta P} < 0$$

since U' is declining ($U(P)$ is a concave objective) and $U'(0) < 0$. Now what is optimal in the longrun? The boundary solution, $e^* = 0$, implies the state dynamics,

$$\dot{H} = -\delta H,$$
$$\dot{P} = \gamma P - \pi H,$$

which have the origin as the only steady state. This steady state is a saddlepoint (the eigenvalues are γ and $-\delta$), which is reached along the saddle, $H = (\gamma + \delta)P/\pi$. Therefore, the following policy results from joining the interior and the boundary ones:

Proposition 3 *Given a feasible solution, i.e., a sufficient rent and a not too large initial population, the objective of the interior policy $e^* > 0$ from (12) is to drive the states to the saddle, $H = (\gamma + \delta) P/\pi$, at which the decline of both states starts and continues, $H \to 0$ and $P \to 0$, until the elite can spend the entire rent on own consumption.*

This confirms the above finding from the analysis of the elite's stationary objective (20), namely, that the elite's objective is to drive the size of its population down to zero in order to keep the entire, then also uncontested, rent. Investment into education is only an instrument to achieve this. Of course this drastic policy stresses first, admittedly, the limits of our model. Secondly, it highlights again why the papers of Boucekkine et al (2016) and Boucekkine et al (2019a) have to rely on a theory of a handover by the elite. Thirdly, our extension for population growth hardens this task, and maybe, also the hearts of the elite.

2.3 Optimal Timing of Abdication

Since there need not be either a feasible nor an optimal solution running up to infinity, the elite has to, and therefore will, exercise the option to quit in finite time (T) in particular if facing unstoppable population growth. From the above we know that quitting will be definitely optimal if population exceeds $\hat{P}$ introduced above since there is nothing then left for the elite for consumption. The following optimization problem captures this decision problem of an elite that considers quitting if the going gets rough and to little is left for consumption,

$$\max_{e(t)\geq 0,T} \int_0^T \exp(-\rho t) \ln(R - (b-a) H - eP - zP) dt, \quad (21)$$

$$\dot{H} = \beta eP - \delta H, \; H(0) = H_0, \; H(T) \text{ free}, \quad (22)$$

$$\dot{P} = g(h) P, \; P(0) = P_0, \; P(T) \text{ free}. \quad (23)$$

The change to finite and optional terminal time changes nothing in terms of the first order condition (11) - (14) except for adding the boundary conditions to the costates,

$$\lambda(T) = 0,$$
$$\mu(T) = 0,$$

which imply immediately that the boundary policy must apply for $t \to T$, i.e., no investment into education, $e^*(t) = 0$ for $t \to T$. Assuming the boundary strategy and applying the transversality conditions, the condition for optimal stopping,

$$\mathcal{H} = \ln(R - (b-a) H - eP - zP) + \lambda(\beta eP - \delta H) + \mu(\gamma P - \pi H),$$

$$\mathcal{H}(T) = \ln(R - (b-a)\,H(T) - zP(T)) = 0,$$

i.e., terminal consumption utility is zero.

Proposition 4 *Even allowing for optimal stopping does not allow for an interior optimal policy at least close to the optimal termination date. Therefore, the optimal policy of the elite is no investment into education for $t \to T$ and to leave when utility from consumption turns zero due to the necessary transfers to the growing population.*

Apparently, this outcome seems to be the reason why a salvage value (in current value terms), $S = \sigma H(T)$, is introduced in the papers of Boucekkine with different coauthors. This means economically and politically that the elite must have some stakes in the future regime in order to invest in at least some development, here human capital. Accounting for population suggests that less remains for the elite if handing over to a larger population, e.g.,

$$S = \sigma \frac{H}{P}, \tag{24}$$

is the simplest version of such a salvage function. This extension changes only the objective in (21) - (23),

$$\max_{e(t)\geq 0,T} \int_0^T \exp(-\rho t) \ln(R - (b-a)\,H - eP - zP)dt + \exp(-\rho T)\,\sigma \frac{H(T)}{P(T)}, \tag{25}$$

However, we skip this analysis because we consider an explicit two stage framework in the next section.

2.4 Penalizing larger population

In order to allow longrun ruling by the elite (and to apply standard techniques), we (have to) subtract a penalty, e.g. κP, from the elite's objective,

$$\max_{e(t)\geq 0} \int_0^\infty \exp(-\rho t)\left[\ln(R - (b-a)\,H - eP - zP) - \kappa P\right] dt. \tag{26}$$

This penalty is not only introduced for the above formal reason but also in order to account for conceivable economic and political constraints: a larger population is more difficult and also more costly to contain and to suppress by any elite.

Proposition 5 *Low discount rates, more precisely, discount rates below the maximal population growth rate, $\rho < \gamma$ ensure a stable longrun policy. Even higher discount rates, $\rho > \gamma$, allow for stable outcomes and* *positive population but only for large rents R. Comparative statics are as expected: a higher penalty (κ) lowers population (for $\rho < \gamma$) while higher rents (R) as well as a higher productivity (a, at least for $\gamma > \rho$) allow for a larger stationary population. The effect of higher discounting is negative (as expected) iff*

$$\beta(b-a)+\delta+2\rho-\gamma>0.$$

The qualitative implications on human capital are the same since $H_\infty = \gamma P_\infty/\pi$.

Proof. Setting up the Hamiltonian (again using the symbol $\mathcal{H}$),

$$\mathcal{H}=\ln(R-(b-a)\,H-eP-zP)-\kappa P+\lambda\,(\beta eP-\delta H)+\mu g\,(H/P)\,P, \tag{27}$$

and deriving the first order optimality conditions, the optimal control remains as in (12), and the costate equations are,

$$\begin{aligned}\dot{\lambda}&=(\rho+\delta)\,\lambda+\pi\mu+\frac{b-a}{R-(b-a)\,H-eP-zP},\\ \dot{\mu}&=(\rho-\gamma)\,\mu+\kappa+\frac{z+e}{R-(b-a)\,H-eP-zP}-e\beta\lambda.\end{aligned}$$

The following canonical equations system is derived for an interior solution, $e^*>0$ from (12),

$$\dot{H}=\beta\,(R-(b-a)\,H-zP)-\frac{1}{\lambda}-\delta H, \tag{28}$$

$$\dot{P}=\gamma P-\pi H, \tag{29}$$

$$\dot{\lambda}=(\rho+\delta+\beta\,(b-a))\,\lambda+\pi\mu, \tag{30}$$

$$\dot{\mu}=\kappa+(\rho-\gamma)\,\mu+\beta z\lambda. \tag{31}$$

This system has a unique steady state,

$$H_\infty=\frac{(\gamma-\rho)\,(\rho+\delta+\beta\,(b-a))+\beta\pi\,(\kappa R+z)}{(\gamma\,(\delta+\beta\,(b-a))+\beta\pi z)\,\kappa}\frac{\gamma}{\pi}, \tag{32}$$

$$P_\infty=\frac{(\gamma-\rho)\,(\rho+\delta+\beta\,(b-a))+\beta\pi\,(\kappa R+z)}{(\gamma\,(\delta+\beta\,(b-a))+\beta\pi z)\,\kappa}, \tag{33}$$

$$\lambda_\infty=-\frac{\kappa\pi}{(\gamma-\rho)\,(\rho+\delta+\beta\,(b-a))+\beta\pi z}, \tag{34}$$

$$\mu_\infty=\frac{\kappa\,(\rho+\delta+\beta\,(b-a))}{(\gamma-\rho)\,(\rho+\delta+\beta\,(b-a))+\beta\pi z}. \tag{35}$$

Therefore, positive steady states, $P_\infty>0$ and $H_\infty>0$, result if either $\gamma>\rho$ such that the shadow price of human capital is negative ($\lambda_\infty<0$);

or also if $\rho > \gamma$ but then only if $\beta\kappa\pi R$ is sufficiently large.
The comparative static properties of the stationary population (and thus stationary human capital) follow from elementary partial differentiation of (33), for example,

$$\frac{\partial P_\infty}{\partial \kappa} = -\frac{(\rho+\delta)(\gamma-\rho)+\beta((b-a)(\gamma-\rho)+z\pi)}{\kappa^2(\delta\gamma+\beta(\gamma(b-a)+\pi z))} < 0 \text{ for } \gamma > \rho,$$
$$\frac{\partial P_\infty}{\partial R} = \frac{\beta\pi}{\gamma(\delta+\beta(b-a))+\beta\pi z} > 0,$$
$$\frac{\partial P_\infty}{\partial \rho} = -\frac{\beta(b-a)+(\delta+2\rho-\gamma)}{(\delta\gamma+\beta(\gamma(b-a)+\pi z))\kappa} < 0 \text{ unless } \gamma > \beta(b-a)+\delta+2\rho,$$
$$\frac{\partial P_\infty}{\partial a} = \frac{(\gamma\rho(\gamma-\rho)+\beta\pi(\rho z+\gamma\kappa R))\beta}{\kappa(\gamma\delta+\beta(\gamma(b-a)+\pi z))^2} > 0 \text{ at least for } \gamma > \rho.$$

The Jacobian,

$$J = \begin{pmatrix} -\beta(b-a)-\delta & -\beta z & 1/\lambda_\infty^2 & 0 \\ -\pi & \gamma & 0 & 0 \\ 0 & 0 & \beta(b-a)+\rho+\delta & \pi \\ 0 & 0 & \beta z & (\rho-\gamma) \end{pmatrix}, \tag{36}$$

has the (four) eigenvalues,

$$ev_{12} = \frac{1}{2}\left(-\xi \pm \sqrt{\xi^2+4\zeta}\right),$$
$$ev_{34} = \frac{1}{2}\left(2\rho+\xi \pm \sqrt{\xi^2+4\zeta}\right),$$

in which

$$\xi := \beta(b-a)+\delta-\gamma,\ \zeta := \delta\gamma+\beta((b-a)\gamma+z\pi) > 0.$$

Saddlepoint stability results if two and only two of the four eigenvalues are negative. The first and the third eigenvalue must be positive. The second is for sure negative (even if $\xi < 0$), and the fourth iff,

$$(2\rho+\xi)^2 < \xi^2+4\zeta \iff \rho^2+\rho\xi < \zeta$$
$$\iff (\rho-\gamma)(\rho+\delta+\beta(b-a)) < \beta z\pi$$

Assuming $\rho < \gamma$ then the unique steady state[1] must be a saddlepoint (the second and fourth eigenvalue are negative and the other two are

[1] Of course in order to meaningful, the steady states of population and human capital must be positive. The stability properties extend arithmetically to negative but meaningless solutions.

positive). The case of $\rho > \gamma$ still allows for stability, but only if the coverage of the subsistence level is sufficiently large, more precisely,

$$z > \frac{(\rho - \gamma)(\beta(b-a) + \delta + \rho)}{\beta\pi}.$$

If this condition is not met, the unique steady state turns unstable that can be only reached along a one-dimensional manifold in the state space. ■

The stability finding in Proposition 5 is surprising in the light of Proposition 2: *the implicit penalty for population of* (zP) *subtracting from the elite's consumption in order to cover subsistence consumption of the population does not lead to a stationary outcome. However, the minor addition of an explicit and linear penalty renders a longrun stable outcome, and this for sure for discount rates below the maximal growth rate of population.* At least arithmetically, we can explain this. The penalty parameter κ appears now in the costate equation (31) which allows for steady states of the costates different from zero and therefore for a longrun interior policy.

Therefore, an elite, which is not too impatient but sufficiently cynical about its own population, is able to stir its course towards a stationary population and keep thereby its rule. Indeed, the assumption that the elite accounts for endogenous population growth as in (8) stipulates implicitly a farsighted elite. However, this assumption may be not applicable to many resource dependent countries as the ruling elites 'must take the money and run', as Adelman quipped about OPEC countries (see also Wirl (2009) about positive objectives of OPEC politicians).

3 Adding a second stage: Going abroad

Given the difficulties or even the infeasibility of keeping the power in the longrun and the difficulties to obtain at least some stakes in the future (democratic) regime, dictators and a small part of the elite have the option, and almost always use it, to amass money abroad (A) and to leave if the going gets rough. For example, the Iranian revolution in 1978 led many people linked to the Shah regime to emigrate and not only the Shah and his family left Iran. Idi Amin of Uganda did the same when leaving for exile in Saudi Arabia.

After leaving the country with no stakes left at 'home', the elite or the ruler exercises a standard Ramsey program:

$$V(A_0) := \max_c \int_0^\infty e^{-\rho t} \ln C dt, \tag{37}$$

subject to the budget constraint (A denotes the assets held in foreign accounts), the initial condition, and the no-Ponzi-game condition

$$\dot{A} = rA - C, \qquad A(0) = A_0, \qquad \lim_{t\to\infty} A(t)\, e^{-rt} \geq 0, \tag{38}$$

in which $r < \rho$ is the exogenously given interest rate earned in capital markets and A_0 the money accumulated outside the country during the periods of ruling. The solution is well known (see e.g., Barro and Sala-i-Martin (1995)), $A \to 0$ and $C \to 0$ (thus the corresponding costate $\nu \to \infty$) all asymptotically given the canonical equations of the second stage,

$$\begin{aligned}\dot{A} &= rA - \frac{1}{\nu},\\ \dot{\nu} &= (\rho - r)\,\nu.\end{aligned}$$

Therefore,

$$A(t) = A_0 e^{(r-\rho)t},\ C(t) = \rho A_0 e^{(r-\rho)t},\ \nu(0) = \frac{1}{A_0 \rho}, \tag{39}$$

in which the initial levels of consumption, state and costate are determined by the no-Ponzi game condition. This explicit solution allows to determine the value function by integrating over the net present value of future utility,

$$V(A_0) = \int_0^\infty e^{-\rho t} \ln(C(t))\, dt = \int_0^\infty e^{-\rho t} \ln\left(\rho A_0 e^{(r-\rho)t}\right) dt = \frac{\ln(\rho A_0)}{\rho} + \frac{r-\rho}{\rho^2}. \tag{40}$$

Accounting for the value obtainable from the second stage yields the optimal control problem,

$$\max_{e\geq 0, I, T} \int_0^T \exp(-\rho t) \ln(R - (b-a)H - eP - zP - I)dt + \exp(-\rho T)\, V(A(T)), \tag{41}$$

$$\dot{H} = \beta eP - \delta H,\ H(0) = H_0, \tag{42}$$

$$\dot{P} = g\left(\frac{H}{P}\right) P,\ P(0) = P_0, \tag{43}$$

$$\dot{A} = rA + I,\ A(0) = 0, \tag{44}$$

and all terminal values of the states are free. While expenditures for education must be non-negative, investment can be positive or negative. If $I > 0$, the rulers move money abroad if $I < 0$, then they draw on their foreign account or take foreign credits. That is, from the very beginning, the rulers must take into account their definite finite and often rather short time of ruling. Therefore, they start transferring money subject to the political constraint of avoiding an uprising. Given population growth, the elite will presumably transfer money to its foreign accounts from the beginning and may be even larger at the beginning given a growing population.

Proposition 6 *Assuming a feasible solution, the elite's optimal policy is:*
(i) to stop investing into education (presumably long) before leaving ($e^* = 0$ for $t \to T$),
(ii) to stay as long as the resource rent R contributes to pay for the elite's consumption (i.e., as long as $R > \Theta$) and leaves when this contribution vanishes ($R - \Theta \to 0$ for $t \to T$),
(iii) to enjoy continuity in consumption across the two stages, which means that the consumption towards the end is financed from revenues abroad, and,
(iv) but already an implication from (iii), to stop investing abroad (long) before leaving, i.e., negative investment, more precisely, $I(T) = -C(T)$.

Proof. Defining the Hamiltonian,

$$\mathcal{H} = \ln(R-(b-a)H-eP-zP-I)+\lambda(\beta eP-\delta H)+\mu(\gamma P-\pi H)+\nu(rA+I), \tag{45}$$

the first order conditions for solutions are: The Hamiltonian maximizing conditions for the expenditures for education,

$$e^* = \max\left\{\frac{R-(b-a)H-zP-I}{P} - \frac{1}{\beta\lambda P}, 0\right\}, \tag{46}$$

is similar to (12). It is for the second control investment (I) with the optimal level,

$$\begin{aligned} \mathcal{H}_I &= -\frac{1}{R-(b-a)H-eP-zP-I} + \nu = 0 \\ &\Longrightarrow I^* = R-(b-a)H-eP-zP-\frac{1}{\nu}. \end{aligned} \tag{47}$$

The costates evolve according to,

$$\dot{\lambda} = (\rho+\delta)\lambda + \mu\pi + \frac{b-a}{R-(b-a)H-e^*P-zP-I}, \tag{48}$$

$$\dot{\mu} = (\rho-\gamma)\mu + \frac{e^*+z}{R-(b-a)H-e^*P-zP-I} - \beta\lambda e^*, \tag{49}$$

$$\dot{\nu} = (\rho-r)\nu, \tag{50}$$

(using e^* as shortcut for the optimal control from (46)), and finally for optimal stopping requires,

$$\mathcal{H} = \rho V = \ln(\rho A) + \frac{r - \rho}{\rho} \text{ at } t = T. \tag{51}$$

Defining with

$$MU = \frac{1}{C} = \frac{1}{R - (b - a) H - eP - zP - I}$$

the elite's marginal utility of consumption (i.e., the benefit from being able to spend an additional $ for own consumption given the logarithmic utility), then

$$MU = \beta\lambda \wedge MU = \nu.$$

Starting with the second equation, the elite is indifferent between spending or saving (foreign assets, of course) this incremental $. The first condition equates marginal utility to the marginal benefit from larger human capital (λ) multiplied by the efficiency of education investments (β). As a consequence, $\nu = \beta\lambda$, i.e., the two costates differ only by the efficiency of investment into education. However, this characterization holds only along the interior solution, i.e., $e^* > 0$.
Given our assumption that the elite has access to all its assets transferred abroad, we can apply the theory of two stage dynamic optimization, Makris (2001), see also Tomiyama (1985), Tomiyama and Rosanna (1989), in which the two stages are linked by continuity conditions, value matching and smooth pasting. These conditions apply only to the state carried forward, namely the assets,

$$\nu(T) = V'(A(T)) = \frac{1}{\rho A(T)},$$

and simplifies the boundary conditions of the other costates in stage 1,

$$\lambda(T) = 0 \wedge \mu(T) = 0.$$

They imply immediately that the elite spends again nothing on education before leaving,

$$e^*(t) = 0, \ t \to T.$$

Therefore, reductions in educational investments are a strong signal that elite considers leaving. The continuity conditions are,

$$\nu_1(T) = \nu_2(T), \tag{52}$$
$$\mathcal{H}_1(T) = \mathcal{H}_2(T), \tag{53}$$

in which the subscripts (which can be dropped for the costate ν due to above) refer to the two stages.

$$\mathcal{H}_1(T) = \ln(C(T_-)) + \nu(T)(rA(T) + I(T)),$$
$$C(T_-) = R - (b-a)H(T) - zP(T),$$
$$\mathcal{H}_2(T) = \ln(C(T_+)) + \nu(T)(rA(T) - C(T_+)).$$

Applying the "smooth pasting" condition (52) implies $C(T_+) = C(T_-)$ and thus

$$I(T) = -C(T). \tag{54}$$

due to "value matching" (53). ■

The optimal stopping conditions imply that the elite stays as long as the rent R can contribute to its consumption. However, it has to rely increasingly on foreign assets to pay for its consumption (negative investments, $I < 0$ for $t \to T$). Furthermore, it will smooth its consumption level across the two stages, because otherwise an arbitrage opportunity would arise. Therefore not only lacking investment in education, but increasing reliance on money from abroad signal that an elite considers leaving.

The property of continuous consumption depends crucially on some of the assumptions. It is eliminated for switching costs, since the elite's exodus is for sure costly, so that the elite will face a discontinuous drop in consumption. Furthermore, with international banks not anymore protecting accounts fed from stolen money, the above derived strategy becomes risky as mentioned in the introduction (e.g., the quoted article in *The Economist* reports that the auction revenues of \$ 27 millions from sports cars seized from Mr. Obiang, the son of the president of Equatorial Guinea were returned). Indeed, this exit option is only possible for rulers retaining some goodwill when leaving, i.e., what is called a golden handshake in the case of managers. Let $V(A)$ again denote the value function of the above Ramsey program and $p(H,P,A)$ the probability (or share) that the former rulers expect to keep after the revolution and when leaving. Therefore define with

$$\Pi(H,P,A) := p(H,P,A)\,V(A),$$
$$\Pi_H > 0,\ \Pi_P < 0,\ p_A < 0 \text{ yet } \Pi_A > 0$$

the expected net present value payoff of the ruler at the moment going into exile. The assumption $\Pi_A > 0$ is for sure violated for large values of A, but accumulating large amounts such that the expected benefit declines is clearly suboptimal. This change in assumptions, does not change the above state and costate equations in their dynamics but affects the boundary conditions,

$$\lambda(T) = \Pi_H > 0,$$
$$\mu(T) = \Pi_P < 0,$$
$$\nu(T) = \Pi_A > 0,$$

in which case the elite may continue investing in education until the very end but consumption need not remain continuous.

4 Concluding remarks

Departing from the interesting papers of Boucekkine et al (2016) and Boucekkine et al (2019a) on how an elite might run a resource exporting economy, this paper introduced a few modifications. Firstly, we question the productive use of the resource in domestic industry and secondly, we account for population growth, which seems crucial for many countries to which such socio-politico-economic scenarios are applicable. The major upshot of the different setups considered in this paper is that an elite will have a hard time in most cases to survive. Only a far sighted and cynical elite with negative concerns about a larger population will be able to sustain its rule in the longrun and only if it can stabilize its population.

Offering the elite some stakes in the future government could induce them to take domestic issues, here modelled as investing into education, into account. In this sense, the opportunities offered in the second stage, the after life of the elite, can cast an important and positive shadow over the first phase when it rules but that is not granted. Or put the other way round. Only an elite that is sufficiently farsighted, patient and has some stakes in the country's future could lead to a peaceful handover. However, our analysis indicates that this mitigation policy of an elite is difficult and impossible in some cases in our framework.

Partially to our own surprise, we are able to characterize the elite's optimal intertemporal policies. The policies derived from our models reveal the "bare bone" and thus drastic intentions of a greedy elite, which are presumably masked and mitigated by additional considerations or further constraints. Therefore, extensions and improvements are not only possible but necessary in order to understand such topical political events. Obvious candidates are: Broader objectives that include social benefits (however, this is more likely for autocrats worrying about their remembrance in the future books of history than for a diffuse elite); a threshold function (i.e., a well educated and also relatively rich population raises less demands) that could explain the transition in particular in oil exporting countries from subsidies to taxes; accounting for strategic issues applying (dynamic) game theory (Boucekkine, et al (2019a) include a cooperative handover which is ultimately reduced to endogenize the salvage value); for uncertainty, e.g., using Ito processes, possibly combined with jumps, instead of deterministic differential equations. The dramatic drop in oil prices in March 2020 to almost $20 per barrel (for Brent) from above $60 in December 2019 is a very recent reminder of the uncertainty associated with resource revenues and also for the need of taxation in oil exporting countries. In addition, an integration of the objectives and possibilities of individuals as addressed in Kuran (1989) should be part of such an analysis.

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