Dynamic economic models of optimal law enforcement

Abstract

Since Becker's (1968) seminal work on crime and punishment economists see a task in the optimal allocation of resources to reduce illegal behaviour. In some follow-up studies Becker's approach, which is essentially *static*, has been extended by including *intertemporal* aspects. It turns out that efficient law enforcement in a dynamic context is a sophisticated task revealing some important and new aspects of optimal crime control. In particular, we will stress that *optimal control theory* and *dynamic games* are tools being suitable for investigating *dynamic* extensions of law enforcement.

One issue of the present paper is to show how the influence of reference groups to micro-behaviour may result in multiple equilibria. This 'density-dependence' and other inherent non-linearities imply the existence of thresholds separating basins of attractions for optimal paths. Instead of providing a systematic framework we illustrate our approach by several interesting examples of optimal law enforcement. In particular, our game-theoretic approach of law enforcement contains only a collection of preliminary ideas and unsolved examples rather a than general competitive approach to crime and punishment. We hope, however, that this material is useful for future work in a more systematic optimal dynamic law enforcement.

1. Introduction

'Crime' is a heterogeneous set of phenomena that are not only of serious social consequences but also economically very important. Criminal activities range widely from murder and burglary to tax evasion and environmental offenses. Crimes generate damages and harm and put billions of dollars in to the pockets of offenders. Significant public and private resources are spent in order to prevent offenses and to apprehend and convict offenders. This raises the question, how many resources and how much punishment should be used to enforce law. In his *economic* approach to crime and punishment, Becker (1968) put it more strangely: 'How many offenses should be permitted and how many offenders should go unpunished?'

Since Becker's work on crime and punishment, economists see a task in the optimal allocation of resources to reduce illegal behaviour. In particular, this *economic approach* should help to determine an efficient allocation of a given budget to apprehend offenders, to treat them, and to prevent offenses.

Although 'crime' is an economically important activity, it is often elusive from an eco-nomist's point of view. The lack of reliable data for reasons whatsoever suggest the necessity of modeling in the economic of crime.

Crime control generally has also received a considerable amount of attention from the *operations research* community and quantitative methods generally, including all of the criminal career modeling (see, e.g., Blumstein and Cohen, 1973, Blumstein et al., 1978, and Blumstein and Nagin, 1981), selective incapacitation, and recently the trend toward longitudinal individual level analyses (see, e.g., Leung, 1995). We also refer to two interesting surveys of Maltz (1994, 1996).

Our main reference, however, is the path-breaking analysis of Gary S. Becker and his followers. According to Becker (1968), the authorities have to determine the amount of resources to prevent offenses and to apprehend offenders. Moreover, for those convicted the punishment which fits the crime has to be ascertained. In particular, Becker tries to find those expenditures on law enforcement and punishments that minimize the social loss. This loss is the sum of damages, costs of apprehension and conviction, and costs of carrying out the punishments. The analysis of the optimality conditions yields numerous interesting insights into efficient control of illegal behaviour.

Several other scientists took up Becker's ideas and extended them in different ways. Let us briefly mention some selected follow-up work of Becker.

In a model of optimal enforcement by Malik (1990) offenders can engage in activities that reduce the probability of being caught and fined. As in other extensions (such as the one by Polinsky and Shavell (1991) where wealth varies among individuals, or the model by Bebchuk and Kaplow (1993) who consider the possibility that individuals are not all equally easy to apprehend) the optimal fines turn out to be less than those proposed by Becker. Polinsky and Shavell (1992) find that the optimal fine equals the harm, properly inflated for the chance of not being detected, plus the variable enforcement cost of imposing the fine.

In contrast with Becker, Akiba (1991) shows that an increase in the subjective probability of apprehension (or severity of punishment) does not necessarily lead to an unambiguously negative effect on crime. And Shavell (1990) tries to answer the question if one should also punish attempts and argues that the punishment of attempts increases deterrence by expanding the set of circumstances in which sanctions are imposed.

Becker's approach and most follow-up models in the literature on optimal punishment theory are confined to a *static* set-up. However, the severity of „crime" we will face tomorrow depends, at least in part, on the law enforcement strategy choosen today.

Leung (1991) showed that Becker's result that the optimal fine should be a multiple of the social costs is no longer valid in a dynamic environment. In contrast to the existing deterrence models which are based on a static framework and thus have to ignore recidivism (which is, in fact, a serious shortcoming), Leung (1995) can - through his micro-dynamic approach - incorporate recidivistic behaviour into his more general deterrence model. Among other things, the analysis confirms the familiar result that an increase in the certainty of punishment is more deterrent-effective than an increase in the severity. Such micro-dynamic extensions of Becker's static framework provide a useful platform for various research possibilities, e.g. that punishment depends on the offender's prior criminal record rather than on the offense rate at arrest time, problems of recidivism, etc.

Davis (1988) models on offender's choice of optimal crime rate if an increase in this rate lowers the expected time until detection. He studies implications of his model for the optimal enforcement of laws.

The paper is organized as follows. In section 2 we present a fairly general intertemporal optimization model providing a framework for an appropriate dynamic extension of Becker's static economic modeling of crime and punishment. As it has been demonstrated in several studies, economic modeling of crime and its control often leads to multiple equilibria. A more detailed mathematical formulation of the model is deferred to an appendix. In section 3, 'density-dependence' is identified as an important cause for nonlinearities implying multiple equilibria. In particular, the existence of separating thresholds or critial levels is discussed. Section 4 contains a (certainly non-exhaustive) sample of such effects of law enforcement which seem to be surprising (at least at the first look). An economic analysis of crime and punishment has to take into consideration that offenders act as rational agents. The complete paradigm amounts to a *dynamic game* between a law enforcement agency, offenders and victims. Section 5 illustrates the differential game approach by several examples. Finally, sec-tion 6 concludes with some remarks which might be valuable for future work in the dynamic economic modeling of crime and law enforcement.

2. Optimal Law Enforcement in an Intertemporal Setting

Let us start with some rather general observations on intertemporal optimization.

Virtually every optimization model starts with three questions:

- *What* should we do?

- *How* can we do it?

- What are the *constraints* to reach our goal?

The first point refers to the target which one tries to reach. The second question is that of the decision possibilities. The third ingredient deals with the impact of these decision variables on the reachability of a specific aim.

Thus, we *firstly* determine the social costs which arise for a law enforcement agency. These costs are the sum of the damages, costs of apprehension and conviction, and costs of carrying out the punishments imposed. These three components of the social loss generated by offenses have been discussed in detail by Becker (1968); see also below.

Second, let us turn to decision instruments. The authority's main economic decision variable are its expenditures on policy, courts etc. which help to determine the probability that an offense is uncovered and that the offenders are apprehended and convicted. In the simplest version the size of the punishment for those convicted and the form of the punishment are assumed to be constant.

The *third* issue are the constraints describing how the instrumental variables influence the social costs. In the dynamic case this refers to modeling the impact of the decision instruments on the change of the number of offenses. In the basic model (described below) this means the effect of law enforcement expenditures on the conviction probability and from those further to the number of offenses[1].

In particular, the following *dynamic* extension of Becker's *static* supply function of offenses[2] has been proposed by Caulkins (1990, 1993a) and others in the context of a macro-dynamic description of the movement of dealers into and out of local drug markets under police enforcement. Comparing the dealers with firms and illicit drug markets with industries, where free entry and exit ensures zero long-run profit, it is proposed that the rate of change of offenders depends on the expected utility from illegal activity compared with that from legal work. To model such a framework it is assumed that the potential criminals become offenders as soon as their individual utility expected from committing a crime exceeds the (average) income

[1] By assuming that each offender commits a constant number of offenses per unit time we may identify the number of offenses with those of the offenders.

[2] Becker (1968) assumes a static supply function relating the number of offenses by any person to his/her probability of conviction and to the punishment. In reality, however, the number of criminals at a certain time depends not only on the conviction probability and the fine at that time but also on the law enforcement in the past.

from an alternative, but legal activity. If their utility is smaller than the reservation wage, criminals will lower or even stop the number of offenses[3].

In addition to that, the dynamics of offenders is reduced by the rate of apprehended criminals.

Using such an offender's dynamics, Feichtinger et al. (1997) minimize the total discounted stream of social losses (as described above). By applying optimal control theory (whose importance in a dynamic approach of crime and punishment is discussed at the end of this section) they are able to prove an interesting 'threshold behaviour' of optimal law enforcement policies. In particular, this means that there exists a critial level for offenses, denoted by N_c, in the following sense. If the initial number of offenders N(0) is *above* N_c, then there is long-run 'high' equilibrium which is gradually approached (both from below and above as long as N(0) is greater than the threshold N_c). In economic terms, this means that the intertemporal trade-off between the damage from offenses and the law enforcement and punishment costs yields an upper (interior) equilibrium. This result answers the question 'how many offenses should be permitted' posed by Becker (1968) and mentioned at the begin of section 1. In addition to Becker, our dynamic analysis provides the optimal time path of law enforcement. It turns out that its structure makes economic sense.

However, if N(0) is *below* the critical level N_c then it is optimal to eradicate crime, i.e. it pays to enforce until the illegal market collapses. The steady-state equilibrium is at the (lower) boundary, and the law enforcement expenditures increase first, but finally decrease (see Fig. 1). For details see Feichtinger et al. (1997) [4]. A brief description of the model is given in the appendix of the present paper.

Fig. 1 about here

The very reason for this threshold-dependent optimal enforcement policy is a special non-linearity originating in the dependence of the conviction probability on the law enforcement expenditure *per offender* (see also the discussion in section 3)[5]. In most cases, it is the convexity of the Hamiltonian with respect to the state variable(s) implying this interesting result. However, it has been recently stressed, that the economically important threshold property is compatible with strict concavity (Wirl and Feichtinger, 1998).

---

[3] Note that the driving mechanisms of a such a system dynamics exhibits a certain similarity with the well-known replicator dynamics (see, e.g., Hofbauer and Sigmund, 1998).

[4] The occurence of multiple equilibria which are separated by thresholds is quite common in economic models. Skiba (1978) discussed such critical values, but his proof is incomplete. A first proof of existence was given by Dechert and Nishimura (1983); see also Long et al. (1997) as well as Wirl and Feichtinger (1998).

[5] Such thresholds occur in several economic models of crime; see Kort et al. (1995) and Tragler et al. (1997, sect. 6).

The model sketched above may be seen as a first step to extend Becker's static approach to an intertemporal setting. A more realistic analysis has to take into consideration that the law enforcement agency has available a diverse array of interventions with which it seeks to mitigate the consequences of crimes. Let us just mention two of them in addition to enforcement by police and courts: treatment of offenders and prevention. The relative efficacy of these instruments depends on the pattern in which the criminal activity considered varies over time.

It is an interesting fact that the use of illicit drug shows an inherent *epidemic* pattern. Behrens et al. (1997ab) have pointed out that for a number of illicit drugs a period of quiescence is typically followed by rapid escalation, a plateau, finally, and a gradual decline of drug consumption. It is quite plausible that the *optimal mix* of interventions will be different for differ-ent levels of drug use. An important research question is how the optimal mix of instruments might vary over time. For a given budget, we could ask whether the ratio of spending financial resources on prevention and spending on treatment should be higher at the beginning of a planning period than later on.

To answer this question one has to specify, among other issues, how imitation of offending depends on the available array of interventions. Preventive measures must interfere with the positive feedback effect of a network of juvenile offenders. Generally, it is a hard task to model both the 'inflow' and the 'outflow' of the offenders. Clearly, prevention dampens the initiation to drug use, while treatment increases the outflow of offenders, and enforcement does both. To specify the functional relationships in the dynamic supply of offenders one has to know how the socio-economic mechanisms are working. Beside of that, psychological, institutional, cultural and other variables will influence the number of offenders over time. A good example of the state of the art on the impact of enforcement and treatment on illicit drug consumption has recently been given by Tragler et al. (1997). The work of these authors illustrates how a more realistic system dynamics than the aforementioned replicator-like one can be identified and validated (at least partially) by empirical data.

In what follows we will describe a solution procedure which is well-suited for obtaining results on optimal enforcement polices as well as on the optimal mix of instruments of controlling offenses. The most efficient instrument in the tool-kit of intertemporal optimization models for the analysis of dynamic economic models is the maximum principle of Pontryagin and his associates. The main advantage of this solution procedure is its powerful capability to yield important *qualitative* insights into the structure of the optimal paths. More specifically, this means that the optimality conditions allow to derive a geometrical, i.e. qualitative des-cription of the shape of the solution trajectories without solving these conditions quantitative-ly (either analytically or, mostly, numerically). The qualitative approach of the maximum prin-ciple also implies the *robustness* of the solution paths in the following specific sense. The structure of the optimal trajectories does only depend on the qualitative properties of the un-derlying functions, like monotonicity, concavity etc. but *not* on the specific form of those mo-del functions.

Let us give an example of this very important fact which represents essentially the main reason of the ubiquitous application of the maximum principle in dynamic economics. In our basic model of dynamic law enforcement (briefly discussed above) the conviction probability depends on the per capita law enforcement expenditures, i.e. on the expenditures for police and courts divided by the number of offenders (or offenses). Clearly, it holds that this proba-bility increases with per capita expenditures. Beyond that, however, it turns out that the struc-ture of the optimal law enforcement policy is similar without respect of the special form of this function. In particular, it can be shown that a concavity shaped conviction probability yields essentially similar solutions as a linear one. Although the concavity assumption seems to be more realistic (decreasing efficiency of marginal conviction probability), a linear proba-bility has been chosen for our analysis, since it is mathematically more tractable (see also Feichtinger et al., 1997).

It is this robustness property of the maximum principle (in the aforementioned specific sense) which guarantees its importance in dynamic socio-economics. The chronical *deficit of empirical* data which prevails particularly in the economic modeling of crime does not allow to estimate neither the specific form of most functional relations nor the parameters. A striking example for this (sad but realistic) fact is provided by the dynamics of illicit drug consumers under the influence of enforcement, prevention and treatment (see Tragler et al. 1997). Although one has estimates of most of the model parameters based on empirical data, the selection of the special functions is often a matter of taste (and experience). Fortunately, the structure of the model solution frequently does not depend on this selection.

To summarize, it must be admitted that we only consider highly stylized models. However, although the dynamic models considered are simplified to an extent that they only are 'caricatures of the reality' they are anything else but useless. On the contrary, their simplicity allows us to exploit their optimality conditions such that substantial qualitative statements on the structure of the solution paths may be derived.

3. How the Macro Level Influences Microbehaviour: An Excursion to Nonlinearity

We might ask for the very reason of the 'threshold behaviour' of optimal law enforcement expenditures in the basic model discussed in section 2. Since the dependence of the optimal paths on the initial condition is due to the existence of *multiple equilibria* the question shifts to this property. Examining our model it is easily seen that the property is due to the depen-dence of the conviction probability on the total number of offenders N (beside those of E). Thus, the *macro* level, N, influences the *individual* conviction probability p.

This idea is of general importance in socio-economics. Before we illustrate this fact, let us very briefly mention the impact of such dependencies of micro-characteristics on the macro environment. It turns out that it is this dependence which creates *nonlinearities* being rich enough to generate complex solution structures. There are virtually dozens of examples in which the macro level influences micro characteristics. Actually, in the literature those effects are well-known. Some decades ago Schelling (1978) has already discussed 'micromotives and macrobehavior'.

In population dynamics and ecology the influence of a reference group or the environment to individual rates is also well-known and denoted as *'density-dependence'*. An interesting example fitting formally in our paradigm is the explanation of the *Easterlin cycle* in a simple nonlinear Leslie model by Samuelson (1976). By assuming that the age specific fertility rates depend negatively on the stocks, i.e. on the number of potential mothers, Samuelson's model is able to generate persistent oscillations.

Let us now focus to the macro-micro impact in the economics of crimes. In his excellent survey on the economics of corruption Andvig (1991) assumes that the utility an individual receives from a given action depends on the choices of others in that individual's reference group. For instance, in an environment where corruption is the norm it would be not rational to 'stay clean'. The general idea is conveyed through what Andvig calls a *Schelling diagram* (see Schelling, 1973, p. 388). Since the application in corruption is extensively described by Andvig (1991, p. 69-75), it is not repeated here. Instead we focus on two questions which Andvig tries to approach by using Schelling's diagram (Andvig, 1991, p. 59):

- Why do corruption levels vary rather strongly across nations and regions?

- How is it possible to explain that not more people are corrupt or clean?

The explanation hinges on the fact already mentioned above that the profitability of engaging in a corrupt transaction depends on the number of other people who do it, i.e. on the size of the reference group (and, more generally, on its structure).

It should be noted that Akerlof's (1980) theory of social customs to explain the existence of involuntary unemployment can be used to explain the persistence of corruption (see Dawid and Feichtinger, 1996a). Again the main assumption is that moral feelings of guilt by breaking the rules or social norms decrease as the number of rule breakers increases.

In his model Akerlof (1980) assumes that there exists a code of honour where the fraction of people believing in the code always adapts towards the fraction of people actually obeying it. A similar approach using social customs may be used to study the phenomenon that on one hand corruption is present in almost any organisation but on the other hand some people al-ways stay honest; see Dawid and Feichtinger (1996a). Bicchieri and Rovelli (1995) show for a dynamic population model that the transition from a corrupt to a honest equilibrium is possible if there are some people in the population who stay honest all times.

Another example for the idea that reference groups may be important in economic behaviour has been suggested by Schlicht (1981). He shows that small changes in profitability of corruption may cause large changes in observed behaviour.

As result of those ideas we conclude that the same kind of agents within the same kind of socio-economic system may through their *interaction* generate different levels of crime. Thus, models including social interactions in the specific sense described above typically possess *multiple equilibria*. This might provide a theoretical explanation of the empirical fact that offense levels and enforcement rates differ regionally and with respect to other characteristics.

Density-dependencies occur not only in biology and demography but also in various fields of sociology and economics. By social interactions, we refer to the idea that the utility an individual receives from a given action depends on the choice of other persons in a reference group. Recently, Brock and Durlauf (1995) developed a stochastic model to describe these spillover effects covering a wealth of charming examples. In the same spirit argue that social interactions can explain large differences in community crime rates.

One of the most interesting research avenues in the economics of crime seems to be the combination of the influences of the macro level to micro behaviour with intertemporal aspects of crime control. The already existing work in that direction suggests that substantial progress could be expected in the design of optimal law enforcement policies. In particular, threshold mechanisms and other inherent non-linearities generate multiple equilibria including boundary equilibria, instabilities and even more complex behaviour.

4. Some Surprising Effects of Law Enforcement

Law enforcement causes several, sometimes surprising and unintended or even contradicting, reactions. Basically, all enforcement efforts are intended to reduce the rate of offense.

The impact of anti-corruption campaigns depends crucially on certain non-linearities which are typical for epidemic processes like the spread of corruption. The threshold structure and multiplicity of equilibria suggest that after strong "cleaning measures" at the beginning of an anti-corruption campaign the momentum may move the system beyond the unstable equilibrium to a "clean" state. On the other hand the instability implies that even small shocks may lead the system dynamics to a high corruption level trap (compare Andvig, 1991).

A mechanism by which strict enforcement can increase crime is when conviction limits future labour market opportunities. Young males commit crimes for whatever reasons, but they typically ‘mature out’ of criminality. However, if while they are young, they are arrested and convicted of crime, particularly a felony, that black market may follow them for the rest of their lives, affecting their ability to get a job. This effect is all the stronger if they serve time in prison. Then, at older ages, crime will be relatively more appealing than it be otherwise would be because their licit labour market earning opportunities are more limited. That is something which can be analysed in a dynamic model.

Focusing on markets of illicit drugs, one might think that enforcement of prohibition will deter individuals from consumption and purchase and, consequently, reduce the trade of drugs. However, repression and an increase of repression, respectively, can, e.g., change the specific way of transaction. Dealers will try to avoid or reduce sales to strangers and, never-theless, create long-lasting vendor-customer relationships. In turn it becomes more difficult for the police to discover drug markets. The risk of high punishments can result in an increa-sed willingness for violence and threats to force maintenance of sales levels.

Benson and Rasmussen (1991), Benson et al. (1992) and Sollars et al. (1994) investigate the relationship among illicit drug use, property crime, police resources, and the allocation of police resources in models using data from the Florida counties. In general, people believe that drug offenders attempting to finance their habits are often responsible for property crimes, thus implying that increasing drug enforcement should reduce property crimes. Increasing drug enforcement might affect the property crime rate (and, by implication, the aggregate crime rate) in a different way, however: this kind of reallocation of given scarce police re-sources results in diminished rate of deterrence to commit property crimes and, as a result, an increase in the numbers of these crimes. Besides, an increase of drug enforcement in general implies higher prices of illicit drugs, requiring users to acquire greater resources which again may lead to increasing rates of offenses (cf. also Braun and Diekmann, 1993). More recently Dworak et al. (1998) developed a dynamic model in which they showed that the amount of drug enforcement expenditures depend both on the parameter settings, mainly the elasticity of demand of the drug and the social costs of drug use, as well as on the initial value of number of users.

Potential customers could be motivated to first-time consumption by offering them prices be-low the common price in the market while, on the other hand, the already addicted customers will face higher prices due to the increased risk of the suppliers. Higher prices can force the consumers of concern to commit thefts or get engaged in drug dealing themselves in order to finance their daily demand of a addictive illegal substance. This "terrible franchising system" (Wichmann, 1993) leads to an increased number of suppliers and, subsequently, to an enlar-ged group of consumers. This "snowball effect" seems to be challenging from a mathematical point of view.

There exist several reports concluding that a higher enforcement rate might lead to an increase of criminal offenses. Caulkins (1993b), e.g., points out that under plausible conditions applying so-called "zero-tolerance" policies (i.e. policies that impose fixed stiff sanctions for pos-session of any positive amount of an illicit drug, no matter how small the peculiar quantity are) can actually encourage users to consume more, not less, than they would if the punish-ment increased in proportion to the quantity.

Braun and Diekmann (1993) describe how a highly restrictive drug policy can result in a higher cumulative demand and lower prices of illicit drugs than a more liberal policy would do. One central problem of drug control is to evaluate the impact of enforcement on the illicit drug market, i.e. on the amount of the transactions, the situation of the addicts, the number of dealers. Braun and Diekmann (1993) provide an interesting survey on inefficiencies on illicit drug markets and negative side effects generated by repressive enforcement. They claim that neither a complete decontrol nor the opposite are optimal options. Concluding we can also say that increased enforcement can strengthen the power of the dealers in a market without lo-wering the amount of drugs consumed.

Another interesting aspect which occurs not only in case of illicit drugs but in all kinds of street crime is the fact that if jurisdiction is relatively tolerant to some type of crime and if neighbouring areas become relatively tougher on this crime, the tolerant jurisdictions will experience increasing crime activity (see Rasmussen et al. (1993) for a drug-related paper). These geographic effects lead to a kind of spatial prisoner’s dilemma which could, for ex-ample, be analysed with the help of Cellular Automata (cf. Nowak and May, 1993).

5. About a Game-Theoretic Approach of Law Enforcement

To obtain a realistic assessment of the enforcement's impact on criminal behaviour the authority has to include the offender's reaction to crime control. A complete economic analysis of crime and punishment should take into consideration the fact that offenders act as (bounded) rational economic agents. Thus, a multi-agent decision model seems to be the appropriate framework instead of the single decision maker (unilateral) approach mostly used up to now. To study the strategic interaction between criminals and law enforcement agents in an intertemporal setting, the theory of dynamic games provides the suitable framework.

To exemplify such a paradigm the representative offender is interpreted as first player whose decision variable is the intensity of his/her criminal activity considered. This player tries to maximize the discounted expected utility stream taking into consideration that the risk of being apprehended and convicted increases with his/her offense rate. The second player, the law enforcement agency, interacts with the criminal individual(s) via the conviction probability which also depends on the expenditures spent for enforcement. As in the optimal control scenario, this agent minimizes the discounted total stream of social losses over a planning period.

For such a game (or similar, more general competitive interactions of offenders and law enforcement agencies) various solution concepts should be calculated (open-loop, feedback, Stackelberg) and compared with each other. Up to now, however, only one-sided partial analyses have been carried out. This means that it has been studied one the one hand how law enforcement intensity depends on a given level of offending, and one the other hand how offending activities react on law enforcement and punishment for the latter (see, e.g. Caulkins 1993b, and Fent et al. 1997). Although such 'partial approaches' might yield valuable insight they are only preliminary steps to a complete game-theoretic analysis[6].

Based on a simple intertemporal optimization of an optimal pilfering thief by Sethi (1979), Feichtinger (1983) considered a differential game of one thief versus the police. In his model a risk-averse offender 'plays' against police whose objective function incorporates convex costs of law enforcement, a one-shot utility at the time of arrest and imprisonment costs. The probability that the offender is arrested at time t depends not only on the offending rate but also on the rate of law enforcement. The analysis delivered some interesting insight into the quali-tative behaviour of Nash equilibria. A remarkable asymmetry of the solutions has been shown: whereas the optimal law enforcement rate always increases, the monotonicity behavior of the thief depends on the constellation of the parameters, mainly on the elasticity of the utility functions.

In Dawid et al. (1996) a conflict between a potential criminal offender and a law enforcement agency is studied. The model is a two-stage extensive form game with imperfect information. It is shown that in equilibrium the offense rate and the law enforcement rate in the first period are always less or equal than the offense rate in the second period. The fact that both offense and enforcement rates are monotonically non-increasing from stage to stage is also established for

[6] A recent extension of the second reference is given by Fent et al. (1998).

multi-stage games, and it is shown that this property disappears if recidivistic behavior is present.

In general the nature of the interaction between criminals and law enforcers implies that the actions of the opposing side are neither easy to determine nor to anticipate. Thus it seems to be appropriate to model these interactions as a game between bounded rational players with a lack of information. Results and techniques from evolutionary game theory may be used to study the evolving behaviour of such a system of interacting individuals. For example, Antoci and Sacco (1995) use the well known replicator dynamics to describe the changing behaviour of a population where each individual can decide in each period whether (s)he will act ho-nestly or corrupt. In order to generate realistic models, it may be necessary to consider systems which can not be dealt with analytically. In this case, Genetic Algorithms or other population based simulation tools could be used to study the behaviour of such a system numerically. An example of this approach has been recently provided by Behrens and Dawid (1996). They model a game theoretic situation between dealers in an illicit open air drug market and crack-down officers within the framework of Genetic Algorithms. These heterogeneous populations rapidly converge towards a homogenous Nash-equilibrium. The success of the crackdown critically depends on the maximum possible enforcement activity and the minimum income of dealers from their illegal activity.

Wirl et al. (1997) analyzed a differential game between a corrupt agent and a law enforcement agency (or the tabloid press with market power). It turns out that the open-loop Nash equilibrium is not unique. The long run strategies are not constant but may follow a persistent limit cycle. The model provides an example of a dynamic game in which complex behaviour holds already in a one state model. In fact, indeterminacy is a generic property of the dynamics game.

Dawid and Feichtinger (1996b) studied a differential game between a (representative) dealer and the drug police. By using rather stylized assumptions they are able to calculate a Markov-perfect equilibrium. It turns out that a foresighted authority should attack the drug problem from the demand side and put much effect in treatment measures.

While law enforcement agents can be subsumed as one player in a first crude approximation this is by no means true for offenders. It is clear that the assumption of the *representative* criminal is an artifact pretending a homogeneity in the offender population which virtually never occurs. Dealers of illicit drugs, addicts, in general criminals are sometimes disorganized and atomized. In that case they cannot implement strategies that involve sacrifices for some members of their group even if they benefit the group as a whole. Even in organized criminality there are mostly several competing organizations (‘gangs’) making the concept of a two-person game elusive.

The conclusion of this state of the art is simply that dynamic games should and will play an important role in the future development of the economics of crime. To conclude this section let us consider the following dynamic game (see Fig. 2).

Fig. 2 about here

A law enforcement authority tries to interfere the interaction between criminals and ‘victims’. Here we have set quotation marks, since, among others, we are particularly interested in so-called victimless crimes. Consider, e.g., the symbiosis of illicit drug dealers and addicts which urgently need each other. Another example of such a positive feedback is corruption where the interaction between bribers and bribes would lead to a persistent growth of this evil without an intervention of the authority in this loop. Even by simplifying to a representative offender and a representative ‘victim’ the resulting three-person sum game is rather complex and too complex to allow a calculation of Markov-perfect solutions without substantial (and mostly unrealistic) simplifications.

A more realistic interpretation of this paradigm is to consider two interacting populations, one of offenders and the other is formed by ‘victims’. They are controlled by a single player, i.e. the law enforcement agency. An appropriate analysis could model the interaction of the two populations in terms of evolutionary games, while the interfering agency is ‘hierarchically above’ the game between the populations.

6. Concluding Remarks and Hints for Further Work

The main message of our brief survey is that ignoring intertemporal aspects of the economics of crime and law enforcement is not suitable anymore due to recent advances in modeling.

An effective analysis has to take into consideration that offenders act as rational agents. The complete paradigm amounts to a game between three competitors, namely offenders, victims and law enforcement agencies.

Inherent nonlinearities, e.g. to the impact of reference groups on microbehaviour, may generate optimal law enforcement strategies which will provide economic explanations of variations in crime frequency. Among others, rather surprising results can be derived in dynamic settings.

By reviewing the existing literature in the economics of crime and punishment, two striking features can be observed. First, there seem to be two different ‘strings’ of papers in law enforcement literature which are virtually not connected to each other. One is in the tradition of the Chicago school of economics and has been originated by Becker’s path-breaking 68’ paper. The other line of research was done by operations researchers and management scientists and is connected mainly with names like Blumstein (see the list of references), Larson (1972) and others. It seems that the separation of both research avenues is rather an institutional matter than a substantial one.

Second, and more important, a newcomer in this field might be surprised about the lack both of *dynamic* aspects as well as *game-theoretic* approaches in the economics of crime. Although there exist already a couple of papers on both issues, neither time nor strategic interactions seem to be mainstream law and economics. The purpose of the present paper is to collect some arguments to change this situation towards a more efficient approach to cope with crimi-nality.

In our opinion it would be an interesting and promising research strategy to apply instruments of dynamic optimization to a dynamic set-up of law enforcement. In section 2 it has been argued that optimal control techniques (in particular the maximum principle) can be applied to this inherently intertemporal field.

Aside from the intertemporal aspects of crime the second important feature of law enforcement which has not yet found due attention, is the *competitive* aspect. An adequate complete analysis of crime control has to take into consideration that law enforcement is not a problem with a single decision maker. Beside of the law enforcement authority (police or court, respectively) the offenders faced with the risk of being apprehended have to decide how intensive their illegal activities should be. Among other things, the optimal enforcement depends on the cost of catching and convicting offenders, the nature of punishment, and the responses of offenders to changes in enforcement etc. Their responses which are important for the question of deterrence are inherently a game-theoretic issue. Another problem which essentially asks for a game-theoretic treatment is the assessment of the effect of various punishment policies on the individual offense rate. Thus, the analysis of Caulkins (1993b) on zero-tolerance policies of drug consumption could be revisited and extended in a (Stackelberg) game framework. Combing the strategic interaction of the offender and the policemen with the intertemporal aspect amounts to the tool-kit of dynamic game theory (see section 5).

Let us now turn to another important possibility to extend the modeling in crime and law enforcement, namely to the heterogeneity of the offender population. Up to now most models in the field consider a homogenous group of criminals or, what is the same, deal with a representative offender.

However, when modeling the behaviour of people towards some special kind of crime one should think of the possibility of splitting the group of offenders into several subgroups characterised by different offending propensities or ‘crime levels’.

Especially in the case of drug control, this inclusion of heterogeneous aspects seems to be crucially important. The non-discriminating attitude of law enforcement agencies towards drug users with regard to what kind of drugs they use (i.e., users of "light" drugs such as mari-juana are usually punished equally heavy as users of "heavy" drugs such as cocaine, heroin, etc.) leads to the unintentional fact that users of light drugs tend to enter the group of heavy users, because the ratio between their costs and their utilities from drug use is too similar to that of heavy users. On the other hand, the familiar arguments of politicians are to stress the enforcement of light drugs, i.e. to prevent an enlargement of the group of novice users. One could also distinguish users of the same drug based upon their frequency of use. A simple dichotomous distinction between light and heavy users of the same illicit substance might help to understand the mechanisms of their contradictive influence to nonusers, which are responsible for the occurrence of a drug epidemic (see Behrens et al., 1997a,b).

A worthwhile task would be the construction of a model measuring the impact of enforcement on the consumption to assess the effectiveness of the whole spectrum of drug control policies (differentiated by the degree of repression). Assume that each non-user has a certain threshold to become a user of light drugs and the same is true for the transition to heavy drugs. Then Granovetter and Soong's threshold model for collective behaviour provides an appropriate framework to describe the dynamics between these three groups (see Granovetter and Soong, 1986). In a recent rational choice approach, Braun (1995a,b) shows that threshold distributions and threshold equilibria may result from benefit-cost distributions and network properties. Since the fact that individual transition behaviour is influenced by threshold mechanisms depending on the individual's social environment is empirically supported, the threshold approach should provide a useful framework for modeling the enforcement dynamics. Braun's threshold idea could yield a basis to compare the efficiency of enforcement policies for light drug control in the Netherlands and Germany or Austria, say.

Appendix

This appendix contains a brief description of the optimal law enforcement model sketched already in section 2.

Consider a law enforcement agency whose aim is to minimize the total discounted stream of social losses

$$\min_{E(.)\geq 0} J[E(.)] = \int_0^\infty \exp(-rt)[D(N(t)) + C_1(E(t)) + C_2(N(t), E(t))]dt. \qquad \text{(A.1)}$$

The number of offenders[7] at time t, N(t), is the state variable, whereas the law enforcement rate E(t) acts as control variable[8]. The discount rate $r > 0$ measures the time preference rate and is assumed to be constant.

As in Becker's (1968) static analysis the social cost can be divided into three components:

- the damage D(N) caused by offenders,
- the cost of apprehension and conviction, $C_1(E)$,
- the cost of punishment, $C_2(N, E)$.

The cost functions D(N) and $C_1(E)$ increase monotonically and are convex with respect to N and E, respectively. Together with $C_2(N, E)$, they are assumed to be sufficiently smooth. For a more detailed discussion see Becker (1968) and Feichtinger et al. (1997).

---

7 By assuming a homogeneous population of offenders we may identify the number of offenses per time unit with that of the offenders.

8 The enforcement rate E transforms the input of manpower, material and capital to an amount of police and court activities per time unit (compare Becker, 1968).

For simplicity the cost functions are specified as follows[9]

$$D(N) = N^2, \quad C_1(E) = \frac{c}{2}E^2, \quad C_2(N, E) = dE.$$

To derive the dynamics of offenders already sketched in section 2 we denote by y the income an offender draws from the illegal activity, and by p the probability that an offense will result in a conviction. Assuming for simplicity a linear utility function, u(y) = y, and a constant fine, f = 1, the expected utility of an offense is given by

$$\mathrm{IE}\,(u) = y - pf = y - p.$$

The key assumption driving the model is that the conviction probability depends on the law enforcement rate per offender, e = E/N, in a concave manner, i.e. p = p(e) with

$$p'(e) > 0, \quad p''(e) \le 0.$$

Now we are able to specify the dynamics of the offenders as follows

$$\dot{N}(t) = \kappa[\beta - p(e(t))] - \alpha p(e(t))N(t), \tag{A.2}$$

where β = y-w, w being the average wage rate of the legal activity. Clearly, β must be positive. The proportionality constant κ adjusts the dimension in the r.h.s. of (A.2). Without loss of generality we set $\kappa = 1$. The rate α refers to imprisonment of the convicted offenders.

Again for simplicity it is assumed that

$$p(e) = pe$$

with constant slope p > 0.

To satisfy $0 \le p(e) \le 1$ we have $0 \le e \le p^{-1}$, i.e. the inequality

$$N(t) - pE(t) \ge 0 \tag{A.3}$$

must be satisfied for all t.

Finally, we introduce a (small) level of undetectable offenders, $\underline{N} > 0$, which adds the inequality

$$N(t) - \underline{N} \ge 0 \tag{A.4}$$

for all t.

---

[9] See Feichtinger et al. (1997) for a justification of the cost function C_2.

To summarize, the authority wants to minimize the total social loss in (A.1) subject to the state dynamics (A.2) for a given initial state N(0) = N_0 ≥ 0. This provides a deterministic op-timal control problem with infinite planning horizon, one state variable (N), one control (E), a mixed path constraint (A.3) and a pure state constraint (A.4). Note that for $C_1'(0) = 0$, which is satisfied for $C_1(E) = (c/2)E^2$, the non-negativity of the control variable E is automatically satisfied.

Some results on the (N,E) phase portrait of this intertemporal optimization model are mentioned in section 2. It can be shown that the qualitative behaviour of the solution, e.g. the existence of a critical point N_c, is rather robust with respect to the specification of the model functions.

List of References

Akerlof, G.A., 1980, A theory of social custom, of which unemployment may be one consequence, *The Quarterly Journal of Economics 94*, 749-775.

Akiba, H., 1991, The deterrent effect reconsidered: the minimum-time approach, *The Journal of Socio-Economics 20(2)*, 181-192.

Andvig, J.C., 1991, The economics of corruption: a survey, *Studi economici 43(1)*, 57-94.

Antoci, A. and P.L. Sacco, 1995, A public contracting evolutionary game with corruption, *Journal of Economics 61(2)*, 89-122.

Bebchuk, L.A. and L. Kaplow, 1993, Optimal sanctions and differences in individuals' likelihood of avoiding detection, *International Review of Law and Economics 13*, 217-224.

Becker, G.S., 1968, Crime and punishment: an economic approach, *Journal of Political Economy 76*, 169-217.

Behrens, D., J.P. Caulkins, G. Tragler, J.L. Haunschmied and G. Feichtinger, 1997a, A dynamic model of drug initiation: implications for treatment and drug control, Forschungsbericht **213** des Instituts für Ökonometrie, OR und Systemtheorie, TU Wien, Dezember (forthcoming in *Mathematical Biosciences*).

Behrens, D., J.P. Caulkins, G. Tragler and G. Feichtinger, 1997b, Controlling the US cocaine epidemic: finding the optimal mix of drug prevention and treatment, Forschungsbericht **214** des Instituts für Ökonometrie, OR und Systemtheorie, TU Wien, Dezember.

Behrens, D. and H. Dawid, 1996, Genetic learning on illicit drug markets: can it lead to a crackdown? Forschungsbericht **206** des Instituts für Ökonometrie, OR und Systemtheorie, TU Wien, Dezember.

Benson, B.L. and D.W. Rasmussen, 1991, Relationship between illicit drug enforcement policy and property crimes, *Contemporary Policy Issues 9(4)*, 106-115.

Benson, B.L., I. Kim, D.W. Rasmussen and T. W. Zuehlke, 1992, Is property crime caused by drug use or by drug enforcement policy?, *Applied Economics 24(7)*, 679-692.

Bicchieri, C. and C. Rovelli, 1995, Evolution and revolution, *Rationality and Society 7(2)*, 201-224.

Blumstein, A. and J. Cohen, 1973, A theory of the stability of punishment, *Journal of Criminal Law and Criminology 64(2)*, 198-207.

Blumstein, A. and D. Nagin, 1981, On the optimum use of incarceration for crime control, *Operations Research 26(3)*, 381-405.

Blumstein, A. et al. (eds), 1978, *Deterrence and Incapacitation: Estimating the Effects of Criminal Sanctions on Crime Rates*, National Academy of Sciences, Wash DC.

Braun, N., 1995a, Individual thresholds and social diffusion, *Rationality and Society 7(2)*, 167-182.

Braun, N., 1995b, The threshold model revisited, submitted to *Journal of Mathematical Sociology*.

Braun, N. and A. Diekmann, 1993, Drogenschwarzmarkt und KonsumentInnensituation: Einige Ergebnisse der Berner Szenebefragung, *Drogalkohol 17*, 161-182.

Brock, W.A. and S.N. Durlauf, 1995, Discrete choice with social interactions I: theory, Working Paper 95-10-084, Santa Fe Institute.

Caulkins, J.P., 1990, *The Distribution and Consumption of Illicit Drugs: Some Mathematical Models and Their Policy Implications*, Ph. D. Dissertation, M.I.T.

Caulkins, J.P., 1993a, Local drug markets' response to focused police enforcement, *Operations Research 41(5)*, 848-863.

Caulkins, J.P., 1993b, Zero-tolerance policies: do they inhibit or stimulate illicit drug consumption?, *Management Science 39(4)*, 458-476.

Davis, M.L., 1988, Time and punishment: an intertemporal model of crime, *Journal of Political Economy 96(2)*, 383-390.

Dawid, H. and G. Feichtinger, 1996a, On the persistence of corruption, *Journal of Economics* **64**, 177-193.

Dawid, H. and G. Feichtinger, 1996b, Optimal allocation of drug control efforts: a differential game analysis, *Journal of Optimization Theory and Applications 91*, 279-297.

Dawid, H., G. Feichtinger and S. Jorgensen, 1996, Crime and law enforcement: a multistage game, Forschungsbericht **201** des Instituts für Ökonometrie, OR und Systemtheorie, TU Wien, August (forthcoming in to *Annals of Dynamic Games*).

Dechert, W.D. and K. Nishimura, 1983, A complete characterization of optimal growth paths in an aggregated model with a non-concave production function, *Journal of Economic Theory 31(2)*, 332-354.

Dworak, M., G. Feichtinger, G. Tragler and J.P. Caulkins, 1998, On the effect of drug enforcement on property crime, Forschungsbericht **215** des Instituts für Ökonometrie, OR und Systemtheorie, TU Wien, Jänner.

Feichtinger, G., 1983, A differential games solution to a model of competition between a thief and the police, *Management Science 29(6)*, 686-699.

Feichtinger, G., W. Grienauer and G. Tragler, 1997, Optimal dynamic law enforcement, Forschungsbericht **197** des Instituts für Ökonometrie, OR und Systemtheorie, TU Wien, September (submitted to *Review of Economic Dynamics*).

Fent, T., G. Feichtinger and G. Tragler, 1998, A dynamic Stackelberg game of offending and law enforcement, Forschungsbericht **219** des Instituts für Ökonometrie, OR und Systemtheorie, TU Wien, Juni.

Fent, T., M. Zalesak and G. Feichtinger, 1997, Optimal offending in view of the criminal record. Forschungsbericht **198** des Instituts für Ökonometrie, OR und Systemtheorie, TU Wien, Oktober (submitted to *CEJOR*).

Granovetter, M., and R. Soong, 1986, Threshold models of interpersonal effects in consumer demand, *Journal of Economic Behavior and Organization 7*, 83-99.

Hofbauer, J. and K. Sigmund, 1998, *Evolutionary Games and Population Dynamics*, Cambridge University Press, Cambridge.

Kort, P., G. Feichtinger, R.F. Hartl and J.L. Haunschmied, 1998, Optimal enforcement policies (crackdowns) on a illicit drug market, *Optimal Control Applications and Methods* **19**, 169-184.

Larson, R.C., 1972, *Urban Police Patrol Analysis*, MIT Press, Cambridge, MA.

Leung, S.F., 1991, How to make the fine fit the corporate crime? An analysis of static and dynamic optimal punishment theories, *Journal of Public Economics 45(2)*, 243-256.

Leung, S.F., 1995, Dynamic deterrence theory, *Economica 62*, 65-87.

Long, N.V., K. Nishimura and K. Shimomura, 1997, Endogenous growth, trade, and specialization under variable returns to scale: the case of a small open economy, in: B. Jensen and K. Wong ed., *Dynamics, Economic Growth and International Trade*, Michigan University Press.

Malik, A.S., 1990, Avoidance, screening and optimum enforcement, *RAND Journal of Economics 21(3)*, 341-353.

Maltz, M.D., 1994, Operations research in studying crime and justice: its history and accomplishments, in: S.M. Pollack et al., Eds., *Handbooks in OR & MS 6*, 201-262.

Maltz, M.D., 1996, From Poisson to the present: applying operations research to problems of crime and justice, *Journal of Quantitative Criminology 12(1).*

Nowak, M.A. and R.M. May, 1993, The spatial dilemmas of evolution, *International Journal of Bifurcation and Chaos 3(1)*, 35-78.

Polinsky, A.M. and S. Shavell, 1991, A note on optimal fines when wealth varies among individuals, *The American Economic Review 81(3)*, 618-621.

Polinsky, A.M. and S. Shavell, 1992, Enforcement costs and the optimal magnitude and probability of fines, *Journal of Law & Economics 35(1)*, 133-148.

Rasmussen, D.W., B.L. Benson and D.L. Sollars, 1993, Spatial competition in illicit drug markets: the consequences of increased drug law enforcement, *Review of Regional Studies 23(3)*, 219-236.

Samuelson, P.A., 1976, An economist's non-linear model of self-generated fertility waves, *Population Studies* 30, 243-247.

Schelling, T.C., 1973, Hockey helmets, concealed weapons, and daylight saving: a study of binary choices with externalities, *Journal of Conflict Resolution 17(3)*, 381-428.

Schelling, T.C., 1978, *Micromotives and Macrobehavior*, Norton, New Nork.

Schlicht, E., 1981, Reference group behaviour and economic incentives: a remark, *Zeitschrift für die gesamte Staatswissenschaft 137*, 125-127.

Sethi, S.P., 1979, Optimal pilfering policies for dynamic continuous thieves, *Management Science 25(6)*, 535-542.

Shavell, S., 1990, Deterrence and the punishment of attempts, *Journal of Legal Studies 19(2)*, 435-466.

Skiba, A.K., 1978, Optimal growth with a convex-concave production function, *Econometrica 46(3)*, 527-539.

Sollars, D.L., B.L. Benson and D.W. Rasmussen, 1994, Drug enforcement and the deterrence of property crime among local jurisdictions, *Public Finance Quarterly 22(1)*, 22-45.

Tragler, G., J.P. Caulkins and G. Feichtinger, 1997, Optimal dynamic allocation of treatment and enforcement in illicit drug control, Forschungsbericht **212** des Instituts für Ökonometrie, OR und Systemtheorie, TU Wien, Dezember.

Wichmann, S., 1993, Gesetze der Ökonomie, *Wirtschaftswoche 47(44)*, 50-59.

Wirl, F. and G. Feichtinger, 1998, Conditions for and existence of unstable equilibria in concave intertemporal optimizations, Working Paper, Otto-von-Guericke University of Magdeburg.

Wirl, F., A. Novak, G. Feichtinger and H. Dawid, 1997, Indeterminacy of open-loop Nash equilibria: the ruling class versus the tabloid press, in: Y. Ben-Haim, *Uncertanty: Models and Measures*. Akademie-Verlag, 124-136.

List of captions:

Fig. 1: Phase portrait of the state-control space, i.e. of the (N-E)-diagram. The basins of attraction of the low ‘equilibrium (instable focus) and the ‘high’ equilibrium (saddle-point) is separated by the critical level N_c.

Fig. 2: A stylized scheme of a three-person non-zero sum dynamic game between a law enforcement agency, the population of offenders and the ‘victims’.

Fig. 1

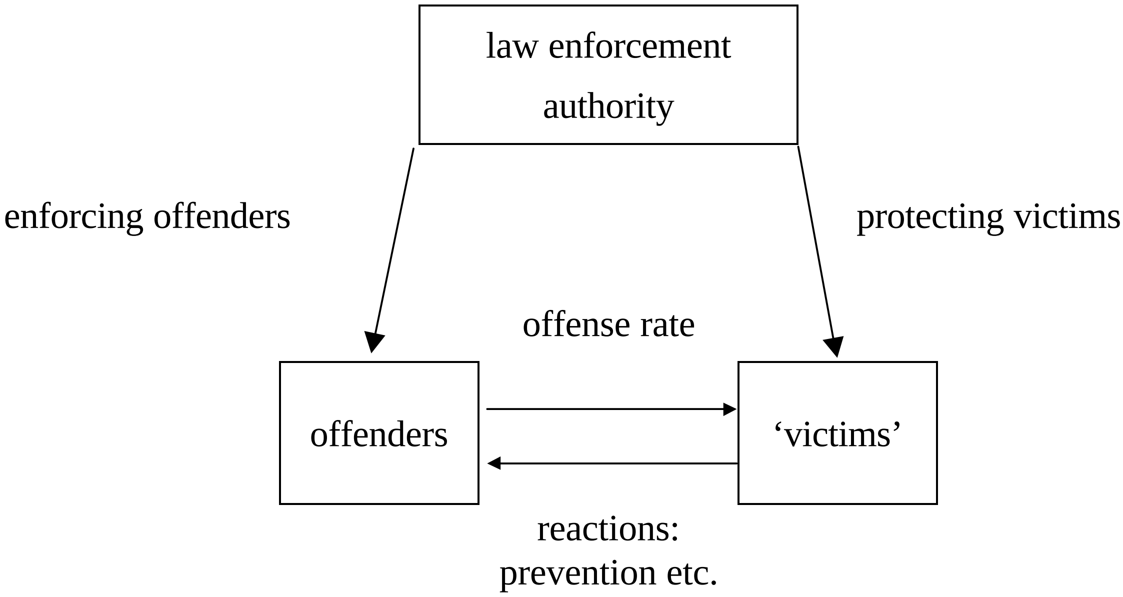
law enforcement
authority
enforcing offenders
protecting victims
offense rate
offenders
‘victims’
reactions:
prevention etc.

Fig. 2