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Working Paper No. 20

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Production, Distribution and Insecurity of Food: A dynamic framework*

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*This paper arose out of ongoing collaborative work on interactive simulation models on Population, Environment, Development and Agriculture (subsequently referred to as PEDA) in Sub Saharan African countries. The project is coordinated by the UN Economic Commission for Africa and currently simulation models are developed for various Sub-Saharan African countries at the International Institute of Applied System Analysis (IIASA) in Laxenburg, Austria. The authors are grateful to W. Lutz and S. Sherbov, the main investigators of the PEDA approach at IIASA, for helpful comments and discussions.

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Abstract

We study the impact of food distribution on the steady-state portion of food-insecure people in a stationary population. By applying a descriptive model we illustrate the positive feedback between food insecurity, low productivity in production and inequalities in food distribution. Under these assumptions multiple steady states of the population distribution may result that differ from each other in the share of food-insecure people.

Keywords: Inequality, food insecurity, multiple steady states, population distribution, less developed countries.

JEL classification: C62, I32, Q18

1 Introduction

As stressed in the work of Amartya Sen (1981), the distribution of food is at least as important as the total production of food in explaining food insecurity. 'Starvation is the characteristic of some people not *having* enough food to eat. It is not the characteristic of there *being* not enough food to eat. While the latter can be a cause of the former, it is but one of many *possible* causes.'(Sen, 1981, p. 1)

The debate on the ultimate 'carrying capacity' of the Earth's land and water resources as provoked by the unprecedented growth in the world's population over the past five decades is a good example where the importance of food distribution is ignored. This debate 'overlooks a crucial aspect of the problem, the world's absolute food supply is almost certainly sufficient for six billion or more people now and in the future, yet some 841 million people – nearly one sixth of the world's population – are chronically malnourished today.' (UNFPA, 1996, p.1)

There exists now abundant empirical evidence, on the global as well as the local and household level, that food is neither produced nor consumed equitably. For instance, in Sub-Saharan African countries women often produce up to 90 per cent of all food that is consumed by their families. However their entitlement to consumption is often the lowest within the family. (UNFPA, 1996)

The aim of this paper is to discuss a minimal model — in the sense that it has the lowest possible number of state variables, namely one — that provides a descriptive framework to relate food insecurity to food production and food distribution on the aggregate macroeconomic level. Food security status is solely determined by the food distribution that is assumed to be historically given. However, since food production depends on the population distribution (the share of food-secure versus food-insecure people) the amount of food to be distributed will vary over time.

Our objective is to analyse the dynamic planar system that describes the share of the food-insecure and food-secure population, respectively. We demonstrate that multiple steady states may emerge depending on the functional form of the food distribution function. The latter result has to do with the phenomena of 'history dependence'. This means that the initial population distribution will determine the long-run equilibrium population distribution.[1]

The remainder of the paper is organised as follows. In section 2 we discuss the model, and in section 3 we present the dynamics and apply numerical bifurcation analysis. We conclude with a general discussion of our model framework.

2 The Model

At each time point t, population P_t consists of two groups denoted by P_t^S and P_t^I with $P_t = P_t^S + P_t^I$. A person belongs to group P_t^S or P_t^I, respectively, if he or she receives a level of food that exceeds (falls short of) $\tilde{y}$. In what follows

[1] In a substantively different but methodological related context, Kremer and Chen (1999) show that the positive feedback between fertility differentials and income inequality may lead to multiple steady states of the long-run population distribution.

we denote the threshold level $\tilde{y}$ as the minimum level of calories necessary to be food secure. In order to concentrate on the food distribution mechanism as it effects food security status we assume a stationary level of the total population, i.e. $P_t = P$.[2]

Similarly to Dasgupta and Ray (1987) we postulate that the production of food requires labour and land as input factors. In food production there are constant mutual returns to scale in labour and land. We assume that land L is given for the economy in fixed quantity (see Dworak et al., 2000, for a variable stock of land). This implies that for any given level of total factor productivity there are decreasing returns to labour in the production of food. We assume food production to have a Cobb Douglas form. Given these implicit assumptions production of food is

$$Y_t = L(h^I P_t^I + h^S P_t^S)^\beta; \qquad L \equiv 1, 0 < \beta < 1 \tag{1}$$

where h^I and h^S denote constant values of labour efficiency of the food-insecure and food-secure population, with $h^I < h^S$. This latter assumption relies on the fact that 'a person's consumption intake affects his productivity'. (Dasgupta and Ray, 1987, p. 177)

At the end of each time period the total amount of food available Y_t is distributed among the population. We postulate a historically given food distribution function which is represented by a Lorenz curve. This curve plots cumulative shares of food $L(F(z))$ as a function of cumulative population

[2] By assuming a stationary population, the total population will be equal to a fixed constant during each time period. Setting this constant equal to 100 allows for the convenient notation for sub-populations in terms of percentage. The case of a non-stationary population is presented in Dworak et al. (2000).

shares $F(z)$ when individuals are ranked in increasing order of the food z that they receive. Food distribution together with the total amount of food Y_t to be distributed in each time period determine the share of food-secure P^S_{t+1} and food-insecure people P^I_{t+1}, respectively, in the following period.

Recalling some basic mathematical properties of the Lorenz curve we can derive an analytical expression for the share of the food-insecure population P^I_t in each time period.[3] We exploit the fact that the slope of the Lorenz curve at any point $F(z)$ is inversely proportional to per capita food production $y = Y/P$ and proportional to the corresponding food level z (see Appendix 1 for the derivation of this result)

$$\frac{dL(F(z))}{dF(z)} = l(F(z)) = \frac{1}{y}z. \tag{2}$$

Assuming further that we can analytically solve for the inverse of the derivative of the Lorenz curve, (2) can be written as

$$F(z) = l^{(-1)}\left(\frac{z}{y}\right). \tag{3}$$

The corresponding level of food z_{max} for which $F(z_{max}) = 1$ holds indicates the maximum level of food in the economy for which 100% of the population receives food less than z_{max}. Equation (3) then implies that the maximum level of food will be constrained by the prevailing level of per capita food production y and the functional form of the Lorenz curve. Unless the economy is in a stationary state, per capita food production y and henceforth the maximum level of food z_{max} will vary over time.

[3]To simplify the notation we shall skip the time argument in the subsequent mathematical derivations.

Recalling that $\tilde{y}$ indicates the threshold level of food a person needs to be food secure, equation (3) evaluated at $z = \tilde{y}$ gives the proportion of the food-insecure population as a function of the level of per capita output y in the previous period. When $\tilde{y}$ exceeds the maximum value of food z_{max}, the entire population will be food insecure in the next period and the share of food-insecure population will be equal to one.

The dynamic evolution of the share of food-insecure people is therefore given by the first order difference equation

$$P^I_{t+1} = \begin{cases} l^{(-1)}(\frac{\tilde{y}}{y_t})P & \text{if } \tilde{y} \leq z_{max} \\ P & \text{otherwise.} \end{cases} \tag{4}$$

with per capita food production $y_t = y(P^I_t)$ being a function of the population distribution of the previous period and the constant P denoting the stationary total population level.

3 Dynamics and bifurcation analysis

For the numerical analysis we postulate the Lorenz curve[4]:

$$L(F(z)) = (F(z))^\alpha \qquad \alpha > 1. \tag{5}$$

By using this specific form of the Lorenz curve, equation (3) reduces to

$$F(z) = \left(\frac{z}{\alpha y}\right)^{1/(\alpha-1)}. \tag{6}$$

[4]See Chotikapanich (1993) for alternative functional forms of the Lorenz curve.

Recalling that equation 4 allows us to solve for for the equilibrium level $\bar{P}^I$ of the food-insecure population, we have:

$$\bar{P}^I = h(\bar{P}^I) = \begin{cases} P(\frac{\tilde{y}}{\alpha y(\bar{P}^I)})^{1/(\alpha-1)} & \text{if } \tilde{y} \leq \alpha y(\bar{P}^I) \\ P & \text{otherwise.} \end{cases} \quad (7)$$

Proposition: *Equation* (7) *determines at least one and at most three different stationary population distributions* $(\bar{P}^I, \bar{P}^S = 100 - \bar{P}^I)$.

Proof (graphical sketch)[5]: The long-run distribution of the population is given by the equilibria of the map $h(P^I) : [0, P] \to [0, P]$ (Figure 1). h is strictly positive if there is no food-insecure population and it monotonically increases with the number of food-insecure people, becoming a horizontal line if the level of the maximum food entitlement $\alpha y(P^I)$ is less than $\tilde{y}$. The shape of h implies that the implicit equation (7) will have at least one and at most three equilibria, as illustrated in Figure 1.

Recalling that the difference between the level of the food-insecure population (as represented by the 45^o line) and the function $h(.)$ is equal to the share of the population that becomes food secure each period, an equilibrium will be stable if $h(P^I)$ intersects the 45^o-line from above (e.g., at point A). In this case any deviation to the left of the equilibrium implies that the size of the food-insecure population is below that of the population that falls short of the subsistence requirement. Hence the share of the food-insecure population will increase and converge to the equilibrium from the left. On the other

[5] A formal proof of the multiplicity of the equilibria and their stability is given in Appendix 2.

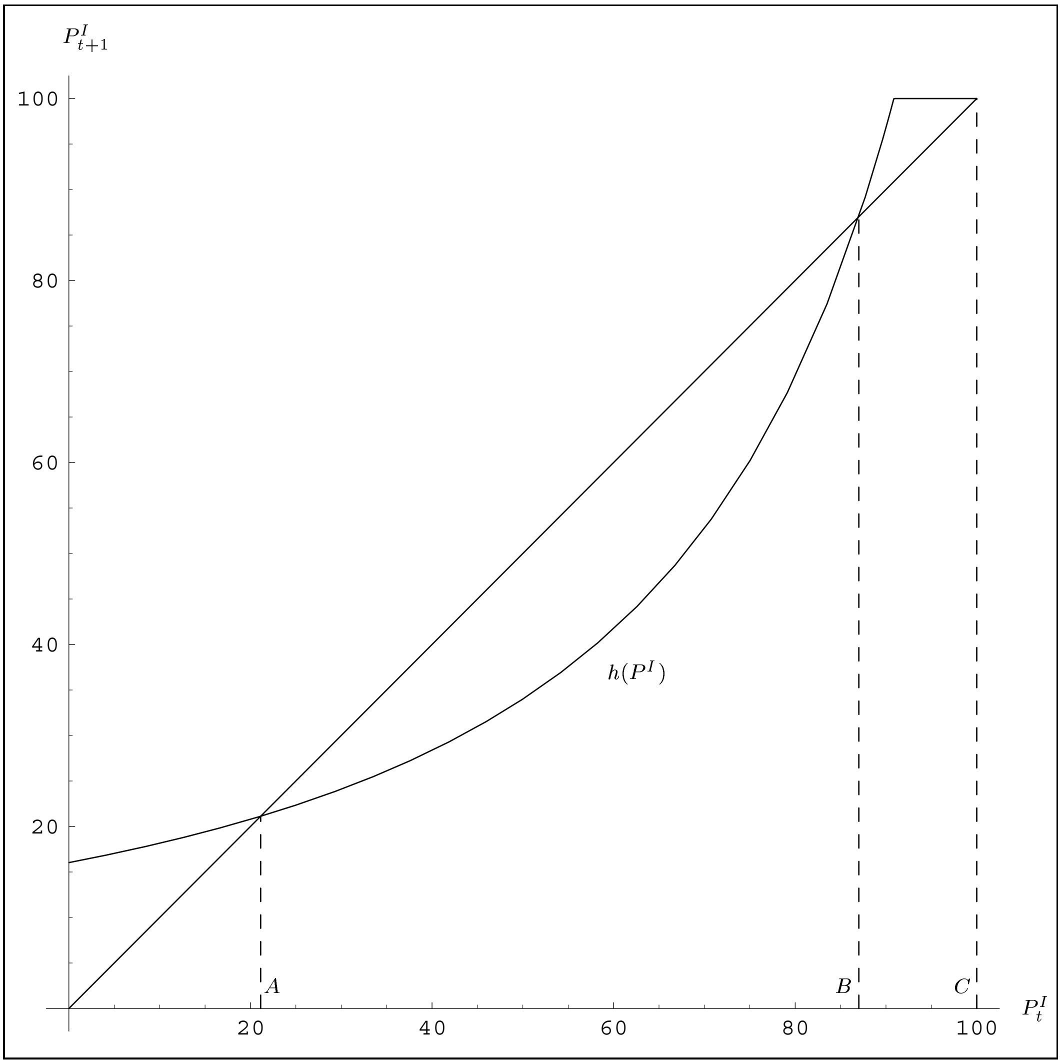

Figure 1: Map $h(.)$ for the parameter values $\alpha = 1.5, \beta = 0.8, h^I = 0.2, h^S = 0.8, P = 100, \tilde{y} = 0.2$.

hand, any deviation to the right of the equilibrium implies that the share of the food-insecure population exceeds the share of the population that falls short of the subsistence level and henceforth the share of the food-insecure population will decrease and approach the equilibrium from the right. On the basis of these considerations one can easily verify that the equilibria A and C are stable while the equilibrium B is unstable.

The existence of one unstable and two stable equilibria implies that history will determine the long-run population distribution.[6] If the initial share of the food-insecure population falls to the right of the unstable equilibrium B, the equilibrium C, where the entire population is food insecure, will be approached. For initial shares of the food-insecure population to the left of the unstable equilibrium B, the lower stable equilibrium A will be approached. This ends the graphical exposition of the proof of the proposition.

Besides history, the degree of inequality in the food distribution function, as represented by the parameter α, will also influence the long-run distribution of the population. A change in the parameter α is less straightforward, as is evident from equation (7). Intuitively an increase in the degree of inequality has two effects. Ceteris paribus it will increase the share of the food-insecure population as more food is distributed to the upper classes. But as more food is distributed to the upper classes the maximum food entitlement in the economy as given by αy will increase and henceforth the

[6] In a substantively different but methodological related context, Kremer and Chen (1999) show that the positive feedback between fertility differentials and income inequality may lead to multiple steady states of the long-run population distribution.

probability that the entire population will be food insecure will decline.

Plotting the long-run value of the percentage of the food-insecure population against the degree of inequality in the food distribution α (Figure 2) confirms these considerations[7]. For low values of α, history (in terms of the initial population distribution) will determine the long-run population distribution where the dotted line separates the domain of attraction to either equilibrium. Obviously, if there is no persisting inequality in the food distribution, i.e., $\alpha = 1$, all people will be either food secure $\bar{P}^I = 0$ or food insecure $\bar{P}^I = 1$, depending on whether the stationary value of per capita food production $y(\bar{P}^I)$ is higher or lower than the subsistence requirement $\tilde{y}$.[8] As the degree of inequality increases, the maximum level of food in the economy αy increases so that at least part of the population will be food secure and the equilibrium where all the population is food insecure no longer exists. At the same time, an increase in the degree of inequality will increase the share of the food-insecure population.

The existence of multiple equilibria allows for the possibility of an hysteresis effect as α is varied. Suppose, we start from an equilibrium configuration of the population where everyone is food insecure (e.g. at $\alpha = 1.5$ in Figure 2) and increase the parameter α. The equilibrium will remain at $\bar{P}^I = P$ until the equilibrium vanishes at α_1 causing a jump to the lower stable equilibrium.

[7]For a more mathematical argument see Appendix 3.

[8]Note that for $\alpha = 1$ everyone obtains the mean food level and equation (7) reduces to

$$\bar{P}^I = h(\bar{P}^I) = \begin{cases} 0 & \text{if } \tilde{y} \leq y(\bar{P}^I) \\ P & \text{otherwise.} \end{cases}$$

If we then decrease α again the equilibrium stays on the lower equilibrium branch even if we pass the value α_1. This lack of reversibility as a parameter is varied is called hysteresis.

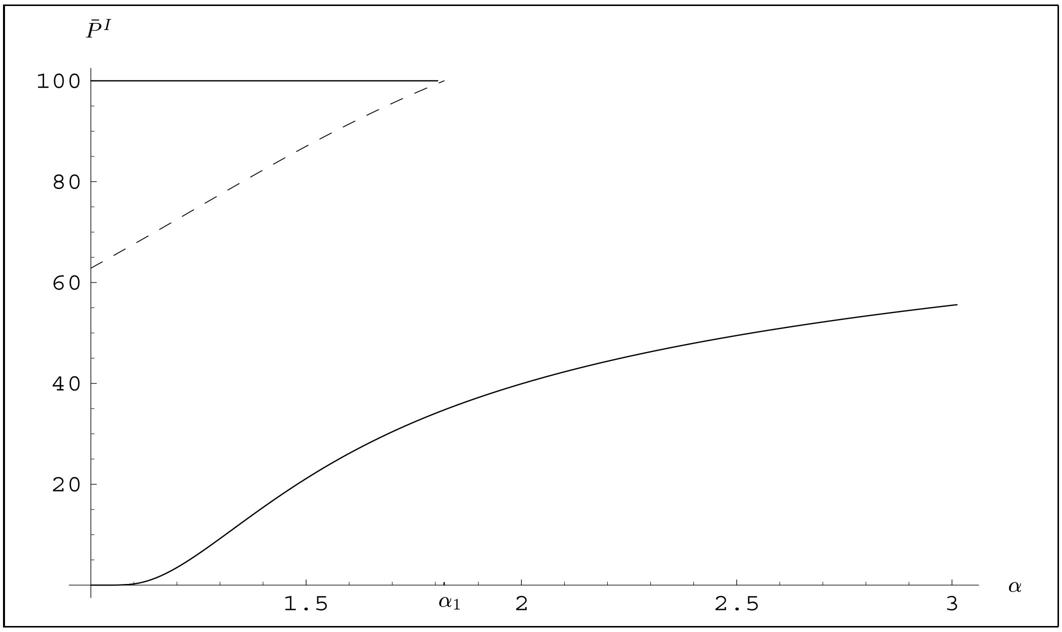

Figure 2: Bifurcation diagram with respect to the parameter α. All other parameters are set as in Figure 1.

4 Conclusions

The role of food distribution in determining food security status is undisputed in the literature. However, as we have illustrated in this short note, the initial population distribution may matter as well. A mere change towards a more equal food distribution may not be the panacea for reducing food insecurity. To increase the productivity of food production and hence total output is of equal importance, particularly so in a situation where the initial share of food-insecure people is high and hence food production low.

Though, our mathematical framework relates to the dynamic link between food production, food distribution, and food insecurity our set up is more general. It offers a framework for studying how various population distributions may originate depending on the mechanism that defines the membership to a specific group.

For instance, the sub-populations could represent students who either pass or fail an exam, with the threshold $\tilde{y}$ representing the minimum number of points needed to pass the exam. The dynamic variable 'time' is replaced by the number of exams taken. At each exam the overall performance of all students will determine the mean number of points obtained (as given by our variable y) and consequently determine the share of students who pass the exam. Like with the food production function we can assume differing efficiency parameters for students in their exams. The assumption of a historically given 'point distribution' function for each exam may in fact be quite a realistic assumption in this context.

Finally, we would like to add that the work presented in this short note is part of an ongoing project on modeling the interactions between changes in the population size and distribution, natural resource degradation, agricultural production, and food security (see Lutz et al., 2000). Relaxing the assumption of a stationary population and introducing the dynamics of the resource stock permits even more complex dynamics such as, e.g., a limit cycle.

Appendix 1

To determine the slope of the Lorenz curve we proceed as follows.

Given an income distribution function with probability density function $f(x)$, the horizontal axis of the Lorenz curve is given by the cumulative distribution function

$$F(z) = \int_0^z f(x)dx$$

and the vertical axis is given by the first moment distribution function

$$L(F(z)) = \frac{1}{y}\int_0^z xf(x)dx = \frac{1}{y}\left[zF(z) - \int_0^z F(x)dx\right] \tag{8}$$

where y denotes the mean income (i.e. $y = Y/P$) and the second identity has been derived by applying partial integration (see Atkinson, 1970 and Lam, 1988). Differentiating equation (8) with respect to z yields

$$\frac{dL(F(z))}{dz} = \frac{1}{y}zf(z). \tag{9}$$

Furthermore, the following condition holds

$$\frac{dL(F(z))}{dz} = \frac{dL(F(z))}{dF(z)}\frac{dF(z)}{dz} = \frac{dL(F(z))}{dF(z)}f(z). \tag{10}$$

Combining equations (9) and (10) yields

$$\frac{dL(F(z))}{dF(z)}f(z) = \frac{1}{y}zf(z), \tag{11}$$

or equivalently

$$\frac{dL(F(z))}{dF(z)} = \frac{1}{y}z. \tag{12}$$

The last equality establishes the assertion that the slope of the Lorenz curve is inversely proportional to the mean income y.

Appendix 2

In the following paragraphs, we provide a formal proof of the multiplicity of the equilibria and their stability for the map h(.) as defined in equation (7).

The map $h(P^I) : [0, P] \rightarrow [0, P]$ has **at least one** stationary state as determined by $\bar{P}^I = h(\bar{P}^I)$. This follows from the fact that each interval possesses the fixed point property (Dunshirn et al., 1996).

To determine the exact number of stationary states and their stability, we define the map $\tilde{h}(P^I) := h(P^I) - P^I$, where the zeros of $\tilde{h}(P^I)$ are equal to the solution $\bar{P}_I$ of equation (7)). The map $\tilde{h}(P^I)$ is twice continuously differentiable on the interval $[0, P]$ except at the point $\tilde{P}^I$ where the condition $\tilde{y} = \alpha y(\tilde{P}^I)$ holds. Solving the latter expression for $\tilde{P}^I$ yields:

$$\tilde{P}^I = \frac{1}{h^S - h^I}\left[h^S P - \left(\frac{\tilde{y}}{\alpha}P\right)^{\frac{1}{\beta}}\right].$$

Furthermore, $\tilde{h}$ is strictly positive for $P^I = 0$ and reaches a unique minimum at

$$\breve{P}^I = \frac{h^S}{h^S - h^I}P - \frac{1}{h^S - h^I}\left(\frac{\tilde{y}P^\alpha}{\alpha}\left(\frac{\beta}{\alpha - 1}\frac{1}{h^S - h^I}\right)^{\alpha-1}\right)^{\frac{1}{\alpha+\beta-1}}.$$

Moreover, for $P^I < (>)\breve{P}^I$, $\tilde{h}(P^I)$ is strictly monotonically decreasing (increasing) in the number of food-insecure population.

To determine the number of zeroes of the map $\tilde{h}$ we first distinguish whether (1) $\tilde{P}^I > P$, (2) $\tilde{P}^I < 0$ or (3) $0 < \tilde{P}^I < P$ holds. For each of these alternatives we then differentiate between the alternatives (a) $\breve{P}^I > P$, (b) $\breve{P}^I < 0$ and (c) $0 < \breve{P}^I < P$.

1. $\tilde{P}^I > P$ implies that $\tilde{h}(P) < 0$.

   (a) If $\breve{P}^I > P$, then $\tilde{h}(P^I)$ is decreasing for $0 \leq P^I \leq P$. Since $\tilde{h}(P^I)$ is strictly positive for $P^I = 0$ and strictly negative for $P^I = P$ there will exist at least one zero in the interval $[0, P]$ according to the 'intermediate value theorem'. Since $\tilde{h}(P^I)$ is strictly montonic decreasing for $P^I \in [0, P]$, the zero is unique. Moreover this unique stationary state $\bar{P}^I$ is stable since $|h'(\bar{P}^I)| < 1$.

   (b) $0 \leq \breve{P}^I \leq P$ together with the fact that $\tilde{h}(P) < 0$ implies that $\tilde{h}(\breve{P}^I)$ has to be negative. A similar reasoning as in case (a) then implies the existence of a unique fixed point between zero and $\breve{P}^I$ which is stable since $|h'(\bar{P}^I)| < 1$. Moreover we can rule out any further zero of $\tilde{h}$ in the interval $[\breve{P}^I, P]$ since it holds that $\tilde{h}(\breve{P}^I) < 0$ and $\tilde{h}(P) < 0$ and $\tilde{h}$ is strictly monotonically increasing in this interval.

   (c) If $\breve{P}^I < 0$ holds then $\tilde{h}(P^I)$ would be strictly monotonically increasing for $0 \leq P^I \leq P$. However this would be a contradiction to $\tilde{h}(0) > 0$ and $\tilde{h}(P) < 0$ and hence we can rule out alternative (c).

   Summing up, if $\tilde{P}^I > P$ there exists a unique stationary state which is stable.

2. If $0 < \tilde{P}^I \leq P$, then $\tilde{h}(P^I) = P - P^I$ for $\tilde{P}^I \leq P^I \leq P$ with $\tilde{h}(\tilde{P}^I) > 0$. Consequently, there exists a stationary state at $\bar{P}^I = P$, which is stable,

since for all initial values $P_0^I \in [\tilde{P}^I, P]$ the sequence P_t^I approaches P.[9]

It remains to analyze $\tilde{h}$ for zeros on the interval $[0, \tilde{P}^I]$, where

$$\tilde{h}(P^I) = P\left(\frac{\tilde{y}}{\alpha y(P^I)}\right)^{1/(\alpha-1)} - P^I.$$

(a) If $\breve{P}^I > \tilde{P}^I$, then $\tilde{h}(P^I)$ strictly monotonically decreases for $0 \leq P^I \leq \tilde{P}^I$. Since $\tilde{h}(0) > 0$ and $\tilde{h}(\tilde{P}^I) > 0$, there does not exist a zero of $\tilde{h}$ between zero and $\tilde{P}^I$, otherwise it would contradict the strict monotonicity of the function $\tilde{h}$. Consequently, the remaining fixed point, which is stable, is unique.

(b) $0 \leq \breve{P}^I \leq \tilde{P}^I$. In this case, one has to distinguish whether $\tilde{h}(\breve{P}^I)$ is greater, equal or less than zero, i.e.

  i. $\tilde{h}(\breve{P}^I) > 0$, then $\tilde{h}(P^I) > 0$ for $0 \leq P^I \leq \tilde{P}^I$, since $\breve{P}^I$ is a minimum on $[0, \tilde{P}^I]$. Hence, there does not exist any stationary state between zero and $\tilde{P}^I$.

  ii. $\tilde{h}(\breve{P}^I) < 0$, then $\tilde{h}(P^I)$ strictly monotonically decreases for $0 \leq P^I \leq \breve{P}^I$, where $\tilde{h}(0) > 0$ and $\tilde{h}(\breve{P}^I) < 0$. According to the 'intermediate value theorem' there exists a zero in $[0, \breve{P}^I]$. Hence, there exists a fixed point $\bar{P}^I$ with $|h'(\bar{P}^I)| < 1$. Furthermore, $\tilde{h}(P^I)$ strictly monotonically increases for $\breve{P}^I \leq P^I \leq \tilde{P}^I$, where $\tilde{h}(\breve{P}^I) < 0$ and $\tilde{h}(\tilde{P}^I) > 0$. Consequently, there exists also a fixed point in $[\breve{P}^I, \tilde{P}^I]$, where $|h'(\bar{P}^I)| > 1$, i.e. this fixed point is unstable.

  iii. $\tilde{h}(\breve{P}^I) = 0$. Since $\tilde{h}$ is strictly monotonically decreasing (increasing) to the left (right) of $\breve{P}^I$, then $\breve{P}^I$ is the unique zero

[9] The stability criterion of the linearization does not apply, since $|h'(\bar{P}^I)| = 1$.

in $[0, \tilde{P}^I]$. Furthermore, this stationary state is semi-stable, which means it is only approached by initial values to the left of the fixed point.

Summing up, there exist two stable and one unstable stationary state in between and in the non-generic case a stable and a semi-stable fixed point, respectively.

(c) $\breve{P}^I < 0$, the $\tilde{h}(P^I)$ strictly monotonically increases for $0 \leq P^I \leq \tilde{P}^I$, where $\tilde{h}(0) > 0$ and $\tilde{h}(\tilde{P}^I) > 0$. Consequently, there does not exist any stationary state between zero and $\tilde{P}^I$. Hence, the remaining stationary state $\bar{P}^I = P$ is unique.

3. $\tilde{P}^I \leq 0$, then $\tilde{h}(P^I) = P - P^I$ for $0 \leq P^I \leq P$. Hence, the unique stationary state is located at $\bar{P}^I = P$. As already shown above, this stationary state is stable.

This ends the proof of the proposition and we conclude that there **exist at least one stable stationary state and at most three stationary states, where two of them are stable.**

Appendix 3

In order to determine how the stationary states depend on parameter values, and in particular on the parameter of inequality in the distribution of food, we use the implicit function theorem. For the fixed points of the map $\tilde{h}$ it holds that $\tilde{h}(\bar{P}^I) = 0$. Applying the implicit function theorem the marginal change of the equilibrium value $\bar{P}^I$ due to a marginal change of the parameter of inequality, α, is given by

$$\frac{\partial \bar{P}^I}{\partial \alpha} = -\frac{\frac{\partial \tilde{h}}{\partial \alpha}}{\frac{\partial \tilde{h}}{\partial \bar{P}^I}}.$$

The denominator of the right hand side is less (greater) than zero if the fixed point is stable (unstable).

The lower stable fixed point and the unstable fixed point, if existing, are less than $\tilde{P}^I$. In this case, the numerator is positive or negative depending on whether the corresponding fixed point $\bar{P}_I$ is less or greater than a threshold level $\hat{P}_I$, which is defined as:

$$\hat{P}_I = \frac{h^S P - \left[\frac{P\tilde{y}}{\alpha} \exp\left(\frac{\alpha-1}{\alpha}\right)\right]^{1/\beta}}{h^S - h^I}.$$

It can easily be shown that $\hat{P}_I$ is located between the lower stable and unstable equilibrium. Therefore, the lower stable equilibrium and the unstable equilibrium rise when the parameter α increases.

The upper stable equilibrium value $\bar{P}^I = P$ does not depend on the parameter α. However, the existence of this fixed point, as determined by the condition $\tilde{P}^I \leq P$, hinges on α. The higher α is, the higher is $\tilde{P}^I$ and the less likely this equilibrium will exist.

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