Human capital, technological progress and the demographic transition*

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January 27, 1999

Abstract

We emphasize the importance to consider components of population growth — fertility and mortality — separately, when modeling the mutual interaction between population and economic growth. Our

*Financial support from the Max Kade Foundation is gratefully acknowledged. The paper was presented at the workshop on 'Nonlinear Demography' at the Max Planck Institute for Demographic Research in Germany, Rostock, May 1998. The authors are grateful for comments and suggestions from participants at the workshop. Comments from a referee have essentially helped to clarify the model structure.

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model implies that two countries with the same population growth will not converge towards the same level of per capita income. The country with the lower level of birth and death rates will be better off in the long run. Introducing a spill over effect of average human capital on total productivity our model implies multiple equilibria as illustrated in [2] and [18]. Besides the existence of a low and high level equilibrium — as characterized by low and high levels of per capita output respectively — we show the existence of multiple low level (Malthusian) equilibria. Initial conditions and parameters of technological progress and human capital investment determine whether an economy is capable to escape the low level equilibrium trap and to enjoy sustained economic growth.

1 Introduction

The failure of standard neoclassical growth models (Solow [14]) to account for diverging long run economic growth rates across countries and within countries over time has led to a revival of modern economic growth theory (see Barro and Sala-i-Martin [1]). Endogenous technical progress (Romer [12]) and human capital accumulation (Lucas [8]) are identified as engines of growth, while the role of population is often still attributed a minor role. But recent work on combining endogenous economic growth and endogenous population growth [1] emphasizes the importance of population dynamics to determine the long run dynamics of economic systems. The papers by Goodfriend and McDermott [5] and Kremer [7] present interesting models on the interdependence of population and economic growth over the very long run. While population growth is mainly regarded to hinder economic growth these authors demonstrate even the necessity of population growth to initiate economic growth.

The aim of our paper is to contribute to the understanding of the interdependent dynamics of population and economic growth by emphasizing that *components of population growth* are important elements in the process of economic development. The basic idea is that countries with similar population growth rates may have different combinations of birth and death rates

[1] See e.g. Ehrlich and Lui [4] for an overview on this topic and Strulik [17] on a detailed summary of the interdependence of population and economic growth in neoclassical growth models.

and may thus have very different economic growth experiences. Bloom and Freeman [3] have shown that the association between income growth and population growth depends on the level of birth and death rates, with the relation being slightly negative among countries with high birth and death rates, and positive among countries with relatively low birth and death rates.

While Bloom and Freeman mainly focus on the separate effects of fertility and mortality on the age distribution as related to the growth of the labor force, we assume that components of population growth matter if we introduce education as a factor of production. We refer to education as being embodied in people and therefore education depreciates with the death of people. It certainly matters whether a population growth rate of e.g. 1% stems from a fertility rate of 4% and a mortality rate of 3% or whether it stems from a fertility rate of 2% and a mortality rate of 1%. The latter economy will have a much smaller depreciation of human capital. Consequently economic development does not only depend on the rate of population growth but also on the level of mortality. Though many authors nowadays (see e.g. Lucas [8]) use human capital as a stock of knowledge, which survives the bearer, already Nerlove [10] emphasized human capital, as tied to its bearer, in the explanation of the demographic transition. Nerlove argues that the central feature of human capital is precisely that it dies with the bearer, such that a fall in death rates would greatly enhance returns to investment in human capital. This in turn would increase the nutritional and health inputs per child and further increase the survival probability.

As the framework of our model we assume an aggregate *descriptive* one sector economic growth model.[2] Output in the closed economy is produced with human capital which is composed of the stock of people in the economy together with accumulated embodied education. The stock of education is build up by investment (assumed to be proportional to total output) and depreciates with the prevailing death rate. We abstract from physical capital accumulation since we aim to retain a parsimonious and tractable framework which captures the essential mechanisms of the interrelation between the components of population growth and economic development (see [2], [5] or [7] for a similar reduced form production function).[3] Per capita output in the economy in turn determines the level of fertility and mortality and the

[2]A related model with exogenous mortality and fertility rates, but based on consumer maximization, is given in Jäger and Steinmann [6].

[3]A similar model, but with physical capital included, is presented in Steinmann et al.

investment rate into education. To generate endogenous economic growth we assume a spillover effect of the average level of human capital on total productivity as in Lucas [8].[4]

Instead of building up a population theory based on explicit micro foundations, we assume exogenously specified functions relating per capita income and the components of population growth — fertility and mortality — in course of the demographic transition. Though cross country and/or time series evidence do not indicate any clear correlation between population growth and income growth, a negative correlation between per capita income and the components of population growth is compatible with empirical evidence (see Strulik [17]). Hence, splitting up population growth into its components not only emphasizes the distinct influence of mortality on human capital depreciation, but also facilitates a modeling of the demographic transition as consistent with empirical observations.

Our stylized model of the mutual effects between economic and population variables permits *multiple steady states* in the long run. We term the equilibrium with low and non growing income per capita and high mortality and fertility as the Malthusian trap. Depending on the form of the spillover effect of average human capital on total productivity *multiple Malthusian equilibria* can result as well. An escape from the Malthusian trap occurs once the economy is capable of generating an exponentially growing income per capita and significant decreases of mortality and fertility. Initial conditions and parameters of technology determine whether an economy ends up in a low level equilibrium trap or whether an escape from the Malthusian trap is feasible. Referring to the work of Becker, Murphy and Tamura [2] our model implies that history and luck are critical determinants of a country's growth experience. Additionally we show that the Malthusian low level equilibrium may loose its stability if e.g. the elasticity of savings with respect to income increases. A further result of our model is that two countries with the same population growth rate might exhibit different long run levels of per capita income. The country with the lower death and birth rate will be better off in the long run. This result is in accordance with the empirical findings reported in [3].

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[15].

[4]While the spillover effect in Lucas is not necessary to foster endogenous growth, the spillover effect in our model constitutes the sole engine of growth.

We proceed as follows. In section 2 we present the model and discuss the functional representations of birth, death and investment rates and technological progress. Section 3 summarizes analytical results on long run growth rates. Using *phase space concepts*, we analyze the resulting nonlinear dynamic equations in section 4. Section 5 concludes and summarizes our main results.

2 The model

We consider one sector of production

$$Y_t = H_t^{\alpha} W_t^{\gamma} \tag{1}$$

with production elasticities $0 < \alpha \leq 1$, $\gamma > 0$. The inputs into production of output Y_t are human capital H_t and technology W_t.

Human capital depends on the number of people L_t and the level of education E_t in the economy

$$H_t = L_t^{\epsilon} E_t^{1-\epsilon} \tag{2}$$

with $0 \leq \epsilon \leq 1$. In the literature human capital and education are either equated or education denotes the flow of human capital. Such a notion of human capital excludes the demographic component. We therefore suggest equation (2) to emphasise the components of human capital: population and embodied education. The production of human capital is assumed to exhibit constant returns to scale with respect to both inputs but decreasing marginal productivity of each input separately. To clarify our assumption assume that E represents the knowledge (education) of teachers, researchers, etc. . Even if this 'embodied' education doubles the economy wide human capital will not double if the stock of people L where education can be applied to does not double. On the other hand a doubling of the population will not increase the total stock of human capital in the economy if not accompanied by a doubling of the education E.

The dynamics of labor, education and technology are given by[5]

$$\dot{L} = [b(y) - d(y)]\, L \tag{3}$$

[5] Time arguments are omitted in the following.

$$\begin{aligned} \dot{E} &= s(y)Y - d(y)E \quad (4) \\ \dot{W} &= w(h)W \quad (5) \end{aligned}$$

where $b(y)$, $d(y)$ and $s(y)$ are the endogenous birth rate, death rate and investment rate into education as a function of per capita income $y = \frac{Y}{L}$, and $w(h)$ is the growth rate of technological progress as a function of per capita human capital $h = \frac{H}{L}$.

Logarithmically differentiating (2) and multiplying both sides by the level of human capital H, results in

$$\dot{H} = [\epsilon b(y) + (1-\epsilon)s(y)\frac{Y}{E}]H - d(y)H. \quad (6)$$

As equation (6) illustrates, human capital is accumulated through the birth of people and investment into embodied education and depreciates by the death of people. Whenever $\epsilon > 0$ holds, part of parents human capital, ϵH, is transferred on to the next generation, without incurring any costs. Setting $\epsilon = 0$ implies that human capital can only be augmented by investment into embodied education, while setting $\epsilon = 1$ would imply that children are born with the same human capital as their parents currently possess and the investment into embodied education does not effect the stock of human capital.

Our model specification is closely related to Mankiw, Romer and Weil [9].[6] Similar to [9] we assume only one sector of production. An alternative would be to assume that education and total output are produced by two different production functions, with the latter one requiring less educational input (see e.g. [1], chapter 5). Since we aim to highlight the importance of differences in the components of population growth during economic development we keep the general framework as simple as possible and assume that output can be used on a one-to-one basis for production, consumption and investment.

[6] Mankiw, Romer and Weil [9] present an augmented Solow model of the form $Y(t) = K(t)^{\alpha}H(t)^{\beta}[A(t)L(t)]^{1-\alpha-\beta}$. Setting $\alpha = 0$, i.e. abstracting from physical capital accumulation, and recalling that their variable H corresponds to our variable E, yields a similar model specification we are using. While [9] assume constant returns to the factors that can be accumulated we allow for decreasing, $\alpha < 1$ or constant returns to scale, $\alpha = 1$. Decreasing returns to scale may result from the existence of a non-reproducible input factor (e.g. land).

We define the birth rate, death rate and investment rate into education as in Strulik [18]:

$$b(y) = b_{nat} + \frac{b_{max}}{1 + exp\left[bs(y - yb)\right]} \tag{7}$$

$$d(y) = d_{nat} + \frac{d_{max}}{1 + exp\left[ds(y - yd)\right]} \tag{8}$$

$$s(y) = \frac{s_{max}}{1 + exp\left[-ss(y - ys)\right]}. \tag{9}$$

The constants b_{nat} and d_{nat} are the natural birth and death rates and s_{max} is the maximum investment rate. These values are taken up as per capita output becomes large. For low levels of per capita income, b_{max} and d_{max} are the maximum birth and death rate. yb, yd and ys determine the turning point of the logistic functions and bs, ds and ss determine the slope of the logistic functions.

The shape of (7)-(9) — evaluated at the parameter values given in [18] — is illustrated in Figure 1.a. During the demographic transition, mortality rates decline first with increasing levels of per capita income as modeled by the assumption $y_d < y_b$. Moreover, mortality rates are more sensitive to changes in per capita income, hence $ds > bs$. The shape of $s(y)$ has been derived in [18] from a simple three period OLG model based on the life-cycle hypothesis. Optimal consumption smoothing implies an increase in savings if mortality declines (survival increases). Linking this result with the fact that mortality declines during the demographic transition with increasing per capita income, results in the specified shape of (9). This result is also consistent with the work of van Imhoff [19]. Within the framework of an optimal control model, van Imhoff [19] has illustrated, that a decrease in the birth rate leads to a higher optimal investment in human capital.

To generate economic growth we postulate a spillover of per capita human capital on total productivity.[7] Technology is exogenously given and does not use up human capital. But the ability to utilize technology depends on the individual's skill level in the developing economy as represented by the endogenous growth rate $w(h)$.[8] We assume that the rate of technolog-

[7]A similar spillover effect is modeled by Lucas [8] who defines the production function $AK^{\beta}[uhN]^{1-\beta}h_a(t)^{\gamma}$, with N being the population size, K physical capital, u time spend in labor force, hN effective labor, and h_a the average level of human capital.

[8]We would like to thank the referee for suggesting this interpretation.

ical utilisation initially rises at an increasing rate with increasing levels of average human capital. Eventually the increase in the rate of technological utilisation starts to decline and converges to its long run maximum.[9] This specification of technological progress turns out to be capable of generating multiple Malthusian steady states, in addition to being capable of generating the low and high per capita income equilibria as outlined in e.g. [2] and [18]. More specifically, we assume the following endogenous rate of technological utilisation

$$w(h) = \frac{w_{max}}{1 + exp\left[-ws(h - hw)\right]}. \tag{10}$$

w_{max} denotes the maximum technological progress, hw defines the turning point of the logistic function and ws determines the slope of the logistic function. The shape of the endogenous technological progress — for different values of w_s and h_w — is illustrated in Figure 1.b.

Logarithmically differentiating total output (1) and human capital (2) and substituting in the dynamics of labour (3), education (4) and technological progress (5) results in a two dimensional system of differential equations in the variables per capita output $y = \frac{Y}{L}$ and per capita education $e = \frac{E}{L}$

$$\hat{y} \;=\; \frac{\dot{y}}{y} = (\alpha - 1)[b(y) - d(y)] + \alpha(1 - \epsilon)\hat{e} + \gamma w(h) \tag{11}$$

$$\hat{e} \;=\; \frac{\dot{e}}{e} = s(y)\frac{y}{e} - b(y) \tag{12}$$

with per capita human capital equal to $h = e^{1-\epsilon}$.[10]

It is interesting to contrast the growth rate of per capita education as given by equation (12) with the growth rate of per capita education, assuming a fixed rate of depreciation δ in equation (4)

$$\hat{e} \;=\; \frac{\dot{e}}{e} = s(y)\frac{y}{e} - b(y) + \left[d(y) - \delta\right]. \tag{13}$$

[9] An analogous functional form is used in [2] with respect to the rate of return of human capital investment.

[10] The system of differential equations (11)-(12) is closely related to the dynamic system in [18]. The main difference of our model as compared to [18] is the assumption of the spillover effect of average human capital on total productivity and the relation between the death rate and the depreciation of education.

Assuming a constant rate of depreciation δ (e.g. the average of $d(y)$ over all income levels) results in overestimating (underestimating) the growth rate of per capita education whenever $d(y) > \delta$ ($d(y) < \delta$). Since the death rate declines with per capita income, assuming a constant depreciation rate of embodied education will systematically bias $\hat{e}$ upwards for less developed countries (as characterized by $d(y) > \delta$), while resulting in a downwards bias of $\hat{e}$ in developed countries (as characterized by $d(y) < \delta$).

3 Analytical results

Assuming exogenous rates of fertility, mortality and savings and assuming zero or constant technological progress, permits to analytically solve for the long-run growth rate of per capita output.

3.1 Zero technological progress

From equation (12) it follows, that the growth rate of per capita education $\frac{\dot{e}}{e} = \hat{e}$ is constant if it equals the growth rate of per capita output $\frac{\dot{y}}{y} = \hat{y}$. Substituting $\hat{y} = \hat{e}$ into equation (11) yields the long run growth rate of per capita output (in absence of any technological progress, i.e. $w(h) = 0$)

$$\hat{y} = \frac{(\alpha - 1)}{1 - \alpha(1 - \epsilon)} n \tag{14}$$

with n being the exogenously given population growth rate, $n = b - d$. The long-run growth rate of per capita output is negative in case of decreasing returns to scale, $\alpha < 1$, and it is zero for constant returns to scale $\alpha = 1$.

In case of $\alpha = 1$, $\hat{e} = \hat{y} = 0$ holds and equation (12) yields the following expression for per capita education,

$$e = \frac{sy}{b}. \tag{15}$$

Substituting equation (15) into the equation for per capita output $y = W_0 e^{1-\epsilon}$ (where we have set W_0 as the constant level of technology) yields the steady state level of per capita output

$$y^* = W_0^{\frac{1}{\epsilon}} \left(\frac{s}{b}\right)^{\frac{1-\epsilon}{\epsilon}}. \tag{16}$$

Proposition 1: Two populations with the same exogenous rate of population growth, but differing levels of birth and death rates, will converge towards different levels of per capita output.

Proposition 1 follows from equation (16). Per capita output does not depend on the population growth rate, but only on the birth rate. This result stems from the assumption that education depreciates with the death of people.

Contrasting this result to the assumption that education depreciates at a constant rate δ and recalling equation (13) yields the steady state level of per capita output

$$y^* = W_0^{\frac{1}{\epsilon}} \left(\frac{s}{n+\delta} \right)^{\frac{1-\epsilon}{\epsilon}} . \tag{17}$$

Equation (17) replicates the well known result of neoclassical growth models, that the long run per capita level of output is similar for given parameters of the savings rate s, population growth rate n and parameters of technology as δ and ϵ.

3.2 Constant technological progress

Assuming constant technological progress at the rate w_{max}, the long run growth rate of per capita income is given by

$$\hat{y} = \frac{(\alpha - 1)}{1 - \alpha(1-\epsilon)} n + \frac{\gamma w_{max}}{1 - \alpha(1-\epsilon)}. \tag{18}$$

Equation (18) implies, that even in economies characterized by decreasing returns to scale, $\alpha < 1$, long run growth can be feasible, if the rate of technological progress overcompensates the capital dilution effect of increasing population levels.

In the next section we investigate the long run dynamics of the system (11)-(12), if the demographic and technological growth patterns evolve endogenously. Our interest is to characterize the economic-demographic set of initial conditions for which the economy is either trapped into a Malthusian state of zero per capita income growth or is able to escape and follow a path of long run economic growth.

3.3 Endogenous growth rates of population and technology

Equilibria of the system (11)-(12) are given by the intersection of the isoclines $\dot{e} = 0$ and $\dot{y} = 0$. In the following we only concentrate on equilibria with both variables being positive.[11]

The isoclines $\dot{y} = 0$ and $\dot{e} = 0$ are given by

$$(1-\alpha)[b(y) - d(y)] = \gamma w(h) \tag{19}$$

$$e = \frac{s(y)y}{b(y)}. \tag{20}$$

Substituting (20) into (19) and recalling $h = e^{1-\epsilon}$ yields

$$b(y) - d(y) = \frac{\gamma}{1-\alpha} w\left([\frac{s(y)y}{b(y)}]^{1-\epsilon} \right). \tag{21}$$

In Figure 2 we plot the left hand side and right hand side of equation (21) as a function of per capita income y for the endogenous functions of technical progress as given in Figure 1.b and keeping the endogenous population growth rate as in Figure 1.a. The production elasticities are set to $\gamma = \alpha = \epsilon = 0.5$.

Depending on the shape of technological progress, the model admits *multiple equilibria*. Note, that even no equilibrium might exist if the right hand side in (21) always exceeds the maximum attainable population growth. Of course, in this case the system will always manage to escape the Malthusian trap (since no low level equilibrium trap exists). An equilibrium to the left of $\max_y n(y)$ with $n' > 0$ might be called Malthusian equilibrium or low level equilibrium, while an equilibrium to the right of $\max_y n(y)$ with $n' < 0$ might be called a high-level equilibrium.

Proposition 2: An equilibrium with $n' < 0$ is always characterized by saddlepoint stability. If $n' > 0$, the equilibrium can either be a saddlepoint, a stable or unstable equilibrium or undergo a Hopf bifurcation.

The proof of Proposition 2 is given in the appendix.

[11] (y,e)=(0,0) is an equilibrium we shall not consider, since it is economically not relevant.

4 Numerical Analysis

To illustrate the possibility of multiple equilibria and their stability we numerically solve the system of differential equations (11)-(12) using the program package Mathematica [20].

Figure 3.a and Figure 3.b summarize the dynamics of per capita output and per capita education, given technological progress $w_1(h)$ or $w_2(h)$ respectively. A shift in the threshold level of technological progress, hw, — as illustrated by a shift from technology $w_1(h)$ to technology $w_2(h)$ — does not alter the stability of the two resulting equilibria but alters the level of the equilibria. Comparing Figure 3.a to Figure 3.b, the stable low level or Malthusian equilibrium, A, decreases from $(y^*, e^*) = (407, 216)$ to $(y^*, e^*) = (141, 14)$, while the saddlepoint equilibrium, B, increases from $(y^*, e^*) = (1211, 3931)$ to $(y^*, e^*) = (1765, 10019)$. Economies starting from initial conditions to the left of the stable manifold of the saddlepoint equilibrium B shall end up in a Malthusian trap as represented by the stable equilibrium A. Economies characterized by initial conditions to the right of the stable manifold of the saddlepoint equilibrium B shall escape the Malthusian trap and follow a path of continuous per capita growth. The long-run growth rate is given by equation (18) with $n = b_{nat} - d_{nat}$. Figure 3.a and Figure 3.b indicate that an increase in the threshold level of endogenous technological progress, hw, renders an escape from the Malthusian trap more difficult (in terms of necessary initial levels of y and e) and lowers the long-run per capita output if an escape is not possible. But once an escape from the Malthusian trap is feasible, the long-run growth rate of the economy is independent from the level of hw.

The shape of the stable manifold implies that high levels of per capita education are not sufficient for economic growth. On the contrary, high levels of per capita output can sustain economic growth, independent of the prevailing per capita levels of education. These results are intuitive, since we explicitly assumed that growing levels of per capita output are the driving force during the demographic transition. If per capita output levels are too small, the growth rate of embodied education will be smaller than the growth rate of labor and per capita education declines. For any given level of population growth, embodied education increases if the rate of investment increases and/or the death rate declines, i.e. if economic development takes place.

Changing the rate of technological utilisation from $w_1(h)$ to $w_3(h)$ not only alters the level, but also the number of equilibria as already illustrated in Figure 2. The stability of the resulting four equilibria is illustrated in Figure 4.a and Figure 4.b (which represents an enlargement of Figure 4.a). Similarly to Figure 3.a the stable manifold of the saddle point equilibrium D separates initial conditions from which the economy is trapped to the Malthusian equilibrium from those initial conditions that render an escape from the Malthusian trap possible. The level of $D = (1202, 3875)$ closely corresponds to the level of B in Figure 3.a. But different to Figure 3.a, the Malthusian low level equilibrium is not unique and is either characterized by the stable equilibrium $A = (142, 14)$ or by the stable equilibrium $C = (641, 971)$. The stable manifold of the saddlepoint equilibrium $B = (309, 91)$ separates the regions of attraction to either of these two equilibria. Again, the long-run growth rate of per capita output, if the economy escapes the Malthusian trap, i.e. if the economy starts out to the right of the stable manifold of the saddle point equilibrium D, is given by equation (18) with $n = b_{nat} - d_{nat}$.

As verified in proposition 2, the Malthusian equilibrium — as characterised by $n' > 0$ — must not be stable. Increasing the elasticity of savings with respect to income, e.g. by increasing the parameter ss, might result in a loss of stability of the low level equilibrium. Figure 5.a presents a phase diagram for the same parameter set as given in Figure 3.a, but increasing the value of the parameter ss up to 0.015. While the high level equilibrium slightly changes to $B = (1210, 3980)$, the low level equilibrium reduces to $A = (281, 5.9)$. To visualize the low level equilibrium, Figure 5.b. presents an enlargement of Figure 5.a. The equilibrium A is no longer stable and the unstable manifold of the low level equilibrium collides with the stable manifold of the saddlepoint equilibrium B.[12] Except for economies starting exactly on the stable manifold of the high level equilibrium B, all feasible initial conditions in Figure 5.a. will now lead to sustained economic growth. Contrary to the preceding phase diagrams, Figure 5.a illustrates that history and luck can be supplemented by intentional interventions, such as increasing the elasticity of savings w.r.t. changes in income, in order to escape the Malthusian low level equilibrium.[13]

[12] See Strogatz [16], page 263 for a qualitative similar dynamical system.

[13] Analytically, an increase in the elasticity of savings η_s leads to an increase in the trace

5 Discussion

A distinct feature of the demographic transition is the hump shaped relation between per capita income and population growth during economic development. Mathematically this relationship implies that one rate of population growth can be associated with two different levels of per capita income. To integrate this observation into a simple one sector economic growth model we refer to embodied human capital as an input into production. Human capital is directly linked to the prevailing demographics since it depreciates by the death of people and accumulates through the birth of people and the investment into embodied education. This simple modification of the standard neoclassical growth model is capable to explain the fact that countries with the same rate of population growth do not necessarily converge towards the same steady state per capita output.

But whether an economy actually follows the path of the demographic transition, as characterized by growing per capita output levels, depends on the initial conditions of per capita output and per capita education and the form of technical progress we introduce. More specifically, our model implies multiple equilibria with the possibility of having more than one low level Malthusian equilibrium. While the high level equilibrium, as characterized by $n' > 0$, is always unstable, the stability of the low level equilibrium, as characterized by $n' < 0$, can be stable, unstable or undergo a Hopf bifurcation. We have illustrated that e.g. a higher elasticity of savings w.r.t. income might induce a change in stability of the Malthusian trap. This result implies that not only history and luck, but also intentional interventions might foster an escape from the Malthusian trap.

Since human capital plays such an important role in our model to foster economic growth, the question arises whether our stylized model is compatible with empirical observations. While Rosenzweig [13] is optimistic on this point and states that there is 'no or little doubt that declines in fertility and increases in human capital levels accompany economic development', Pritchett [11] jeopardizes these results. Pritchett has found that cross national data on economic growth do not support the hypotheses that increases in educational capital due to improvements in the educational attainment of the labor force have had a positive impact on the rate of growth of output

of the Jacobian (see Appendix).

per worker. The authors offer different explanations of these observations but principally conclude with 'what may appear to be low returns to schooling may in fact be a low quality environment for applying cognitive skills'. In fact, our model captures this possibility in terms of the form of the spillover effect of average human capital on total productivity. The spillover effect could be so low that our stylized model will never generate economic growth in the long run.

References

[1] Barro, R.J. and X. Sala-i-Martin (1995) Economic Growth, McGraw-Hill Inc.

[2] Becker, G.S., K. Murphy and R. Tamura (1990) Human capital, fertility, and economic growth, Journal of Political Economy 98, 12-37.

[3] Bloom, D.E. and R.B. Freeman (1988) Economic Development and the timing and components of population growth, Journal of Policy Modeling 10 (1), 57-81.

[4] Ehrlich, I. and F. Lui (1997) The problem of population and growth: A review of the literature from Malthus to contemporary models of endogenous population an endogenous growth, Journal of Economic Dynamics & Control 21, 205-242.

[5] Goodfriend, M. and J. McDermott (1995) Early Development, American Economic Review 85(1), 116-133.

[6] Jäger, M. and G. Steinmann (1997) Mortality and Economic Growth, Volkswirtschaftliche Diskussionsbeiträge, Discussion Paper 4, Martin-Luther-Universität Halle-Wittenberg.

[7] Kremer, M. (1993) Population growth and technological change: one million B.C. to 1990, Quarterly Journal of Economics 108, 681-716.

[8] Lucas, R. (1988) On the mechanics of economic development, Journal of Monetary Economics 22, 3-42.

[9] Mankiw, N.G, D. Romer and D.N. Weil (1992) A contribution to the empirics of economic growth, The Quarterly Journal of Economics, 407-437.

[10] Nerlove, M. (1974) Household and economy: toward a new theory of population and economic growth, Journal of Political Economy 82: 200-218

[11] Pritchett, L. (1997) Where has all the education gone? World Bank, Policy Research, Working Paper 1581.

[12] Romer, P.M. (1990) Endogenous Technological Change, Journal of Political Economy 98(5), 71-102.

[13] Rosenzweig, M.R. (1990) Population growth and human capital investment: theory and evidence, Journal of Political Economy 98(5), 38-70.

[14] Solow, R.M. (1956) A contribution to the theory of economic growth, Quarterly Journal of Economics 70, 65-94.

[15] Steinmann, G., A. Prskawetz and G. Feichtinger (1998) A model on the escape from the Malthusian Trap, Journal of Population Economics 11/4.

[16] Strogatz, S.H. (1995) Nonlinear Dynamics and Chaos. With Applications to Physics, Biology, Chemistry, and Engineering, Addison-Wesley Publishing Company.

[17] Strulik, H. (1996) Die Interdependenz von Bevölkerungs- und Wirtschaftswachstum in neoklassischen Wachstumsmodellen, Cuvellier, Göttingen.

[18] Strulik, H. (1999) Demographic transition, stagnation, and demoeconomic cycles in a model for the less developed economy, forthcoming in Journal of Macroeconomics 21/2.

[19] van Imhoff, E. (1988) Optimal investment in human capital under conditions of nonstable population, The Journal of Human Resources XXIV (3), 414-432.

[20] Wolfram, S. (1991) Mathematica: A system for doing Mathematics by Computer, Addison-Wesley, New York.

APPENDIX

The elments of the Jacobian of system (11)-(12)

$$\begin{aligned} \dot{y} &= y\left[(\alpha-1)[b(y)-d(y)]+\alpha(1-\epsilon)(\frac{\dot{e}}{e})+\gamma w(h)\right] = f(y,e) \\ \dot{e} &= s(y)y - b(y)e = g(y,e) \end{aligned}$$

are given by[14]:

$$\begin{aligned} f_y &= \left[(\alpha-1)[b(y)-d(y)]+\alpha(1-\epsilon)(\frac{\dot{e}}{e})+\gamma w(h)\right] + \\ & \quad y\left[(\alpha-1)(b'-d')+\alpha(1-\epsilon)(\frac{\dot{e}}{e})_y\right] \\ f_e &= y\left[\alpha(1-\epsilon)(\frac{\dot{e}}{e})_e+\gamma w_e\right] \\ g_y &= s'y+s(y)-b'e \\ g_e &= -b(y). \end{aligned}$$

Evaluated at equilibria (y^*, e^*) as implicitely defined by

$$\begin{aligned} b(y)-d(y) &= \frac{\gamma}{1-\alpha}w(h) \\ e &= \frac{s(y)y}{b(y)} \end{aligned}$$

the elements of the Jacobian reduce to

$$\begin{aligned} f_y &= y[(\alpha-1)(n')+\alpha(1-\epsilon)\frac{A}{e}] \\ f_e &= y[\alpha(1-\epsilon)(-\frac{sy}{e^2})+\gamma w_e] \\ g_y &= A \\ g_e &= -b \end{aligned}$$

where we have set $n' = b' - d'$ and $A = s[\eta_s + 1 - \eta_b]$ with η_s, η_b denoting the income elasticity of the savings rate and the birth rate, i.e. $\eta_s = \frac{ds}{dy}\frac{y}{s}$ and

[14]Subscripts denote partial differentials with respect to the subscripted argument. A prime denotes the differential w.r.t. the single argument.

$\eta_b = \frac{db}{dy}\frac{y}{b}$. Note, that A is always positive since the income elasticity of the savings rate is positive and the income elasticity of the birth rate is negative.

The stability of the equilibria is determined by the trace, $tr J = f_y + g_e$, and the determinant, $\det J = f_y g_e - f_e g_y$, of the Jacobian. In particular, if $\det J < 0$, the equilibrium is a saddlepoint, while in case of $\det J > 0$, the equilibrium is either stable (if $tr J < 0$), unstable (if $tr J > 0$) or undergoes a Hopf bifurcation (if $tr J = 0$).

The determinant and the trace of the Jacobian, evaluated at the equilibria, are given by

$$\begin{aligned} detJ &= yb(1-\alpha)n' - Ay\gamma w_e \\ trJ &= y[(\alpha - 1)n' + \alpha(1-\epsilon)A/e] - b. \end{aligned}$$

Hence, whenever $n' < 0$, the determinant is negative and the equilibrium will be a saddlepoint. Only in case of $n' > 0$, the determinant of the Jacobian can be positive. In this case, the sign of the trace of the Jacobian determines the stability of the system.

List of Captions

Figure 1: Parameter values:

$b(y)$:	$b_{nat} = 0.015$	$b_{max} = 0.065$	$bs = 0.003$	$yb = 1500$
$d(y)$:	$d_{nat} = 0.01$	$d_{max} = 0.1$	$ds = 0.005$	$yd = 300$
$s(y)$:		$s_{max} = 0.2$	$ss = 0.007$	$ys = 600$
$w_1(h)$:		$w_{max} = 0.05$	$ws = 0.1$	$hw = 10$
$w_2(h)$:		$w_{max} = 0.05$	$ws = 0.1$	$hw = 100$
$w_3(h)$:		$w_{max} = 0.05$	$ws = 1$	$hw = 10.$

a: Endogenous birth rate $b(y)$, death rate $d(y)$, population growth rate $n(y)$ and savings rate $s(y)$.

b: Endogenous growth rate of technological progress $w_1(h), w_2(h), w_3(h)$.

Figure 2: Possible equilibria of the system (11)-(12) for the three different endogenous functions of technological progress as illustrated in Figure 1.b and with the other parameters set at their reference value as given in Figure 1.a. Production elasticities are set equal to $\alpha = \gamma = \epsilon = 0.5$.

Figure 3: Phase space diagram of system (11)-(12) for the reference parameter set as given in Figure 2 and alternative specifications of technological progress.

a: $w_1(e)$

b: $w_2(e)$

Figure 4: a: Phase space diagram of system (11)-(12) for the reference parameter set as given in Figure 2 and setting technological progress equal to $w_3(e)$.

b: Enlargement of Figure 4.a.

Figure 5: a: Phase diagram of system (11)-(12) for the reference parameter set as given in Figure 3.a but increaseing the parameter ss up to 0.015.

b: Enlargement of Figure 5.a.

FIGURE 1.a

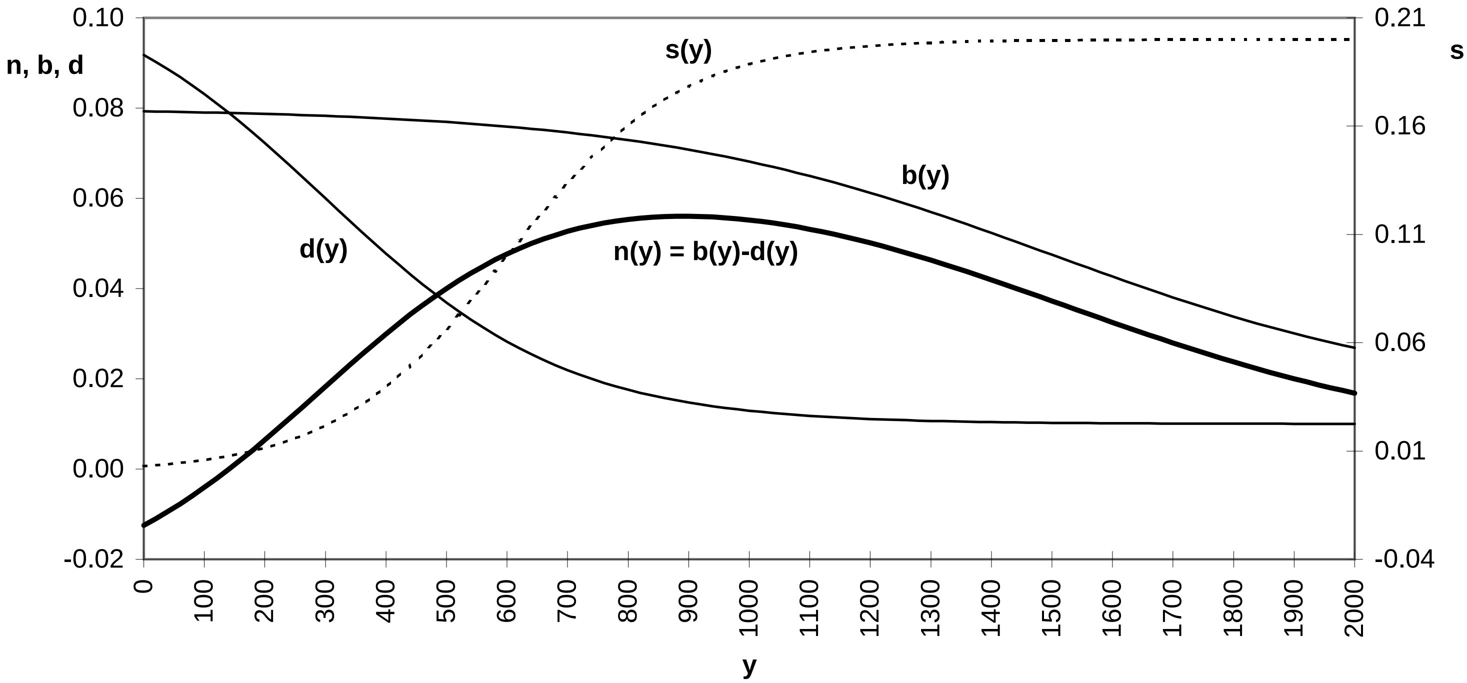


FIGURE 1.b

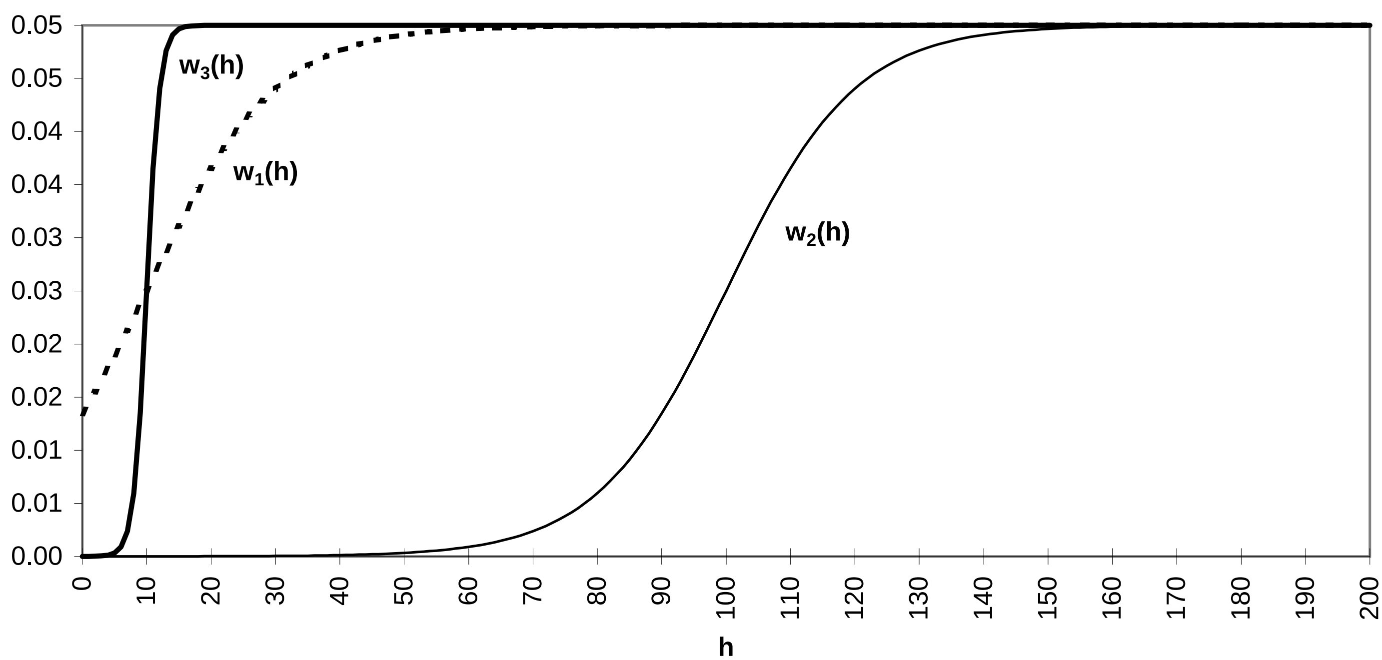
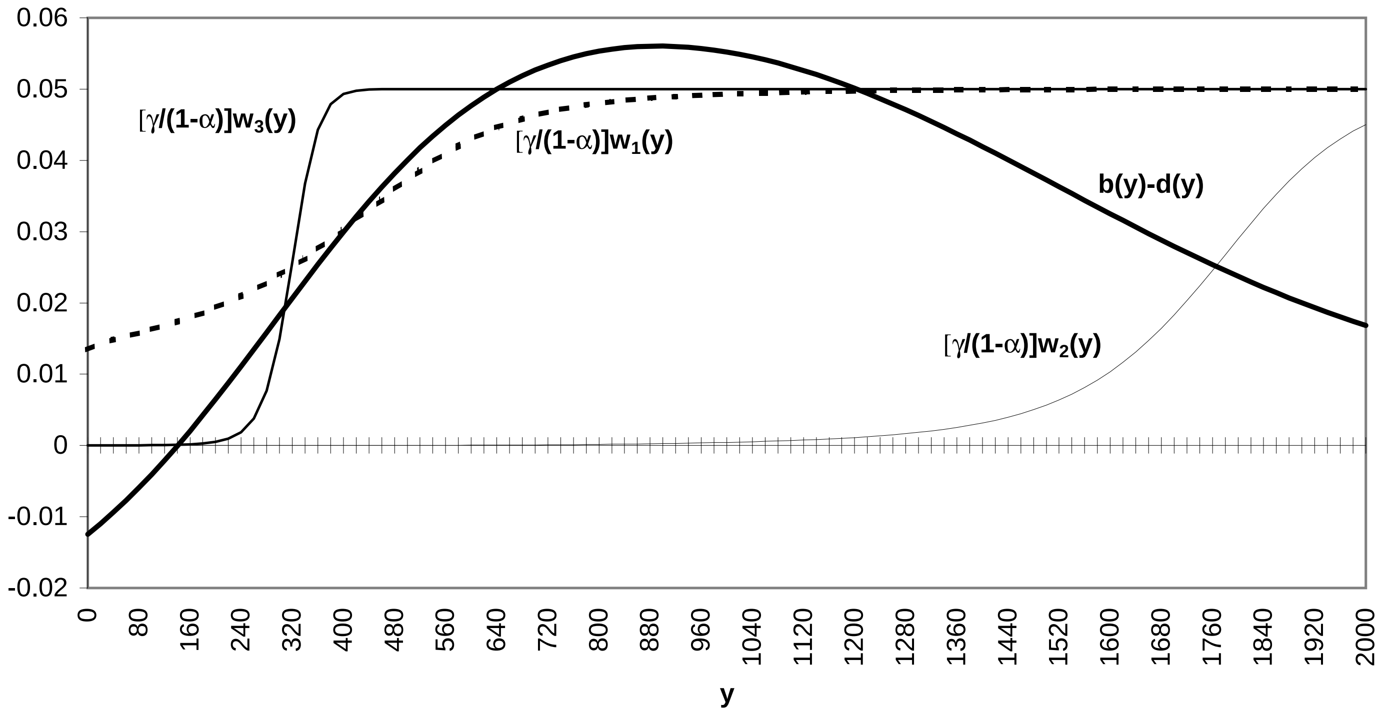


FIGURE 2



0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
[γ/(1-α)]w_3(y)
[γ/(1-α)]w_1(y)
b(y)-d(y)
[γ/(1-α)]w_2(y)
0
80
160
240
320
400
480
560
640
720
800
880
960
1040
1120
1200
1280
1360
1440
1520
1600
1680
1760
1840
1920
2000
y

Figure 3. a

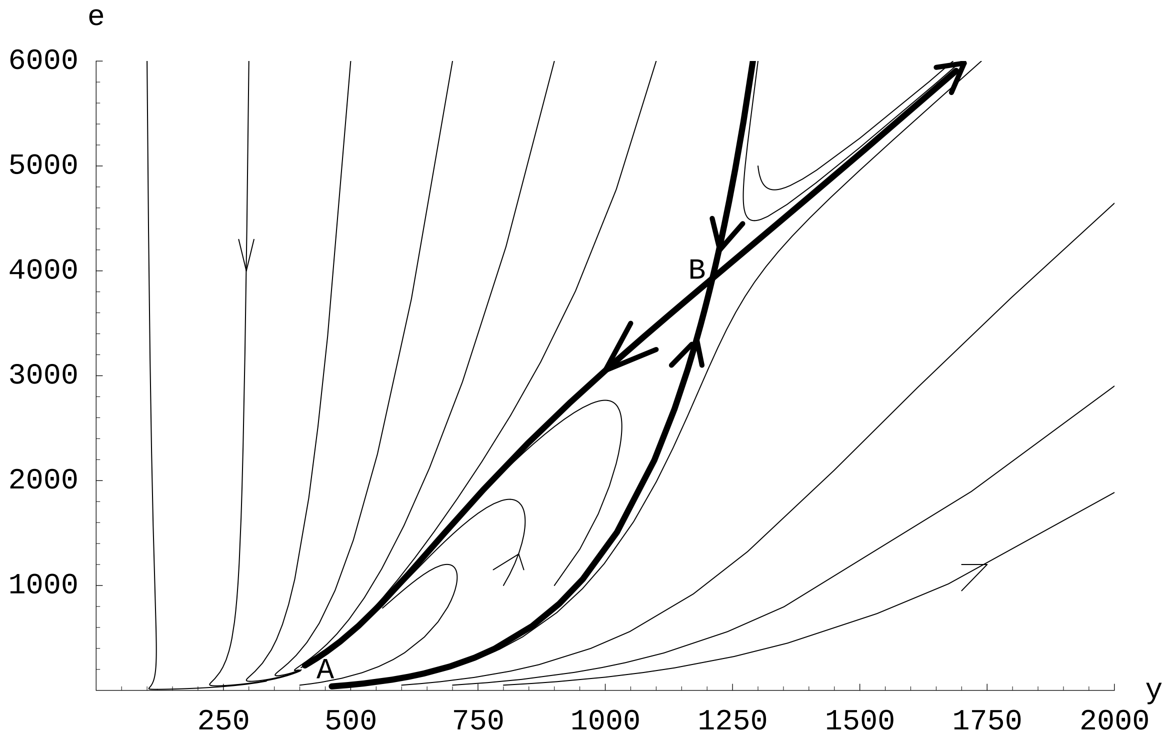

e
6000
5000
4000
3000
2000
1000
B
A
250
500
750
1000
1250
1500
1750
2000
y

Figure 3. b

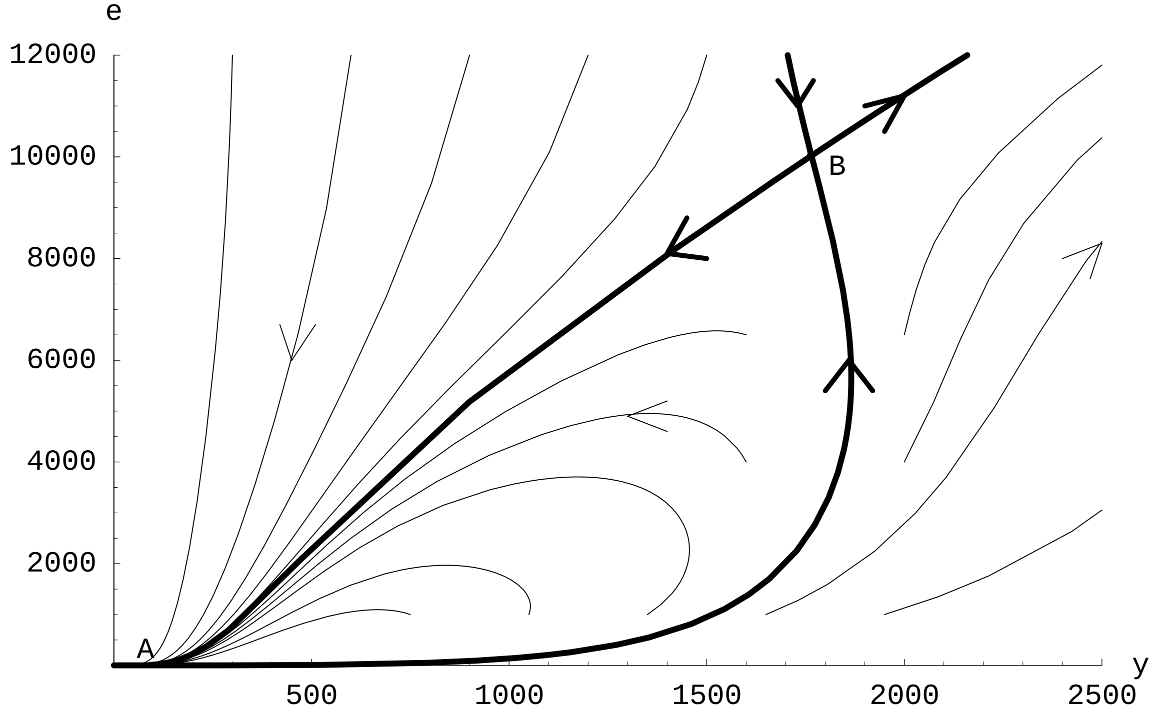


Figure 4. a

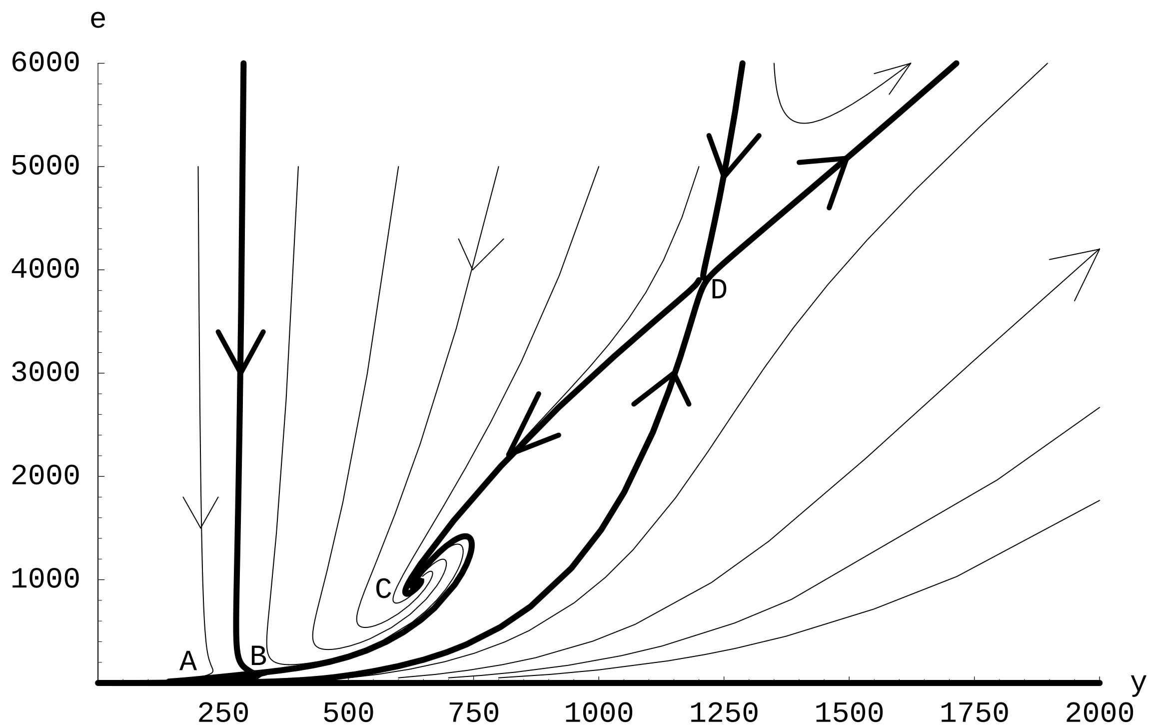

e
6000
5000
4000
3000
2000
1000
A
B
C
D
250
500
750
1000
1250
1500
1750
2000
y

Figure 4. b

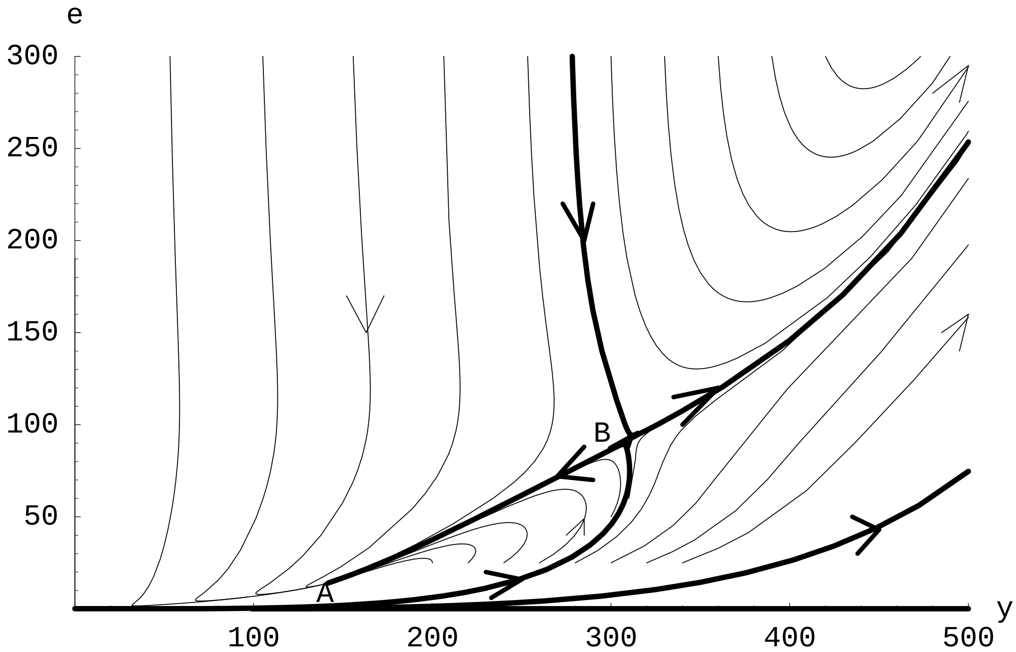


Figure 5. a

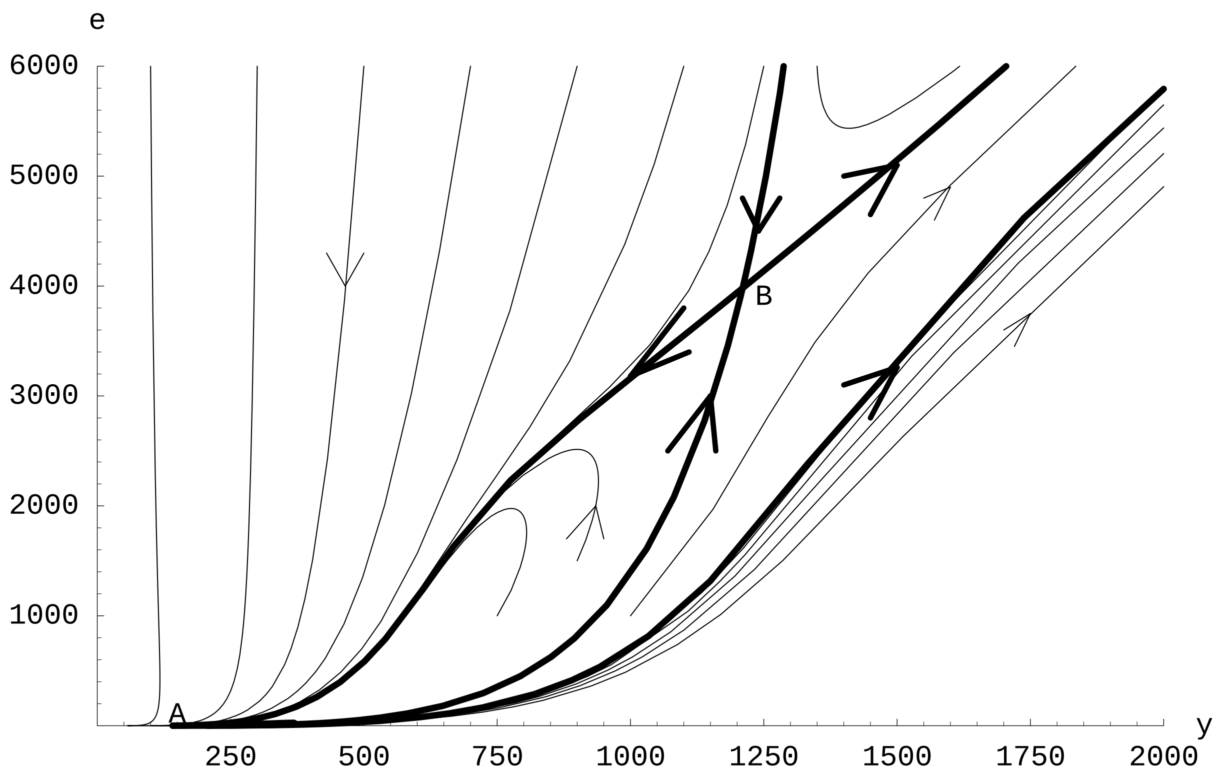

e
6000
5000
4000
3000
2000
1000
A
B
y
250
500
750
1000
1250
1500
1750
2000

Figure 5. b