Reconsidering the dynamic interaction of renewable resources and population growth: a focus on long-run sustainability

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Abstract

Within the framework of a general equilibrium model we study the long-run dynamics of resources and population if the growth rate of resources and population and the share of labor devoted to production are adversely affected by resource scarcity. Our results show that sustainability, i.e. a positive value of resources and population in the long run, essentially depends on the level of per capita resources at which these feedback mechanisms become active. A detailed bifurcation analysis evidences the richness of possible long-run dynamics. Stable, strictly positive values of population and resources can be replaced by limit cycles, where population and resources follow a cyclic path. We can even observe the coexistence of different stable modes of behavior in which the long-run dynamics will be determined by the initial values of population and resources.

Keywords: Renewable resource scarcity, endogenous population growth, sustainability, dynamic models, local bifurcation theory.

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1 Introduction

While exhaustible resources have been at the center of research in environmental economics for some time, it is the alarmist news about scarcity of renewable resources that has overshadowed much of the academic and scientific discussion during the last years. As has been highlighted in recent papers by Homer-Dixon et al. (1993) and Maxwell and Reuveny (2000), in the 21st century most renewable natural resources in developing countries are expected to become increasingly scarce. Continued population growth together with increases in output growth are regarded as the driving forces of this development. As a result, deforestation, air and water pollution, water scarcity, scarcities of agricultural and grazing land, dwindling of fish stocks, etc. will likely become more severe in the near future.

In contrast to nonrenewable resources, the use of which is controlled largely by market prices, renewable resources are very often open-access. The overuse of resources if property rights are absent or poorly defined is called the 'first tragedy of the commons' (Hardin, 1968) and can be corrected by restricting access to the resource. But as emphasized by Lee (1990), if the population is not fixed, free access through reproduction constitutes the 'second tragedy of the commons'. It is the latter phenomenon that has recently gained in importance with respect to developing countries. In these countries many societies still depend heavily on the agricultural sector. The combination of population growth and unequal access to land puts severe pressure on the environment in these countries.

For example, economic deprivation in the Philippines has forced landless agricultural laborers and poor farmers to move to the least productive and often most ecologically vulnerable territories, such as steep hillsides. This induces a cycle of falling food production, the clearing of new plots and further land degradation. Hence, population growth and unequal access to good land forces huge numbers of people onto marginal lands and also into cities (Homer-Dixon et al., 1993, p.20).

Resource scarcity may even lead to civil and international strife, as has recently been argued by many authors. Homer-Dixon et al. (1993) report on the outcome of various case studies showing that scarcities of renewable resources are already contributing to violent conflicts in many parts of the developing world. This may become even worse in poor countries, where shortage of water, forests and, particularly, fertile land, coupled with rapidly expanding populations already cause great hardship. Hence, it is in societies that depend on the natural resource base and that do not have the wealth to move towards alternative means of production where resource scarcity may lead to violent conflicts.

Soysa et al. (1999) examine the post-Cold War pattern of conflict with a focus on the role of agriculture and show that countries that are highly dependent on agriculture are more likely to have conflict. However, these results do not support a causal link between economic dependence on agriculture and the incidence of armed conflict. Again, the missing link is poverty and the lack of physical, human, and social capital. The conflict itself is just the manifestation of the incapacity of social and political systems to handle such crises.

In a series of recent papers, the scarcity of natural resources as it interacts with population dynamics has been modeled by referring to simple 'mathematical cartoons' (see Cohen, 1995 and Prskawetz, 2000 for a short review of some of these models). Such models aim to capture stylized facts observed in the past and thereby gain a better understanding how resource and population dynamics may interact under various institutional settings. Two of the most prominent recent examples are the model of Easter Island (Brandner and Taylor, 1998) and the model of the Tsembaga society of New Guinea (Anderies, 1998). The lack of any institutional changes is generally held to be responsible for the resource collapse in Easter Island that occurred around 1400 AD. On the other hand, the Tsembaga society of New Guinea serves as an example where institutional settings such as a ritual cycle in which pigs are killed and warfare is initiated serve to re-establish a sustainable equilibrium of population and resource stocks.

Brandner and Talyor's model has recently been extended by Reuveny and Decker (2000), who introduce endogenous technical progress, and by Anderies (2000), who relax the assumption of a constant labor force participation rate in the agricultural sector. However, in both cases, the system dynamics can change dramatically, thereby making a system collapse even more likely. These results are best summarized in Anderies (2000, p. 410): "When individual agents can increase the rate of exploitation of their resource base in an effort to meet a minimum demand, the time scale upon which institutional adaption can occur is drastically shortened." Another interesting application of the Easter Island model is presented in Maxwell and Reuveny (2000). The authors refer to recent research on resource scarcity as it may initiate conflict in developing countries and extend this causal link by studying the repercussion of conflict situations on resource and population dynamics. Most interestingly, two of the suggested feedback mechanisms are already implemented in the model of the Tsembaga society of New Guinea. These are the increase in death rates and the reduction in the number of people devoted to production during periods of resource scarcity. In the Tsembaga society, death rates increase during periods of warfare and pig husbandry diverts part of the labor force (mostly women) from production. Hence, both models share the common feature that the ritual cycle (in the Tsembaga society) and the conflict situation (in the model by Maxwell and Reuveny, 2000), respectively, act to re-establish the long-run equilibrium between population and resource dynamics.

With the exception of the papers by Anderies (1998, 2000), the possible long-run behavior of population and resource dynamics — as it depends on the various parameters of the system dynamics and the institutional settings in particular —

are studied by referring to alternative simulation runs. A more systematic investigation of the possible long-run dynamic behaviour of population and resource growth can be obtained, however, by applying bifurcation theory. This amounts to plotting the long-run value of population and resources and indicating their stability as a function of various system parameters. Such an approach is particularly helpful for locating various long-run scenarios of population and resource growth in the parameter space.

The aim of this paper is to present one such detailed analysis of population-resource interactions. We shall demonstrate these techniques with an analysis of a variant of the model presented in Maxwell and Reuveny (2000). The model consists of two system equations that endogenously determine the stock of resources and population. A conflict situation is modeled to arise when per capita resources fall below an exogenously given threshold level. As suggested in Maxwell and Reuveny (2000) such a conflict situation may feed back on population growth and resource dynamics. First, part of the labor force will be devoted to the conflict situation and will thus not be available for production. Second, as conflict is initiated by resource scarcity, it is straightforward to assume that death rates will increase in a conflict situation. Third, conflict may damage natural resources even further. To model these various feedback mechanisms Maxwell and Reuveny (2000) assume that resources and population follow two different system dynamics, depending on whether a conflict situation prevails or not. We, in contrast, postulate that these changes in the labor force participation rate, the death rate, and the growth rate of resources, as induced by a conflict situation, are 'smooth' functions of the prevailing level of per capita resources. Within this simple mathematical framework we then investigate the dynamic interaction between population growth, resource degradation, and environmentally induced conflicts. In particular, we demonstrate that by applying local bifurcation theory one can delineate conditions that yield sustainable future time paths of population and resource growth from conditions in which either population or resources become extinct. Using these results we can then determine the specific feedback mechanisms that are required to yield a sustainable future time path of population and resource stocks. Alternatively, our analysis allows for a better understanding of why some societies may have experienced a resource and population collapse while others have managed to escape the vicious circle of resource degradation and population growth.

The paper is organized as follows. We present a variant of Maxwell and Reuveny's model (2000) in Section 2 and shortly summarize the choice of the system parameters in Section 3. For the baseline scenario, where we assume no feedback mechanism from resource scarcity on population and resource dynamics, we present analytical results in Section 4. In Section 5 we then focus on a detailed study of the long-run behavior of population and resource dynamics as it depends on the parameters and specific feedback mechanisms. We conclude our paper with a discussion of our results in Section 6.

2 The model

As the framework of our analysis we choose a general equilibrium model of population and resource dynamics as outlined in Brander and Taylor (1998). At each point in time, an individual derives utility $u(t)$ from harvest $h(t)$ and a composite consumption good $c(t)$, where utility is assumed to be of Cobb-Douglas form

$$u(h(t), c(t)) = h(t)^{\beta} c(t)^{1-\beta}. \tag{1}$$

Choosing the composite consumption good $c(t)$ as numeraire and assuming that each individual is endowed with one unit of labor that earns the wage rate $w(t)$, the individual faces the budget constraint

$$p(t)h(t) + c(t) = w(t) \tag{2}$$

where $p(t)$ denotes the price of the harvest good. Similarly to Maxwell and Reuveny (2000) we postulate that, during a conflict situation, part of the labor force will be withdrawn from production. Denoting γ (with $0 \leq \gamma \leq 1$) as the fraction of the labor force devoted to production (and $1 - \gamma$ as the fraction of the labor force devoted to conflict) the individual budget constraint can be written as

$$p(t)h(t) + c(t) = \gamma w(t). \tag{3}$$

Upon maximising utility (1) subject to the budget constraint (3), the optimal demand for harvest and the composite consumption good are:

$$h(t) = \frac{\gamma\beta}{p(t)} w(t) \tag{4}$$

$$c(t) = \gamma(1-\beta) w(t). \tag{5}$$

Total labor force in the economy $\gamma L(t)$ is divided up between production of the composite consumption good $L^C(t)$ and harvesting $L^H(t)$ where $\gamma L(t) = L^C(t) + L^H(t)$.[1]

The production of the composite consumption good is described by a linear technology with labor $L^C(t)$ being the only input, $C(t) = L^C(t)$, while the harvest function $H(R(t), L^H(t))$ depends on the available resource stock $R(t)$ and labor force $L^H(t)$. Assuming that all markets are competitive, we can solve for

[1] Capital letters denote aggregate variables as opposed to small letters, which refer to per capita levels.

the zero profit condition in both sectors. The choice of $C(t)$ as the numeraire together with the condition of zero profits in the composite consumption good sector, $C(t) - w(t)L^C(t) = 0$, means that the wage rate $w(t)$ will be equal to one. Free movement of labor between the two sectors implies that the wage in the harvest sector will be equal to one as well. Moreover, zero profits in the harvest sector $p(t)H(R(t), L^H(t)) - L^H(t) = 0$ implies

$$p(t) = L^H(t)/H(R(t), L^H(t)) = L^H(t)/H(t). \tag{6}$$

Equating aggregate supply $H(t) = H(R(t), L^H(t))$ and aggregate demand $L(t)h(t) = L(t)\frac{\gamma\beta}{p(t)} = L(t)\frac{\gamma\beta}{L^H(t)/H(t)}$ yields the general equilibrium levels of the labor force $L^H(t)$ and $L^C(t)$: $L^H(t) = \gamma\beta L(t)$ and $L^C(t) = \gamma(1-\beta)L(t)$. Hence, in a state of equilibrium, the total labor force will be split up in a fixed proportion between production of the composite consumption good and the harvest.[2]

To solve for the price level $p(t)$ we have to postulate the shape of the harvest function $H(t)$. While Maxwell and Reuveny (2000) assume a Cobb Douglas-type production function as in Prskawetz et al. (1994), we postulate a Monod-type harvesting function:[3]

$$H(t) = H(R(t), L^H(t)) = \frac{eR(t)L^H(t)}{fL^H(t) + R(t)} = \frac{e\gamma\beta R(t)L(t)}{f\gamma\beta L(t) + R(t)} \tag{7}$$

where e and f are constant parameters. Substitution of (7) into the equation of the equilibrium price level (6) yields

$$p(t) = \frac{fL^H(t) + R(t)}{eR(t)}. \tag{8}$$

Note that, similarly to Maxwell and Reuveny (2000), the specific form of the harvest function implies that utility will be a function of per capita resources $R(t)/L(t)$.[4] As noted in Maxwell and Reuveny (2000), a linear harvest function

[2]As recently illustrated by Anderies (2000), the relaxation of the constant labor proportion assumption is fundamentally destabilizing and may yield more dramatic system dynamics.

[3]By choosing a Monod-type function, we assume that per capita harvest is bounded from above when per capita resources tend to infinity. This assumption seems to be more realistic than the Cobb Douglas type production function, since it postulates that the total per capita amount that can be harvested tends to some upper limit even if per capita resources tend to infinity. Note that the specific functional form of the harvest meets the standard concavity properties of production functions, i.e. $H_R > 0, H_L > 0$ and $H_{RR} < 0, H_{LL} < 0$. See the Appendix for a more detailed derivation and discussion of the specific functional form as it relates to prey-predator systems in biology.

[4]Substituting the price level (8) into the optimal individual demand for harvest (4), this result follows.

as postulated in Brander and Taylor (1998) would imply that utility depends only on the total stock of resources and not on per capita levels.

To complete the model we need to state the dynamic evolution of the resource stock and the population. We postulate the same dynamics as suggested in Brander and Taylor (1998) and Maxwell and Reuveny (2000):

$$\dot{R} = g(R(t)) - H(t) = rR(t)\,[1 - R(t)/K] - H(t) \tag{9}$$

$$\dot{L} = [b - d + \phi H(t)/L(t)]\,L(t) \tag{10}$$

where $H(t)$ is the harvest function as given by (7).

The net growth of the renewable resource $\dot{R}$ is affected by two counteracting factors: indigenous biological growth $g(R(t))$ and the harvest $H(t)$. Indigenous growth is modeled by the logistic growth function in which the coefficient K determines the saturation level (carrying capacity) of the resource stock (i.e. K is the stationary solution of R if the resource is not degraded) and parameter r (the intrinsic growth rate) determines the speed at which the resource regenerates.

The population growth rate is assumed to be Malthusian. In the absence of any harvest $H(t)$, the population would decline to zero as modeled by $b-d < 0$. With increasing stock of per capita harvest $H(t)/L(t)$ the rate of population growth increases, where ϕ measures the sensitivity of population growth to increases in per capita harvest.

To model the occurrence of conflict and its feedback on population and resource dynamics we refer to Maxwell and Reuveny (2000). We postulate that conflict is initiated if per capita resources fall below some threshold level, i.e. if $\frac{R(t)}{L(t)}$ falls short of a threshold level $\bar{v}$.[5] This assumption can be justified by the fact that per capita resources enter the utility of each individual and conflict may be initiated if the utility is diminished.[6] As motivated in Maxwell and Reuveny (2000), the outbreak of conflict will feed back on the dynamics of population and resources. In addition to diverting part of the labor force from production, as already modeled by the parameter γ, two further 'channels' through which conflict will alter the long-run dynamics of population and resources are stated in Maxwell and Reuveny (2000). In conflict situations, the death rate d might increase and the intrinsic growth rate of resources r may be reduced. The first assumption can be modeled by assuming that the death rate is equal to d if there is no conflict, whereas it increases to ηd with $\eta > 1$ in the case of conflict. Similarly, we can assume that the intrinsic growth rate of resources is equal to r if there is no conflict but decreases to $(1-\theta)r$ with $0 < \theta < 1$ if there is conflict.

[5]We denote the threshold level of per capita resources by $\bar{v}$ so as not to confuse per capita resources with per capita harvest h.

[6]A similar mechanism is present in the model of the Tsembaga society (cf. Anderies, 2000, p. 402), where the pig-to-person ratio determines the warfare intensity.

Allowing for conflict situations, the dynamic evolution of the resource stock and the population change to:

$$\dot{R} = (1-\theta) r R(t) \left[1 - R(t)/K\right] - H(t) \tag{11}$$
$$\dot{L} = \left[b - \eta d + \phi H(t)/L(t)\right] L(t). \tag{12}$$

While Maxwell and Reuveny (2000) assume a discontinuous change in the dynamics once conflict sets in (i.e. if $R(t)/L(t)$ falls short of a threshold level $\bar{v}$), we model the parameter η, θ and γ as continuous functions of per capita resources and thereby assume a continuous change in the dynamics of population and resources. More specifically, Maxwell and Reuveny (2000) assume that the dynamics of resources and population are described by system (11)-(12) with $\eta = 1, \theta = 0, \gamma = 1$ if there is no conflict and by system (11)-(12) with $\eta > 1, 0 < \theta < 1, 0 < \gamma < 1$ in a conflict situation. An exogenous threshold of per capita resources $\bar{v}$ determines the switch between the two systems. In contrast, our strategy of modeling the parameters as continuous functions of per capita resources means that the dynamics of resources and population are described by only one system of two differential equations - but with the parameters η, θ and γ as an endogenous function of per capita resources:

$$\dot{R} = \left[1 - \theta(\bar{v}, \frac{R(t)}{L(t)})\right] r R(t) \left[1 - R(t)/K\right] - H(t) \tag{13}$$
$$\dot{L} = \left[b - \eta(\bar{v}, \frac{R(t)}{L(t)}) d + \phi H(t)/L(t)\right] L(t). \tag{14}$$

Recall the fact that conflict is initiated if per capita resources fall short of some given exogenous threshold level $\bar{v}$ and that conflict in turn increases the death rate, reduces the growth rate of resources, and lowers the share of the labor force devoted to production. Combining these assumptions, we postulate the following functional form for η, θ and γ:

$$\eta = 1 + \frac{\eta_{max} \bar{v}^{p_1}}{\bar{v}^{p_1} + \left(\frac{R(t)}{L(t)}\right)^{p_1}} \tag{15}$$
$$\theta = \frac{\theta_{max} \bar{v}^{p_2}}{\bar{v}^{p_2} + \left(\frac{R(t)}{L(t)}\right)^{p_2}} \tag{16}$$
$$\gamma = \gamma_{min} + (1 - \gamma_{min}) \frac{\left(\frac{R(t)}{L(t)}\right)^{p_3}}{\bar{v}^{p_3} + \left(\frac{R(t)}{L(t)}\right)^{p_3}}. \tag{17}$$

The constants η_{max}, θ_{max}, and γ_{min} denote the maximum impact that conflict will exert on the death rate, the intrinsic growth rate of resources, and the share of the

Insert Figure 1 here.

labor force devoted to production. As per capita resources tend towards extinction the functions (15), (16), and (17) tend towards $\eta = 1 + \eta_{max}$, $\theta = \theta_{max}$, and $\gamma = \gamma_{min}$, respectively. If per capita resources tend towards infinity, however, these functions are reduced to $\eta = 1$, $\theta = 0$ and $\gamma = 1$. The threshold level of per capita resources $\bar{v}$ determines the turning point of the logistic functions, and it is called the half saturation constant. It represents the level of per capita resources at which the logistic functions obtain half their maximum value. The constants p_1, p_2 and p_3 determine the slope of the logistic functions (see Figure 1).

If p_1, p_2, and p_3 are very large, these endogenous functions of η, θ and γ assume such an abrupt change in the value of η, θ, and γ in their simulations and discussions. Our framework allows for smooth transitions between a situation of no conflict and a situation of conflict.

3 Parametrization

Except for the baseline scenario — where we assume no feedback from resource scarcity on the parameters that effect population and resource growth and the share of the labor force devoted to production — we need to rely on numerical tools to investigate the long-run behavior of system (13)-(14).

Our choice of parameter values follows the suggestions in Brander and Taylor (1998) and Reuveny and Decker (2000), who aimed at replicating the history of Easter Island. Assuming a 10-year period interval, we set $N = b - d = 0.07$ and $r = 0.2$. These values imply that population will decline by 7 percent per decade if there are no resources left, and resources will increase by 20 percent if the carrying capacity tends to infinity.[7] Note that these values for N and r will be altered once we introduce the feedback mechanism from resource scarcity on population and resource growth. For the preference parameter β, which measures the share of labor devoted to harvesting, we choose 0.3. We set the parameter $\phi = 0.2$, which together with the values of $e = 2$, $f = 8$ implies a positive population growth rate if the total stock of resources is about three and a half times as large as the population size. This latter assumption is more restrictive than the parameter setting in Brandner and Taylor (1998). The carrying capacity is set to $K = 20$ and determines the scaling of the magnitudes of resources and population.

[7] We have experimented with various values of r and opted for this extreme value to highlight the possible spectrum of population and resource dynamics.

Insert Figure 2 here.

4 Analytical Results

For the baseline scenario we can analytically derive the equilibrium values of population and resources. Recalling equations (9) and (10), the equilibria are given by the intersection of the isoclines

$$\dot{R} = 0: \quad L = \frac{r(1-R/K)R}{E-rF(1-R/K)}, \quad R = 0 \tag{18}$$

$$\dot{L} = 0: \quad L = \frac{-N+\phi E}{NF}R, \quad L = 0 \tag{19}$$

with $N = d - b$ and $E = e\beta, F = f\beta$. It can easily be verified that the shape of the isocline $\dot{R} = 0$ will depend on the sign of the term $E - rF$. If this term is greater than zero the isocline will be hump shaped as in Figure 2.a, while it will have an asymptote at $R = K(1 - E/(Fr))$ if the term is less than zero (Figure 2.b).

The isocline $\dot{L} = 0$ is a straight line and its slope depends upon the sign of the term $\phi E - N$, which represents the maximum per capita growth rate of the population corresponding to an infinite amount of resource (see equations (7) and (10) with R tending to infinity). To allow the population to grow under such conditions, $\phi E - N$ is assumed to be positive, so that the isocline $\dot{L} = 0$ has a positive slope.

It follows from these considerations that there will always exist two trivial equilibria as given by $(R, L) = (0, 0)$ and $(R, L) = (K, 0)$, and there will be one strictly positive equilibrium $(R^*, L^*) = (K(1 - \frac{\phi E - N}{\phi r F}), \frac{\phi E - N}{NF}R^*)$ at most. Local stability analysis yields the following: the equilibrium $(R, L) = (0, 0)$ is a saddle point while the equilibrium $(R, L) = (K, 0)$ is asymptotically stable if $\phi E < N$ and it is a saddle if $\phi E > N$. Since we restricted the parameter set to the latter case, the equilibrium where $(R, L) = (K, 0)$ will always be a saddle. The strictly positive equilibrium, if it exists, will be either a stable or an unstable node or focus, but it can never be a saddle. (The resulting dynamics of population and resources are discussed in the next section.)

Note that the equilibrium value for resources R^* will depend only on 'behavioral' parameters such as β, N, and ϕ but not on the growth rate of resources r and their carrying capacity K. More specifically, the stationary value of resources decreases the more sensitive fertility reacts to increases in the harvest (ϕ), the more time is spent in harvesting (β), and the smaller the natural decrease in population N.

As the analytical results already indicate, the existence and stability of the interior equilibrium, as well as the stability of the trivial equilibrium $(K, 0)$, will depend on the specific parameters that determine the dynamic evolution of the resource stock and the population. In the next section we shall present a more in-depth analysis of the complexity of the long-run dynamics of population and resource stocks as they depend on the parameters of the system. The results are obtained through a detailed numerical local and global bifurcation analysis (see Champneys and Kuznetsov, 1994 and Strogatz, 1995) of the system (13)-(14).

5 Bifurcation Analysis

5.1 Baseline Scenario

Figure 3 illustrates a detailed picture of the possible long-run dynamics as they depend on the values of the growth rate of resources r and population N for the benchmark case, where we assume no feedback from resource scarcity on the dynamics of population and resources. The various dynamic behaviors are described in separate sketches that show the corresponding stable and unstable equilibria in phase space together with selected time paths.[8] If the natural rate of population decline $N = d - b > 0$ is high, the economy will — independently of the intrinsic growth rate of resources — converge to a situation where the resource stock equilibrates at its carrying capacity but the population vanishes (region 1). (Note that we have ruled out this possibility in the previous section by assuming $\phi E > N$ but include it for the sake of completeness in Figure 3.) Reducing the natural rate of population decline as it corresponds to a leftward movement along a horizontal line in Figure 3, the previously obtained trivial equilibrium $(K, 0)$ becomes unstable and a stable strictly positive equilibrium (R^*, L^*) emerges (region 2). However, the stability of the interior equilibrium will depend on the intrinsic growth rate of resources and population. Other things being equal, a reduction in the growth rate of resources implies that the equilibrium (R^*, L^*) becomes unstable and that a stable limit cycle emerges (region 3). A further decrease in the growth rate of resources inflates the period of the limit cycle and leads to a heteroclinic orbit in region 4, i.e. a situation where both trivial equilibria are saddle points that are connected with each other through a heteroclinic orbit. Trajectories starting outside the heteroclinic orbit tend toward the origin, while those starting inside tend toward the orbit. A further decrease in the growth rate of resources leads to a situation where the strictly positive equilibrium (R^*, L^*) will collide with the trivial equilibrium $(0, 0)$, which is no longer a saddle point — it has become a tangle (region 5). Such an equilibrium constitutes an attractor that is stable, although

[8]The caption in small letters near each bifurcation curve indicates the specific type of the bifurcation. We use the following abbreviations: trans.: transcritial bifurcation, heter.: heteroclinic bifurcation, heter. tang.: heteroclinic tangent bifurcation, hopf$^+$: subcritical Hopf bifurcation, hopf$^-$: supercritical Hopf bifurcation, fold: Fold bifurcation.

Insert Figure 3 here.

not in the sense of Lypanuov. Trajectories starting near the equilibrium $(0, 0)$ tend toward it, but they do not remain near the equilibrium (this is why it is not a local attractor anymore but a global one).

As evidenced by the shape of the various stability domains in Figure 3, there exists a tradeoff between population and resource growth. A higher rate of population growth requires more rapidly growing resources to keep the system sustainable. Our results also indicate that there exists a small region (areas 3 and 4) where continued cycles of population and resource growth may result. The period and amplitude of these cycles give an indication of the sustainability of the system dynamics. Sustainability is endangered the larger the amplitude and period become. These results are intuitive. As the population level increases, the pressure on resources will be exacerbated, and it will become ever more difficult for the resources to regenerate (see Anderies, 2000, p.400, Figure 3, for a similar finding). Moreover, whether a limit cycle is to be preferred to a stable equilibrium will depend on the amplitude and the time scale of the limit cycle, as already noted in Anderies (2000). For instance, if we compare the sketches for region 3 and region 4 it is obvious that a heteroclinic cycle is inferior to a stable equilibrium. Along the heteroclinic cycle, long periods of absence of resources and population alternate with long periods of resources at their carrying capacity and a near absence of population. Finally, a transition from continued cycles (region 4) to a system collapse (region 5) may result if resource growth declines further and/or population growth accelerates.

In summary, the bifurcation diagram as presented in Figure 3 allows us to investigate the specific combination of population and resource growth that may cause the model to shift from a sustainable long-run equilibrium towards a limit cycle and finally to a collapse of the system. Local bifurcation theory is the numerical tool that determines the boundaries between these regions in parameter space.

To see how the long-run dynamics of population and resources will change by including the feedback mechanisms described in Section 2, we start by introducing each of these feedbacks separately while varying the intensity and sensitivity of theses mechanisms.

5.2 Feedback mechanism through death rates

Figure 4 represents a scenario where we start in a situation that corresponds to egion 5 in Figure 3. It introduces, however, a positive feedback from resource scarcity on the death rate. Obviously, if the value of $\bar{v}$ is very low, the long-

Insert Figure 4 here.

run dynamics will not be altered since death rates will only be raised when per capita resource levels are already very low. By increasing $\bar{v}$, however, death rates will increase when resource scarcity is not yet that severe. Consequently, the probability of not ending up in a state where population and resources become extinct (region 5) will decrease. The shape of the bifurcation curve indicates that the extent to which death rates increase in conflict situations, η_{max}, has an impact on the long-run equilibrium as well.

Proposition 1: The more death rates increase as a consequence of resource scarcity and the higher the per capita resource stock at which this positive feedback on death rates sets in, the more likely the system will end up in a sustainable long-run equilibrium in which population and resources are both positive (Figure 4, region 2 and region 6).

The equilibrium value of per capita resources will then determine whether this sustainable equilibrium in region 2 will be sustained by 'continuous' conflict or no conflict.[9]

Figure 5 shows two different phase portraits corresponding to cases where the equilibrium is either in the conflict region (upper figure) or in the no conflict region (lower figure). Though qualitatively similar, these two regions represent very distinct situations. It can certainly be argued that it is preferable to have a situation where stabilization of long-run resource and population levels at positive values can be achieved by having no conflict in the long run. However, the transition to the steady state may well pass through the region of the conflict zone even if the long-run equilibrium level of resources and population is reached in a state where per capita resources are above the threshold level $\bar{v}$. Starting from point 2 in the lower figure, i.e. in a region of no conflict, the trajectory passes the conflict zone until it finally converges towards the no-conflict equilibrium.

Furthermore, note that the possible spectrum of dynamics in Figure 4 becomes richer than in the baseline scenario in Figure 3. In region 6 different stable modes of behavior coexist. If we start inside the unstable limit cycle, the system converges to a positive stationary regime, whereas starting outside, the system converges toward the heteroclinic connection of the two saddle equilibria $(0, 0)$ and $(K, 0)$ characterized by long periods of absence of resources and population alternating with long periods of resource at its carrying capacity, with a near absence of population. Starting outside the heteroclinic connection, the system converges toward resource and population extinction. Such a configuration implies that the

[9]We have a conflict zone (no conflict zone) in regions 2 and 6 of Figure 4 if per capita resources at equilibrium are below (above) the threshold level $\bar{v}$.

Insert Figure 5 here.

Insert Figure 6 here.

long-run dynamics will depend not only on the specific parameter values but also on the initial conditions of population and resources. Hence, the long-run dynamics are 'history dependent' in region 6. Such behavior implies that two societies in which resources and population follow the same dynamic laws may nevertheless diverge in the long run if they start from differing initial values of population and resources. In this case, a one-time change in the levels of resources and population may be preferable to switching to alternative institutional settings as represented by a change in the parameter values of $\bar{v}$ and η_{max}, since the latter policy may even push the system towards region 4 instead of region 2.

5.3 Feedback mechanism through growth rate of resources

Similarly to allowing for a feedback from resource scarcity on the death rate, we can introduce a feedback that directly impinges on the growth rate of resources (Figure 6). While the feedback on death rates constitutes a stabilizing mechanism, the postulated negative feedback on the growth rate of resources will reinforce the impact of resource degradation. Starting at $\bar{v} = 0$ and $\theta = 1$, i.e. a point that lies within region 2 in the baseline scenario of Figure 3, introducing the negative feedback from resource scarcity on the growth rate of resources may cause the economy to end up in region 5, where population and resources become extinct. Similarly to Figure 4, there exists a threshold of $\bar{v}$ below which the introduction of this negative feedback will not have any impact on the long-run equilibrium values. Moreover, in those regions that are characterized by a sustainable long-run value of population and resources (region 2), lower values of $\bar{v}$ are preferable, as they support these long-run values without being in a conflict situation.

Proposition 2: The stronger and the earlier (i.e. already at high values of per capita resources) a conflict situation feeds back on the growth rate of resources, the more likely it is that resources and population will become extinct.

While a conflict situation that only impinges on the death rate will relieve resource scarcity (by decreasing the denominator in the expression of per capita resources R/L), the opposite holds true in the case of a negative feedback effect on resource growth. If a conflict situation reduces the indigenous growth rate of resources (e.g. by depleting soil nutrients), resource scarcity will even be exacerbated. Lower resource growth implies fewer resources and lower levels of per capita resources in general.

Insert Figure 7 here.

5.4 Feedback mechanism through share of labor devoted to production

Figure 7 represents the case where we allow resource scarcity to displace some workers in manufacturing production and also, in particular, in resource harvesting, as they may be devoted to conflict resolution. Similarly to the feedback mechanism that operates through the death rate, the feedback mechanism that reduces the share of productive workers will have a stabilizing effect on the long-run values of resources and population. The lower the value of γ_{min}, i.e. the more people will be displaced from resource harvesting under conditions of conflict, the more likely it is that the system will end up in region 2. The value of $\bar{v}$ will again determine whether the sustainable long-run value of population and resources in region 2 can be obtained through continuous conflict or not. Similarly to Figure 4, there exists a small region in the parameter space in Figure 7 where history determines the long-run behavior of resources and population. More specifically, in region 7 we observe the coexistence of a stable limit cycle and a heteroclinic orbit.

Proposition 3: The more and the earlier (i.e. already at high values of per capita resources) a conflict situation diverts part of the labor force from production to the resolution of the conflict situation, the more likely it is that the system will end up in a sustainable long-run equilibrium.

5.5 Feedback mechanism through death rate and growth rate of resources

As already indicated in Figures 4, 6, and 7, the feedback from resource scarcity on death rates and the share of the labor force displaced from production constitutes a stabilizing feedback, i.e. it may facilitate a positive level of population and resources in the long run. In contrast, the feedback introduced through the growth rate of resources accelerates resource degradation and henceforth convergence of the system towards extinction. In Figure 8 we present a two-dimensional bifurcation diagram that illustrates the long-run dynamics of population and resources if we introduce a stabilizing (η_{max}) together with a destabilizing (θ_{max}) feedback mechanism. As expected, the bifurcation diagram shows the tradeoff between these two mechanisms. If death rates increase as a consequence of resource scarcity ($\eta_{max} > 0$) while at the same time the resource growth rate is reduced to a great extent (θ_{max} high), population and resources will become extinct (region 5). The positive slope of the bifurcation curves illustrates this tradeoff.

Insert Figure 8 here.

As this last example shows, it is not the richness of various feedback mechanisms that will determine long-run sustainability. In fact, we can even observe that societies with the most complex structure of institutional settings are the most vulnerable ones. The reason for this is that the parameter space for which the various feedback mechansims cancel out each other may become dominant (cf. region 5 in Figure 8).

6 Discussion

In this paper we have presented a detailed analysis of the possible long-run dynamics of population and resources based on a recent model by Maxwell and Reuveny (2000). By using local bifurcation analysis we can distinguish between various long-run system dynamics[10] since they depend on alternative parameter combinations. Moreover, we can locate those mechanisms that will yield sustainable levels of population and resources in the long run. Expanding on Anderies (1998, 2000), who already applied local bifurcation theory to study the long-run dynamics of traditional societies, we have introduced two-dimensional bifurcation diagrams. This allows us to trace the system dynamics even if not just one but two parameters of the model change. In particular, we focused on the parameters that determine the sensitivity and extent of feedback mechanisms that are initiated by resource scarcity.

Though we referred specifically to the notion of a conflict situation that may be initiated by resource scarcity and then feed back on the population's death rate, the growth rate of resources, and the share of the labor force devoted to production, the set-up of the model is more general. In particular, it shows how the long-run dynamics of population and resources may change if the growth rates of population and resources depend on the current state of the system. Our work thereby complements the research of Anderies (2000) and Reuveny and Decker (2000), who focused mainly on how institutional settings that connect resource and population growth may change, depending on the state of the system.

As the bifurcation diagrams show, it is mainly the level of resource scarcity at which various feedback mechanisms become active (i.e. the value of $\bar{v}$) that will determine whether a sustainable future path of population and resource levels can be achieved. Hence, sustainability requires not only the mere prevalence of

[10]Despite Keynes's well-known statement 'in the long run we are all dead', it is precisely in models that include natural resources where we need to be concerned with long run strategies. Our suggestion of using bifurcation theory and thereby focusing on long-run dynamics is therefore justified.

stabilizing feedback mechanisms but also that they become active 'early' enough. As population and resources move on different time scales (compare Gragnani et al., 1998 and Milik and Prskawetz, 1996), resource scarcity may be realized 'too late' or not at all. In particular, one should keep in mind the finite lifetime of individuals on which decisions are based. A similar argument is also brought forward by Anderies (2000). He explains the lack of institutional changes by the fact that ecological processes move on a slow time scale as compared to the fast time scale of a human lifetime. By increasing the threshold level of per capita resources at which resource scarcity initiates various feedback mechanisms, our models account for these caveats since the warning about resource scarcity is communicated early enough.

Although we focused here mainly on the various paths of feedback mechanisms once per capita resources become scarce, the preferred strategy should be to investigate mechanisms that may prevent resource scarcity already in the first step. Figure 3, in which we varied the natural decline of population N and the growth rate of resources r, presents such precautionary policies which may prevent resource scarcity from becoming acute. The aim of the papers by Reuveny and Decker (2000) and Anderies (2000) is precisely to study how such precautionary policies may have become institutionalized or, alternatively, how technical progress in the agricultural good sector can change the dynamics.

In addition to merely presenting the possible spectrum of long-run dynamics as they will be realized, depending on the parameters of the system dynamics, our analysis can be used the other way around. That is, it offers the possibility of locating those parameter regions that produce dynamics corresponding to past experiences such as, e.g., the collapse of the Easter Island civilization as opposed to the self-regulating forces prevalent in the Tsembega society of New Guinea. Such results might help us to understand why specific feedback mechanisms have not been established while others have evolved through time, since they may have been targeted to yield sustainable future paths of population and resource growth.

We should also add that our analysis is necessarily restrictive in terms of the possible alternatives once resource scarcity is realized. But the main purpose of our paper has not been to suggest new strategies but to rely on well-established models such as that of Brandner and Taylor (1998) and Maxwell and Reuveny (2000) and to present the technique of local bifurcation theory (which is often very aptly termed 'comparative dynamics') in order to gain a better understanding of the long-run behavior of a system that depends on various parameters.

APPENDIX

The standard Monod function of resource harvesting is given by

$$h = \frac{ax}{b + x} \tag{20}$$

where the harvest h denotes the number of prey x 'eaten' by one predator in one unit of time. The unit of time is split up into a fraction η_s of time devoted to searching for prey, a fraction of time η_r of time devoted to rest, and a fraction η_h of time to handle the prey, where $\eta_s + \eta_r + \eta_h = 1$ holds. It is then assumed that the fraction of time devoted to the search is proportional to h/x, the fraction of time devoted to resting is constant, and the fraction of time devoted to the harvest is proportional to the harvest:

$$\begin{aligned} \eta_s &= \alpha \frac{h}{x} && (21) \\ \eta_r &= \beta && (22) \\ \eta_h &= \gamma h. && (23) \end{aligned}$$

Substituting the fractions of time into $\eta_s + \eta_r + \eta_h = 1$ yields $h\left[\alpha/x + \gamma\right] = 1 - \beta$, which yields

$$h = \frac{\frac{1-\beta}{\gamma}x}{\frac{\alpha}{\gamma} + x} = \frac{ax}{b + x} \tag{24}$$

where the constant a is proportional to the resting time, and b is proportional to the searching time.

The specific harvest function as given by (7) is similar to the standard Monod function, except that the denominator is slightly different. Specifically, the harvest function we use is of the form:

$$h = \frac{ax}{by + x}. \tag{25}$$

It can easily be verified that the specific form of this function can be derived if we assume that the relation between η_r and η_h is similar to the above case, and at the same time assume now that the fraction of search time is proportional to $\frac{h}{x/y}$, i.e. $\eta_s = \alpha \frac{h}{x} y$. This modification implies that the fraction of search time now depends also on the number of predators y, which in some sense reflects competition between predators. The more predators there are, the longer the search time will be. Substituting the new definition of η_s into the time constraint equation $\eta_s + \eta_r + \eta_h = 1$ yields

$$h = h = \frac{\frac{1-\beta}{\gamma}x}{\frac{\alpha}{\gamma}y + x} = \frac{ax}{by + x}. \tag{26}$$

Recalling the definition of the harvest function in (7), the constants a and b are equal to $a\gamma\beta$ and $b\gamma\beta$. Hence, the parameters of the Monod function are scaled by the term $\beta\gamma$, which represents the fraction of time each individual devotes to harvesting.

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Captions to the figures

Figure 1: Shape of logistic functions $\eta - 1$, θ and $1 - \gamma$ with $\eta_{max} = \theta_{max} = 1$ and $\gamma_{min} = 0$, $\bar{v} = 5$ and $p_1, p_2, p_3 = 2, 4, 6, 8$.

Figure 2: Isoclines $\dot{R} = 0$ and $\dot{L} = 0$ for $K = 20, e = 2, f = 8, \beta = 0.3, \phi = 0.2, N = 0.07$ and (a) $r = 0.2$, (b) $r = 0.3$.

Figure 3: Two-dimensional bifurcation diagram with respect to the natural rate of population decline $N = d - b$ and the intrinsic growth rate of resources r. All other parameters are set as in Figure 2. In all five sketches full (empty) dots refer to stable (unstable) equilibria and solid closed trajectories refer to stable cycles.

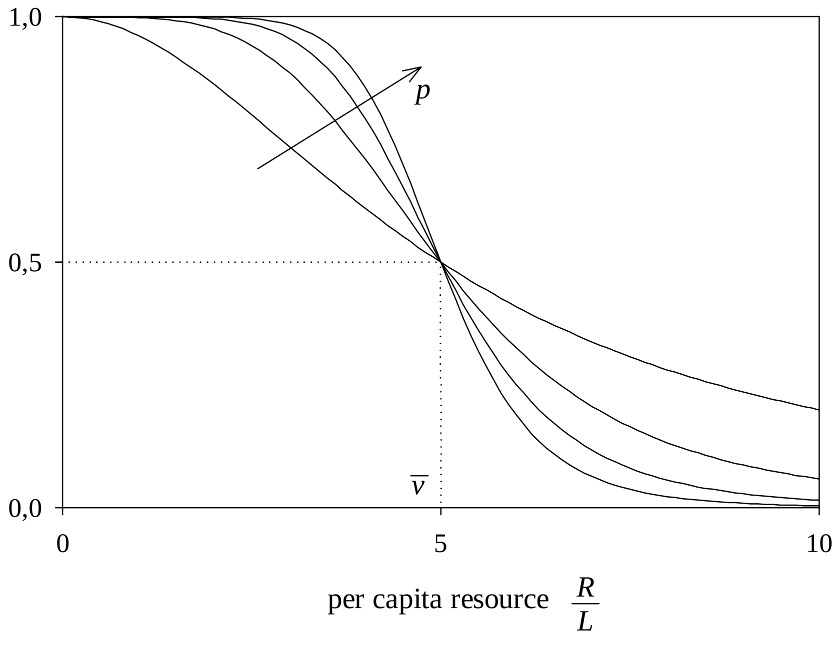
Figure 4: Two-dimensional bifurcation diagram with respect to the threshold level of per capita resources $\bar{v}$ and the maximum level of the death rate η_{max}. All other parameters are set as in Figure 2, with $r = 0.1, N = 0.05$ and $p_1 = 2$. In the sketch that refers to region 6, solid (dashed) closed trajectories refer to stable (unstable) cycles.

Figure 5: Phase portraits with the parameter set that corresponds to region 2 in Figure 4. The upper figure is for $\bar{v} = 9, \eta_{max} = 1$ and the lower figure is for $\bar{v} = 3.5, \eta_{max} = 1.4$.

Figure 6: Two-dimensional bifurcation diagram with respect to the threshold level of per capita resources $\bar{v}$ and the maximum level of the negative feedback on the growth rate of resources θ_{max}. All other parameters are set as in Figure 2 with $r = 0.1, N = 0.09$ and $p_2 = 2$.

Figure 7: Two-dimensional bifurcation diagram with respect to the threshold level of per capita resources $\bar{v}$ and the maximum share of the labor force diverted from production γ_{min}. All other parameters are set as in Figure 2, with $r = 0.15, N = 0.04$ and $p_3 = 2$.

Figure 8: Two-dimensional bifurcation diagram with respect to the maximum level of the death rate η_{max} and the maximum level of the negative feedback on the growth rate of resources θ_{max} setting $\bar{v} = 8$. All other parameters are set as in Figure 2, with $r = 0.1, N = 0.05$ and $p_1 = p_2 = 2$.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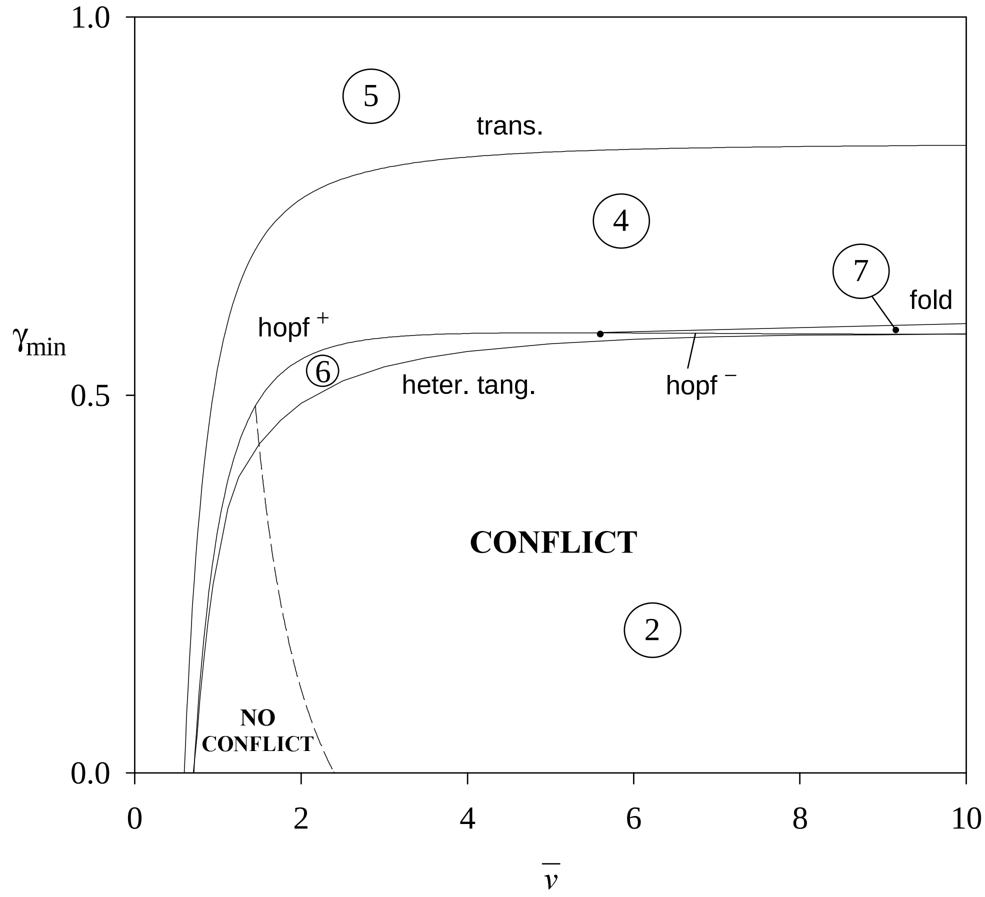

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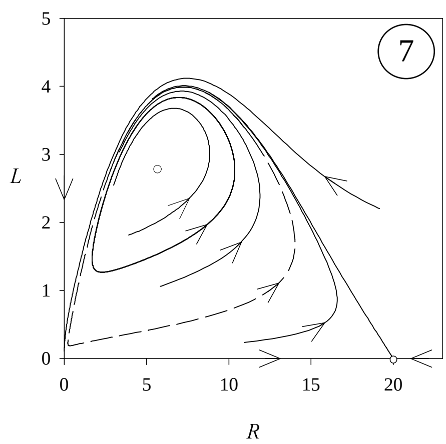

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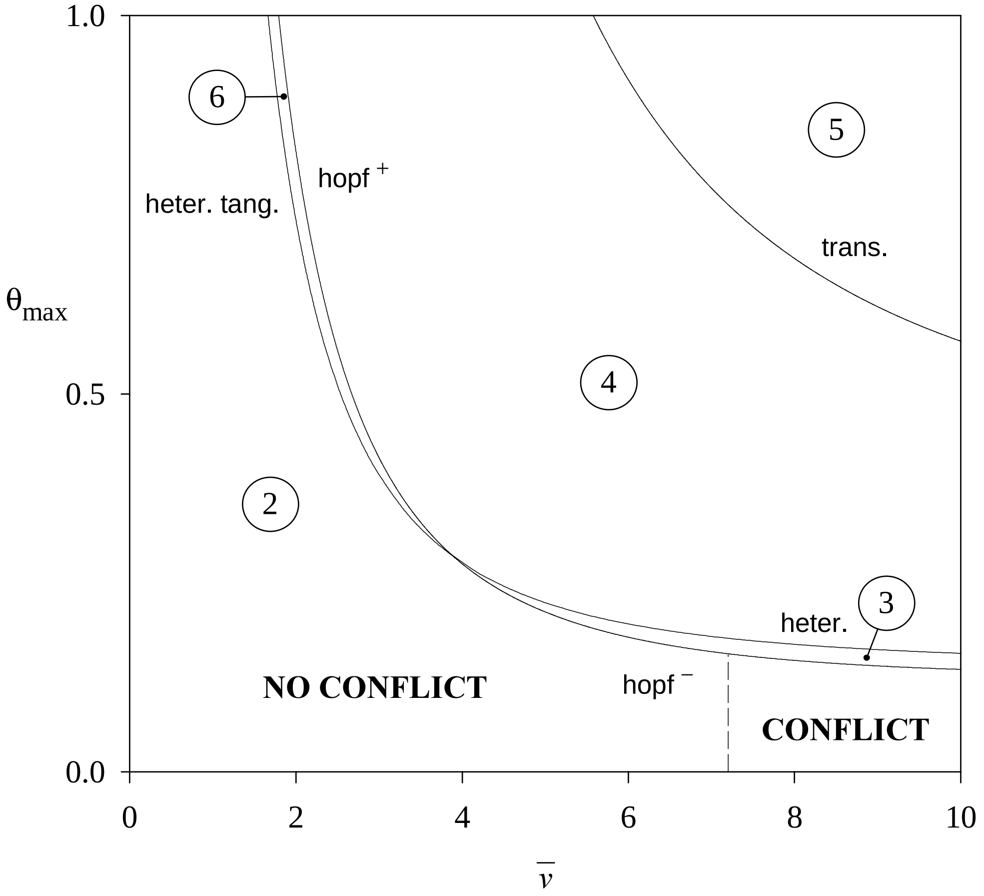

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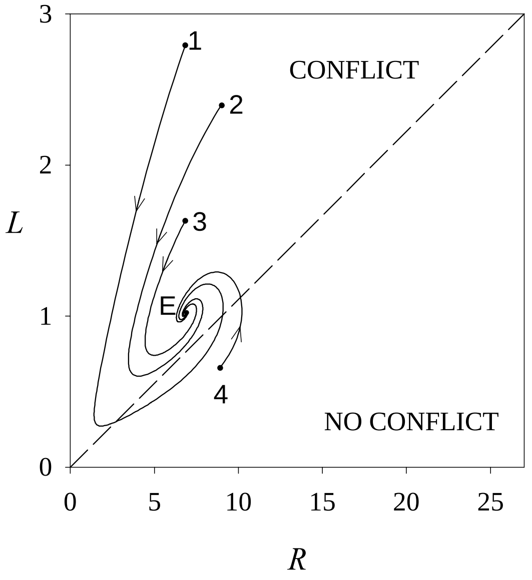
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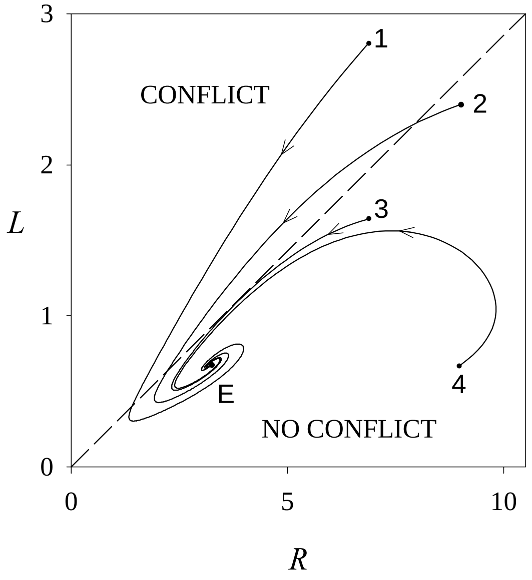
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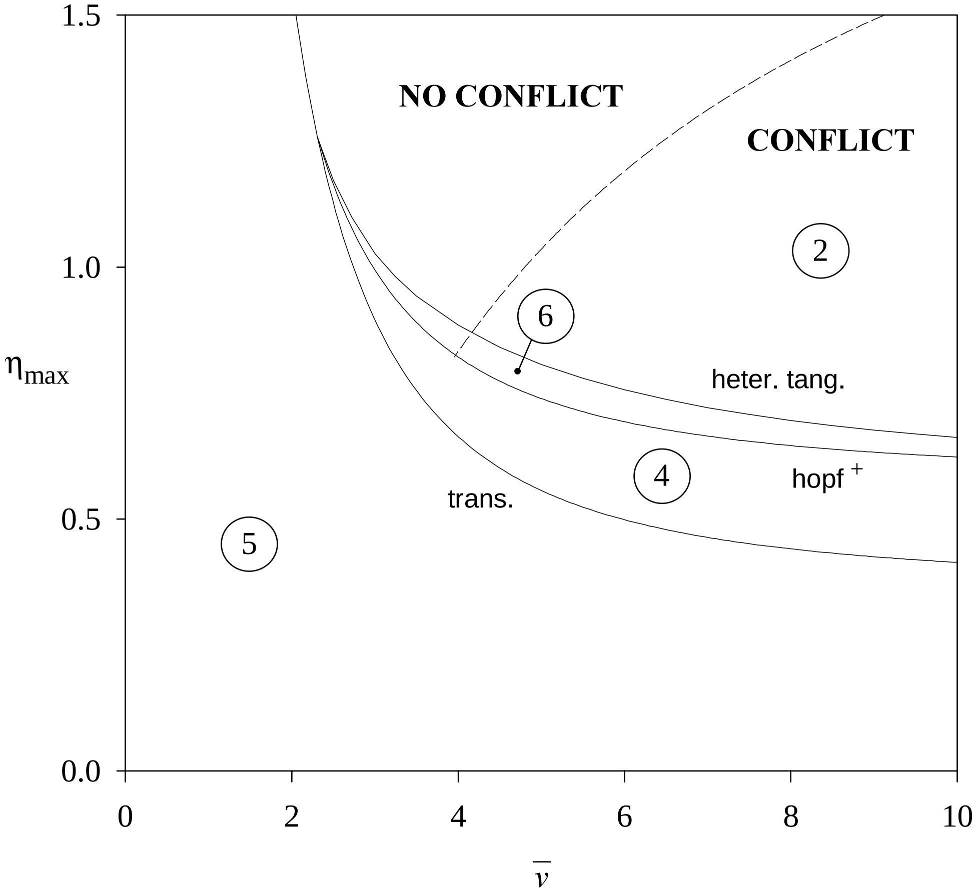
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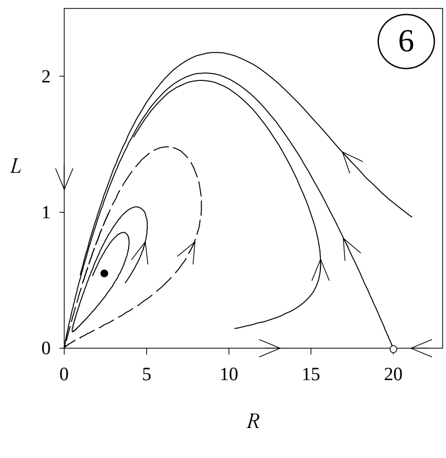
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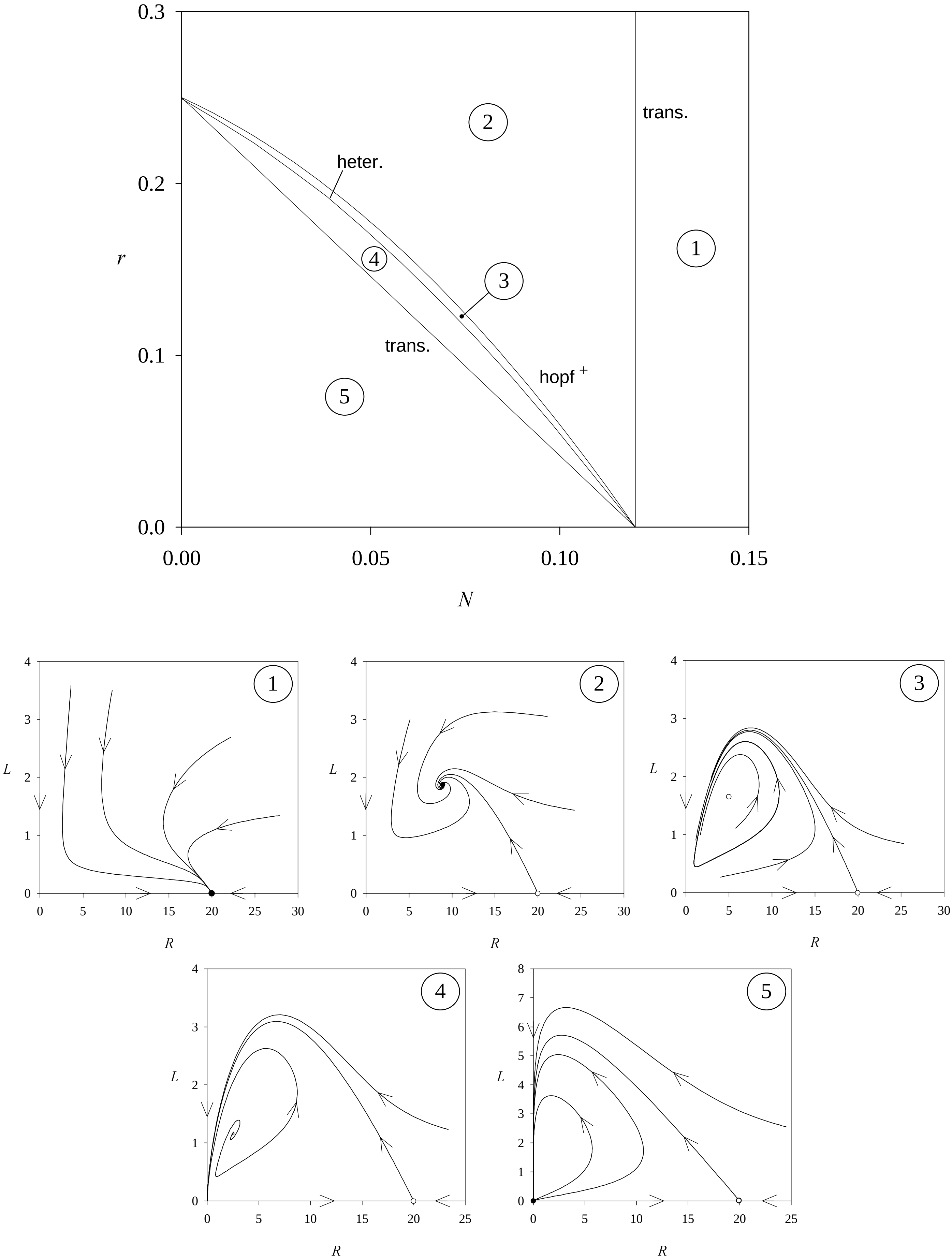

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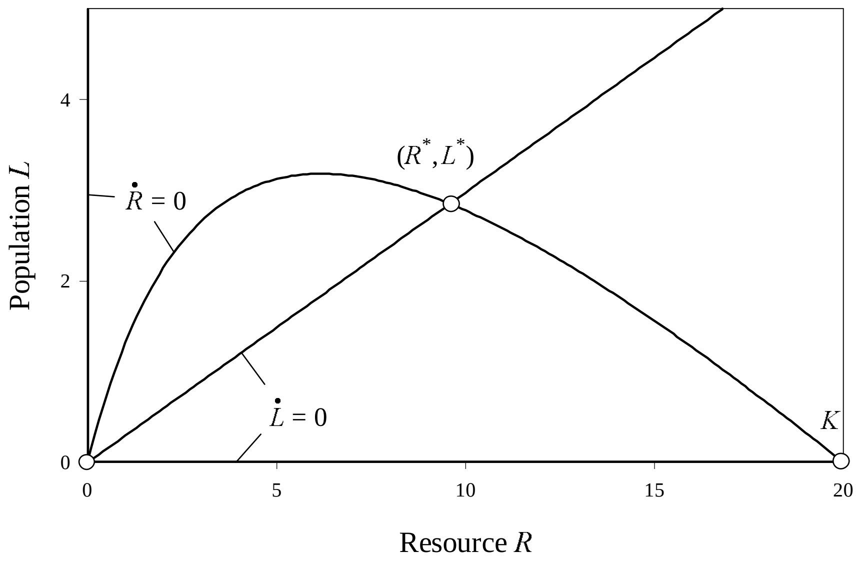

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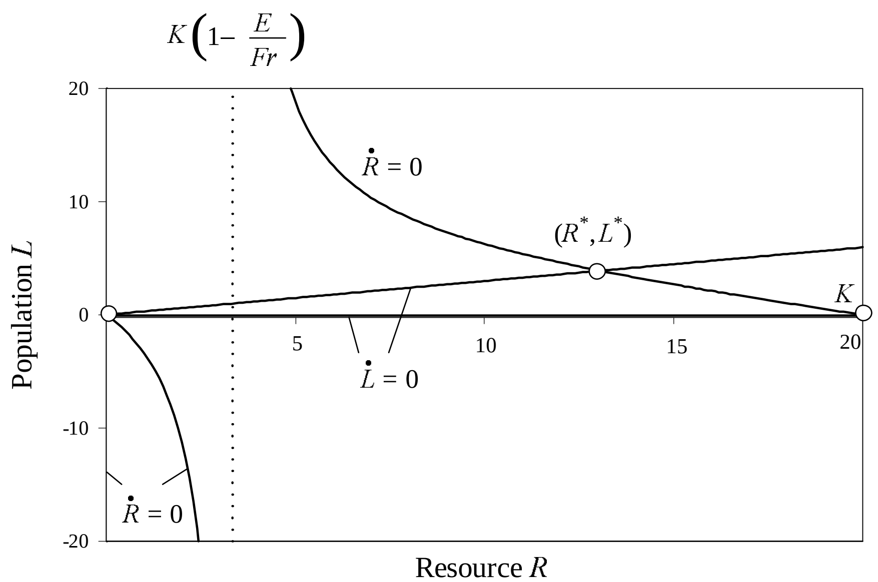

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Population L
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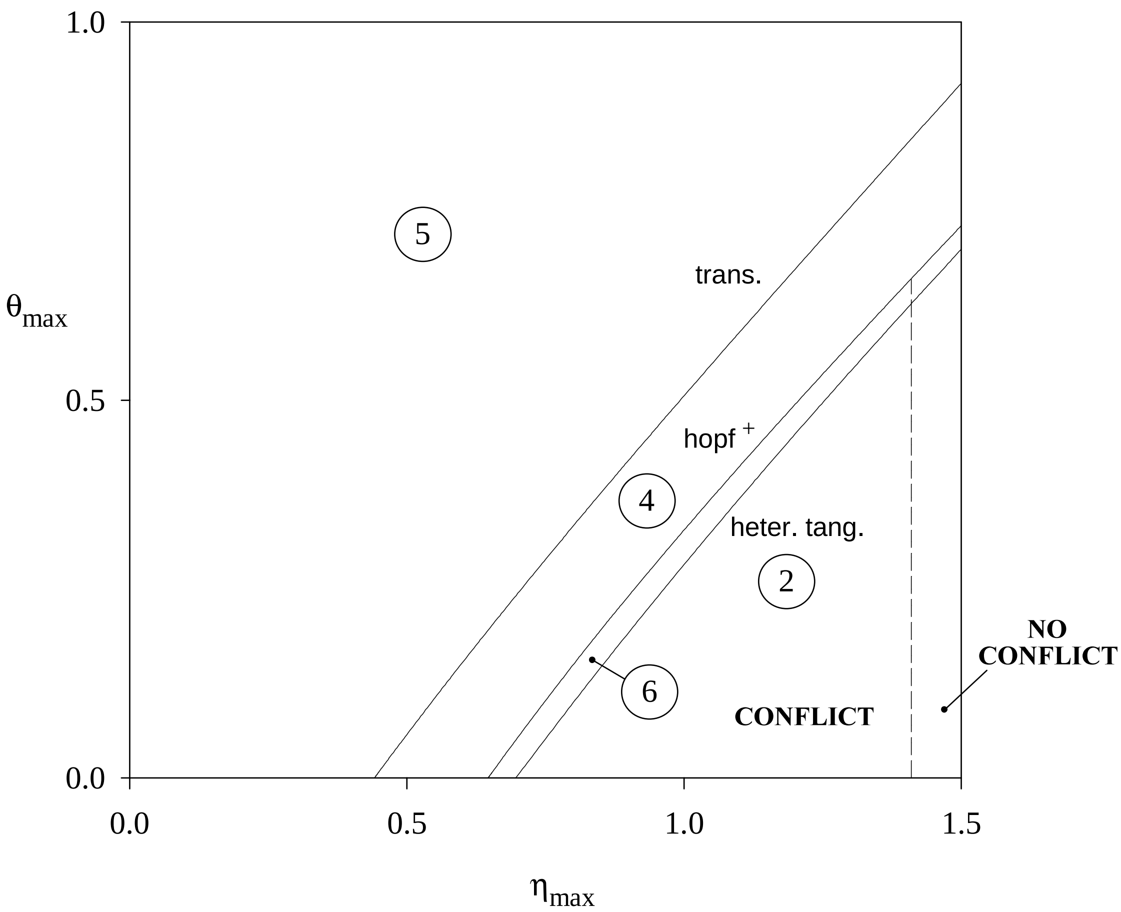
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