Environmental Policy, the Porter Hypothesis and the Composition of Capital: Effects of Learning and Technological Progress[1]

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Abstract

In this paper the effect of environmental policy on the composition of capital is investigated. By allowing for non-linearities it generalizes Xepapadeas and De Zeeuw (Journal of Environmental Economics and Management, 1999) and determines scenarios in which their results do not carry over. In particular, we show that the way acquisition cost of investment decreases with the age of the capital stock is of crucial importance. We also focus more explicitly on learning and technological progress. Among others we obtain that in the presence of learning, implementing a stricter environmental policy with the aim to reach a certain target of emissions reduction has a stronger negative effect on industry profits, which implies quite the opposite as to what is described by the Porter hypothesis.

Keywords: Porter hypothesis, dynamics of the firm, vintage capital stock, environmental policy, technological progress

1 Introduction

The Porter hypothesis says that environmental policy spurs innovation which makes firms better off in the long run, since it increases their competitiveness. This results in a win-win situation in the sense that environmental policy improves both environment and competitiveness. The Porter hypothesis was studied in a very interesting paper by Xepapadeas and de Zeeuw [5], employing a dynamic model of the firm. It was found that a win-win situation will not hold, but the decrease in competitiveness due to the extra environmental costs is mitigated by a modernization effect.

The economic reason for modernization was that modern machines, although being more expensive, are more productive and pollute less. This can be caused by aging or technological progress. In Xepapadeas and De Zeeuw [5] only the effect of aging was taken into account. Although they say otherwise, we actually will show that their model does not contain technological progress.

In this paper we generalize the framework of Xepapadeas and De Zeeuw by allowing for nonlinear functional forms and technological progress. We derive that results could be generalized as long as acquisition costs of investment are concavely decreasing in age. However, in case of a convex dependence of these costs on age, average age of the capital stock can increase when a stricter environmental policy is imposed.

We also investigate the effects of learning. We find that the emission tax increase should be larger to reach a given emissions output reduction, and the effect of the tax increase on profits is more severe. Hence, the Porter hypothesis is rejected strongly in the presence of learning.

The paper is organized as follows. Section 2 presents a generalized version of the model of Xepapadeas and De Zeeuw [5]. Here we also provide the new result that in their model it is optimal for the firm to converge to the steady state. In Section 3 we study the effects of nonlinearities as well as the implications of learning and technological progress. Section 4 concludes. All proofs and technical considerations are collected in the Appendix, where also the precise assumptions are formulated in mathematical terms.

2 The Model

Here we present a slightly extended version of the model of Xepapadeas and De Zeeuw [5]. As in their paper, here it also holds that the age of the machine is denoted by $a \in [0, h]$, so that the maximum age of the machines is h. As usual, calendar time is denoted by t.

Denote by $v(a)$ the output produced by a machine of age a. If it is assumed that an older machine cannot produce more output than a newer one, then $v'(a) \leq 0$. This model feature is taken from Xepapadeas and De Zeeuw [5] who argue that this implies that new machines are more productive because they embody superior technology. However, this argument seems to be incorrect. To see this, note that $v(a)$ is the same for different t. Now consider two points of time: t_1 and t_2 so that $t_2 > t_1$. Then a machine constructed at time t_2 has the same productivity at the same age as a machine constructed at time t_1, i.e. the second machine produces at $t_2 + a$ amount $v(a)$, which is also the amount that the first machine produces at $t_1 + a$. Hence there is no superior technology embedded in the machine constructed at t_2. The conclusion is that, since v is independent of calendar time t, no technological progress is included.

To include technological progress, we impose that productivity increases with every new vintage. To model this, we denote output produced by a machine of age a at time t by $v\left(a\right) f\left(t-a\right)$, where it holds that $f'\left(t-a\right) > 0$ (see also Feichtinger et al. [4], [2]).

In fact, $v'(a) \leq 0$ can still be a sensible assumption, because due to aging machines can get less productive as time passes. However, the opposite, i.e. $v'\left(a\right) > 0$, is also possible since this reflects that due to learning machines get more productive over the years (cf. Feichtinger et al. [4]).

The stock of capital goods of age a at time t is denoted by $K(t, a)$. With p the price of output, the firm's revenue in year t is defined as

$$pQ(t) = \int_0^h pv(a) f\left(t-a\right) K(t, a)\, \mathrm{d}a.$$

It is assumed that markets exist for machines of any age from 0 to h. The cost of buying a machine of age a is given by $b(a)$, with $b'(a) \leq 0$ (older machines cannot be more expensive than newer machines) and $b(h) = 0$ (a machine at the maximum age is not worth anything). Let

$I(t,a)$ be the number of machines of age a bought (if $I(t,a) > 0$) or sold (if $I(t,a) < 0$) in year t. The total cost or revenue to the firm from transactions in the machine market is defined as $b(a)I(t,a) + \frac{1}{2}[I(t,a)]^2$, with the second term reflecting the adjustment costs in buying or selling machines.

Running the machine is costly, and the running costs of a machine of age a are denoted by $c(a)$, $c'(a) \geq 0$. Furthermore, $s(a)$ are emissions of a machine of age a, $s'(a) \geq 0$. As a result of aging, older machines emit at least as much as newer machines (but, analogous to our argument in connection with $v(a)$, not as a "result of cleaner technologies being embodied in the new machines" (Xepapadeas and De Zeeuw [5], p. 168). Here also learning effects may be present resulting in $s'(a) \leq 0$: over time firms learn how to use machines in a more environmental friendly way.

Per unit of emissions the firm has to pay an emission tax τ. As in Xepapadeas and De Zeeuw, we also assume away discounting and depreciation. The resulting dynamic model of the profit maximizing firm is now given by

$$\max_I \int_0^\infty \int_0^h [pv(a)f(t-a)K(t,a) - (c(a) + \tau s(a))K(t,a)] \,\mathrm{d}a\,\mathrm{d}t \qquad (1)$$

$$-\int_0^\infty \int_0^h \left[b(a)I(t,a) + \frac{1}{2}(I(t,a))^2\right] \mathrm{d}a\,\mathrm{d}t$$

$$\text{subject to} \quad \frac{\partial K(t,a)}{\partial t} + \frac{\partial K(t,a)}{\partial a} = I(t,a), \qquad (2)$$

$$K(t,0) = 0, \quad K(0,a) = K_0(a) \geq 0, \qquad (3)$$

$$K(t,a) \geq 0. \qquad (4)$$

This is an infinite horizon optimal control problem with transition dynamics described by a linear partial differential equation (Carlson et al. [1]), in which the investment $I(t,a)$ is considered as a control variable. Formula (3) provides the initial and boundary conditions, which apply at time zero and when the age of the machine is zero.

The objective value in this problem can be infinite. Nevertheless the meaning of optimality is very natural, and will be revealed by Proposition 1 below. To do this we first consider the problem (1)–(3) on a finite time horizon $[0,T]$ instead of $[0,\infty)$, with $T > 2h$.

The Hamiltonian H for this problem is given by:

$$H(t,a,K,I,\lambda) \quad = \quad pv(a)f(t-a)K - [c(a)+\tau s(a)]\,K - b(a)I - \frac{1}{2}I^2 + \lambda I. \tag{5}$$

Consequently (see e.g. Feichtinger et al. [3] for general optimality conditions for age-structured systems), the first-order optimality conditions are

$$\frac{\partial H}{\partial I} = 0, \quad \text{or} \quad I(t,a) = \lambda(t,a) - b(a), \tag{6}$$

$$\frac{\partial \lambda(t,a)}{\partial t} + \frac{\partial \lambda(t,a)}{\partial a} = -\frac{\partial H}{\partial K} = -pv(a)f(t-a) + c\,(a) + \tau s\,(a)\,, \tag{7}$$

$$\lambda(t,h) = 0, \quad \lambda(T,a) = 0. \tag{8}$$

Solving (7) and (8) to obtain $\lambda(t,a)$ and employing (6) yields for $a \in [0,h]$, $t \in [a, T-h]$:

$$I(t,a) = \int_a^h [pf(t-a)v(\alpha) - c\,(\alpha) - \tau s\,(\alpha)]\, d\alpha - b(a). \tag{9}$$

An expression for the stock of capital goods can be derived from (9) and (2), while $t \in [a, T-h]$:

$$K(t,a) = \left(\int_0^a \left[\int_\sigma^h [pf(t-a)v(\alpha) - c\,(\alpha) - \tau s\,(\alpha)]\, \mathrm{d}\alpha - b(\sigma) \right] \mathrm{d}\sigma \right). \tag{10}$$

An implicit assumption for the subsequent analysis is that the values of $K(t,a)$ obtained by the above formulae are all nonnegative, so that the constraint (4) is automatically satisfied. We obtain the following proposition, which describes the meaning of optimality for the infinite horizon problem (1)–(4).

Proposition 1 *For every t the values of the solution $K(t,a)$, $a \in [0,h]$, of problem (1)–(4) on the time horizon $[0,T]$, with $T > t+h$, are independent of T.*

In the remainder of the section we consider the model without technological progress, thus when $f\,(t-a)$ is constant, say $f\,(t-a) = 1$, which is the original Xepapadeas-De Zeeuw framework. Xepapadeas and De Zeeuw [5] limit their analysis to the OSSP (Optimal Steady State Problem, see Carlson et al. [1]) without actually verifying whether the optimal solution will reach this steady state. However, we now show that it is in fact optimal for the firm to reach this steady state at time h, which strengthens the validity of their results.

An important observation is that for $f = 1$ (9) and (10) are time invariant. This means that for $t > h$, implying that $t \geq a$, the steady state with respect to calendar time is reached. We thus can state the following proposition.

Proposition 2 *Consider the model of this section with $f(t-a) = 1$. Then it holds that at most after h years the firm reaches its steady state. For each age $a \in [0, h]$ the capital stock is then time invariant and given by (10). The investment intensity in a capital stock of a given age is also constant over time and given by (9).*

3 Results

In this section we study whether introducing nonlinearities influences the impact of the emission tax on the average age and productivity of the capital stock. Also, we investigate the effects of deterioration, learning and technological progress.

Average age of the optimal capital stock at a fixed time t is defined by

$$g(\tau) = \frac{\int_0^h aK(t,a)\,da}{\int_0^h K(t,a)\,da},$$

while average productivity of the capital stock is given by

$$\pi(\tau) = \frac{\int_0^h v(a)\,K(t,a)\,da}{\int_0^h K(t,a)\,da} \tag{11}$$

(here g and π depend on τ through K).

Xepapadeas and De Zeeuw [5] completely focus on natural deterioration and assume a simple linear specification $v(a) = a_0 + a_1(h-a)$, where a_0 is nonnegative and a_1 is strictly positive. In this case they obtain a monotonic relationship between modernization and productivity in the sense that the lower the average age of the machines the higher the productivity is. Then they prove for linear specifications of $c(a)$, $b(a)$, and $s(a)$ that an increase in the emission tax reduces the optimal average age of the capital stock and increases its average productivity.

The following proposition generalizes Xepapadeas and De Zeeuw's productivity result. It consists of three parts: part (i) generalizes Xepapadeas and De Zeeuw to allow for non-linear functions, part (ii) concerns the learning case, while part (iii) considers technological progress, where we specify a simple linear function for $f(t-a)$:

$$f(t-a) = f^0 + \kappa[t-a], \tag{12}$$

with f^0 and κ being nonnegative constants such that $f^0 > \kappa h$. As it is shown in Feichtinger et al. [2] a specification like this is justified by "Moore's law" which says that the memory and arithmetic power of micro chips develop exponentially over time, if we assume in addition that production is a logarithmic function of technology.

Proposition 3 *At any time $t \geq h$, average productivity of the capital stock rises with an increase in the emission tax when*

(i) $f(t-a) \equiv 1$, $v(a)$ is non-increasing, $c(a)$ and $s(a)$ are non-decreasing, and $b(a)$ is concave;

(ii) $f(t-a) \equiv 1$, $v(a)$ is non-decreasing, $c(a)$ and $s(a)$ are non-increasing, and $b(a)$ is convex;

(iii) there is technological progress specified by (12), $v(a)$ is non-increasing, $s(a)$ and $c(a)$ are non-decreasing, and $b(a) = b[h-a]$.

Moreover, if $v(a)$ is not constant, then the increase of the average productivity is strict.

Hence, if the emission tax rate increases, under each of the conditions (i)–(iii) the firm responds by changing its machine park in such a way that average productivity increases.

The next proposition shows the implications for the optimal average age of the capital stock. Like in Xepapadeas and De Zeeuw we have modernization in cases (i) and (iii), but in case (ii) (learning) average productivity is increased by employing, on average, older machines.

Proposition 4 *At any time $t \geq h$, the optimal average age of the capital stock changes with an increase in the emission tax as follows:*

(i) the optimal average age decreases if $f(t-a) \equiv 1$, $v(a)$ is non-increasing, $c(a)$ and $s(a)$ are non-decreasing, and $b(a)$ is concave;

(ii) the optimal average age increases if $f(t-a) \equiv 1$, $v(a)$ is non-decreasing, $c(a)$ and $s(a)$ are non-increasing, and $b(a)$ convex;

(iii) the optimal average age decreases if there is technological progress specified by (12), $v(a)$ is non-increasing, $c(a)$ and $s(a)$ are non-decreasing, and $b(a) = b[h-a]$.

Moreover, if $v(a)$ is not constant, then the increase (resp. decrease) of the average age is strict.

In both propositions result (ii) applies to the learning case. Learning most likely occurs during the first years that a certain vintage is in operation. This implies that $v'(a) > 0$ for $a < \nu$ and $v'(a) = 0$ for $a \geq \nu$, where ν is a positive constant (cf. Feichtinger et al. [4]). From the propositions it can be derived that under such a specification of $v(a)$, both average productivity and average age are strictly increasing with the emission tax.

Result (iii) in both propositions is in fact the main result of the Xepapadeas-De Zeeuw paper, but it is now derived while allowing for some nonlinearities and modeling technological progress in the correct way. From both propositions it is also inferred that the shape of the acquisition function turns out to be crucial. The economic interpretation is as follows. An increase of the emission tax rate leads to downsizing implying that the firm cuts down on machines (cf. Xepapadeas and De Zeeuw [5]). In case $b(a)$ is concave the firm gains most by removing the older machines since they have the highest marginal cost. In case of convex $b(a)$ the opposite is true, so then the firm prefers to remove the younger machines. This explains why for result (i) $b(a)$ needs to be concave: in case of convex $b(a)$ contrary effects arise, because removal of the younger machines leads to an increase of average age and thus a reduction of productivity. It is straightforward that for result (ii) these arguments work in the opposite direction.

Let us illustrate this for the case where technological progress is present. We replace the linear acquisition cost function $b(a) = b[h-a]$ by

$$b_\gamma(a) = [b - \gamma a][h-a],$$

with $\gamma \in [0,1]$. Note that for $\gamma = 0$ the two functions coincide. We carried out a numerical investigation with specifications $h = 10$, $b = 10$, $v(a) = 200$, $c(a) = 0$, $s(a) = 18$, $\tau = 10$, $f(t-a) = 1 + 0.001[t-a]$, for which the outcome is provided in Figure 1.

We see the crucial role of the shape of $b(a)$: the intuitively plausible outcome of Proposition 3, i.e. average productivity of the capital stock increases with the emission tax rate, does no longer

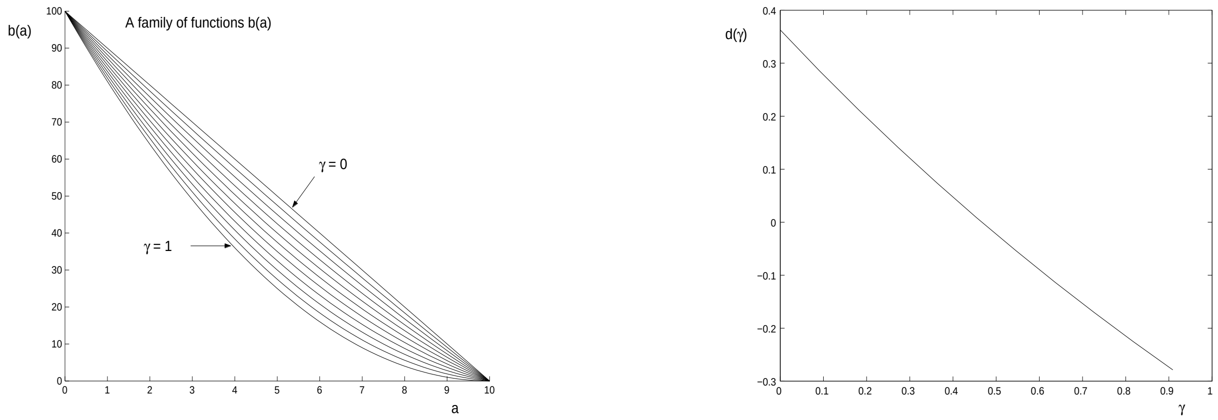

Figure 1: Representatives of the family of functions $b_\gamma(a)$ for different values of γ (left); the difference, $d(\gamma)$, in the average productivity for $\tau = 10$ and $\tau = 0$, as a function of γ (right).

hold in case the acquisition function $b\,(a)$ is sufficiently convex. We see that for $\gamma \geq 0.5$ average productivity starts to decrease ($d(\gamma) < 0$) when the emission tax rate increases.

3.1 Profit Effects of Emission Taxes

To analyze the effect of an emission tax on profits, Xepapadeas and De Zeeuw [5] depart from a framework of an international duopoly. The firm in the home country is subject to an emission tax, while the firm in the other country operates without being bothered by environmental regulation. Following Xepapadeas and De Zeeuw [5] we consider a linear demand schedule and we let capital stock be a function of age and the tax rate while leaving out time:

$$p = \bar{p} - \int_0^h v\,(a)\,K^\tau(a)\,\mathrm{d}a - \int_0^h v\,(a)\,K^0(a)\,\mathrm{d}a,$$

where we attach the superscript τ to the optimal capital stock to indicate its dependence on τ.

The aim is to determine the effect of an emission tax on profits and emissions. To do so, first a benchmark case is defined:

$$v(a) = y, \;\; c(a) = 0, \;\; s(a) = s, \;\; b(a) = b[h - a].$$

The benchmark case is compared to a case where productivity varies with age. Xepapadeas and

De Zeeuw propose the following specification:

$$v(a) = \kappa y + \frac{8}{3h}[1-\kappa]\, y\,[h-a], \tag{13}$$

$$c(a) = 0, \quad s(a) = s, \tag{14}$$

$$b(a) = \left[b + \frac{1}{15}(1-\kappa)^2 p_0 y\right](h-a). \tag{15}$$

It is easy to determine the equilibrium price, which has the form $p = p_1\tau + p_0$. In both the benchmark case and the varying productivity case it holds that

$$p_0 = \frac{\bar{p} + \frac{2}{3} y b h^3}{1 + \frac{2}{3} h^3 y^2}.$$

while

$$p_1 = p_1^b = \frac{\frac{1}{3} y s h^3}{1 + \frac{2}{3} y^2 h^3}, \qquad p_1 = p_1^v = \frac{\frac{1}{3} y h^3 s}{1 + \frac{2}{3}\left[1 + \frac{1}{15}[\kappa-1]^2\right] y^2 h^3}$$

in the benchmark case and in the varying productivity case, respectively. In what follows we assume that, in the absence of emission taxes, the equilibrium price $p = p_0$, is sufficiently large so that

$$p_0 y - c - b > 0. \tag{16}$$

Xepapadeas and De Zeeuw assume that $\kappa \in [0,1)$ in (13),(15), implying that productivity is decreasing with age. Their result is that when the downsizing of the home industry due to stricter environmental policy is accompanied by modernization, the losses in profits are smaller and there are greater gains in emission reductions.

We now show that this result need not hold in case of learning. The latter implies that productivity increases with age, and to model this we just need to assume that $\kappa > 1$. Using this framework the following proposition can be stated for $\kappa \in (1, \kappa^*)$, where $\kappa^* > 1$ is specified in Section 4.3.

Proposition 5 *Let* $\mathrm{d}S^b(\tau)/\mathrm{d}\tau$, $\mathrm{d}S^v(\tau)/\mathrm{d}\tau$, *and* $\mathrm{d}\Pi^b(\tau)/\mathrm{d}\tau$, $\mathrm{d}\Pi^v(\tau)/\mathrm{d}\tau$ *denote the marginal changes in emissions and profits by a stricter environmental policy in the home country, in the benchmark and productivity learning cases, respectively. Then under the assumptions made previously a positive number* τ^* *can be identified such that for* $\tau \in (0, \tau^*]$

$$\frac{\mathrm{d}S^b(\tau)}{\mathrm{d}\tau} < \frac{\mathrm{d}S^v(\tau)}{\mathrm{d}\tau} < 0,$$

and

$$0 < \frac{\mathrm{d}\Pi^b}{\mathrm{d}S^b}|_{\tau=0} < \frac{\mathrm{d}\Pi^v}{\mathrm{d}S^v}|_{\tau=0}.$$

The results in Proposition 5 are significantly different from Xepapadeas and De Zeeuw [5]. Their Proposition 3 claims that

$$\left|\frac{\mathrm{d}S^v(\tau)}{\mathrm{d}\tau}\right| > \left|\frac{\mathrm{d}S^b(\tau)}{\mathrm{d}\tau}\right|, \tag{17}$$

which in fact means that if productivity decreases due to aging, the reduction in emissions due to stricter environmental policy is larger. If learning is present the reverse is true. This implies that if productivity increases due to learning the reduction in emissions due to stricter environmental policy will be smaller. Thus learning mitigates the effect of stricter environmental policy with respect to emissions reduction.

Now, we examine the effect of an emission tax on profit. For $\kappa < 1$ Xepapadeas and De Zeeuw obtained the result that

$$\left|\frac{\mathrm{d}\Pi^v(\tau)}{\mathrm{d}\tau}\right| < \left|\frac{\mathrm{d}\Pi^b(\tau)}{\mathrm{d}\tau}\right|.$$

This implies that in the case of decreasing productivity the effect of a stricter environmental policy on profits is lower. In the case of learning we establish that an emission tax that decreases emissions by one unit, leads to a larger decrease of profit, compared to the benchmark case where learning is not present.

The conclusion is that in the presence of learning a stricter environmental policy can have a more negative effect on the economy: to obtain a given emissions reduction the emission tax increase should be higher, and the resulting negative effect on industry profits is larger.

4 Appendix

4.1 Auxiliary Analysis

We start with some preliminary analysis that will be used in the proofs of the propositions.

Standing assumptions:

(i) f, v, c, and s are not identically zero (except possibly c) monotone[2], nonnegative, and continuously differentiable, f is non-decreasing; (ii) b is twice continuously differentiable, nonnegative and monotone decreasing, $b(h) = 0$; (iii) $K^\tau(t,a) \geq 0$ for all $t \geq 0$, $a \in [0,h]$, and $\tau \in [0, \tau_{\max}]$ (with $\tau_{\max} > 0$), where $K^\tau = K$ is the optimal solution of the problem (1)–(3).

Below $t > h$ will be fixed, therefore $K^\tau(t,a) = K^\tau(a)$ is given by (10).

For a given nonnegative strictly monotone function φ we shall investigate how the φ-average

$$\mathcal{P}_\varphi(\tau) = \frac{\int_0^h \varphi(a) K^\tau(a)\,\mathrm{d}a}{\int_0^h K^\tau(a)\,\mathrm{d}a}$$

is related to the one corresponding to $\tau = 0$. Thus $\mathcal{P}_\varphi(\tau)$ represents the average age if $\varphi(a) = a$, and the average productivity if $\varphi(a) = v(a)$.

Below we shall use the symbol $\prec$ for one of the relations $\leq$ or $\geq$. Moreover, we introduce the notation

$$Q(\varphi, v) = \int_0^h \int_0^a \int_\sigma^h \varphi(a)\hat{f}(a)v(\alpha)\,\mathrm{d}\alpha\,\mathrm{d}\sigma\,\mathrm{d}a, \quad \text{where} \quad \hat{f}(a) = pf(t-a),$$

$$R(\varphi, r) = \int_0^h \int_0^a \int_\sigma^h \varphi(a)r(\alpha)\,\mathrm{d}\alpha\,\mathrm{d}\sigma\,\mathrm{d}a, \qquad B(\varphi) = \int_0^h \int_0^a \varphi(a)b(\sigma)\,\mathrm{d}\sigma\,\mathrm{d}a.$$

We fix an arbitrary $\tau \in (0, \tau_{\max})$. The inequality $\mathcal{P}_\varphi(\tau) \prec \mathcal{P}_\varphi(0)$ can be written as

$$\frac{Q(\varphi,v) - R(\varphi,c) - \tau R(\varphi,s) - B(\varphi)}{Q(1,v) - R(1,c) - \tau R(1,s) - B(1)} \prec \frac{Q(\varphi,v) - R(\varphi,c) - B(\varphi)}{Q(1,v) - R(1,c) - B(1)}$$

Below we shall skip the terms containing c (assuming $c = 0$). These terms are of the same nature as the terms containing v, but with the opposite sign. Therefore for c one should require the properties that $-v$ has. Having this in mind, it is easy to verify that the last inequality is equivalent to the following one:

$$\frac{Q(\varphi,v) - B(\varphi)}{Q(1,v) - B(1)} \prec \frac{R(\varphi,s)}{R(1,s)}. \tag{18}$$

Substitute in (18)

$$v(\alpha) = v(0) + \int_0^\alpha v'(\beta)\,\mathrm{d}\beta, \quad s(\alpha) = s(0) + \int_0^\alpha s'(\beta)\,\mathrm{d}\beta.$$

[2]The terms “monotone”, “increasing”, “decreasing” and “convex” are used in a non-strict sense.

Then it becomes

$$\frac{v(0)Q(\varphi,1)+V(\varphi)-B(\varphi)}{v(0)Q(1,1)+V(1)-B(1)} \prec \frac{s(0)R(\varphi,1)+S(\varphi)}{s(0)R(1,1)+S(1)}, \tag{19}$$

where

$$V(\varphi)=\int_0^h\int_0^a\int_\sigma^h\int_0^\alpha \varphi(a)\hat{f}(a)v'(\beta)\,\mathrm{d}\beta\,\mathrm{d}\alpha\,\mathrm{d}\sigma\,\mathrm{d}a,$$

$$S(\varphi)=\int_0^h\int_0^a\int_\sigma^h\int_0^\alpha \varphi(a)s'(\beta)\,\mathrm{d}\beta\,\mathrm{d}\alpha\,\mathrm{d}\sigma\,\mathrm{d}a.$$

Inequality (19) is implied by the following three relations (skipping further the second argument of Q and R, which is always equal to one):

$$\frac{v(0)Q(\varphi)+V(\varphi)-B(\varphi)}{v(0)Q(1)+V(1)-B(1)} \prec \frac{Q(\varphi)}{Q(1)} \prec \frac{R(\varphi,1)}{R(1,1)} \prec \frac{s(0)R(\varphi,1)+S(\varphi)}{s(0)R(1,1)+S(1)}.$$

This implies the following sufficient condition for the relation $\mathcal{P}_\varphi(s(\cdot)) \prec \mathcal{P}_\varphi(0)$:

$$V(\varphi)Q(1) \prec V(1)Q(\varphi) \qquad \& \qquad B(1)Q(\varphi) \prec B(\varphi)Q(1) \tag{20}$$

$$\& \quad Q(\varphi)R(1) \prec Q(1)R(\varphi) \qquad \& \qquad S(1)R(\varphi) \prec S(\varphi)R(1). \tag{21}$$

The first and the last relations are of the same type, but with reversed sign $\prec$. We separately investigate the first three relations.

1. Changing the order of integration we represent

$$V(\varphi)=\int_0^h G(\varphi;\beta)v'(\beta)\,\mathrm{d}\beta, \quad \text{where} \quad G(\varphi;\beta)=\int_\beta^h\int_0^\alpha\int_\sigma^h \varphi(a)\hat{f}(a)\,\mathrm{d}a\,\mathrm{d}\sigma\,\mathrm{d}\alpha.$$

Also by changing the order of integration we obtain that

$$Q(\varphi)=\int_0^h\int_0^\alpha\int_\sigma^h \varphi(a)\hat{f}(a)\,\mathrm{d}a\,\mathrm{d}\sigma\,\mathrm{d}\alpha = G(\varphi;0).$$

Then

$$V(\varphi)Q(1)-V(1)Q(\varphi)=\int_0^h \Delta(\beta)v'(\beta)\,\mathrm{d}\beta, \tag{22}$$

where

$$\Delta(\beta)=G(\varphi;\beta)G(1;0)-G(1;\beta)G(\varphi;0).$$

We now prove that $\Delta(\beta)$ does not change its sign in $[0,h]$. Assume the opposite. Clearly $\Delta(0)=0$ (since $G(\varphi;0)G(1;0)-G(1;0)G(\varphi;0)=0$), and $\Delta(h)=0$ (since $G(1;h)=G(\varphi;h)=0$). Then $\Delta'(\cdot)$ must have at least two zeros in the open interval $(0,h)$. We calculate

$$\Delta'(\beta)=G'(\varphi;\beta)G(1;0)-G'(1;\beta)G(\varphi;0), \quad G'(\varphi;\beta)=-\int_0^\beta \int_\sigma^h \varphi(a)\hat{f}(a)a\sigma.$$

Since obviously $\Delta'(0)=0$, we conclude that $\Delta'(\cdot)$ has at least three zeros in $[0,h]$. Then Δ'' must have at least two zeros in the open interval $(0,h)$. We have that

$$\Delta''(\beta)=G''(\varphi;\beta)G(1;0)-G''(1;\beta)G(\varphi;0), \quad G''(\varphi;\beta)=-\int_\beta^h \varphi(a)\hat{f}(a)a.$$

Since obviously $\Delta''(h)=0$, we conclude as before that $G'''(\cdot)$ must have at least two zeros in $(0,h)$. We have that

$$\Delta'''(\beta)=\varphi(\beta)\hat{f}(\beta)G(1;0)-\hat{f}(\beta)G(\varphi;0).$$

Since $\hat{f}$ is strictly positive, the equation $\varphi(\beta)G(1;0)=G(\varphi;0)$ must have at least two solutions in $(0,h)$. Since φ is a strictly monotone function, we arrive at a contradiction. This contradiction is caused by the assumption that Δ changes its sign in $[0,h]$. Hence, Δ has a constant sign. To find it we note that $\Delta(0)=\Delta'(0)=0$, so that

$$\operatorname{sgn}\Delta(\beta)=\operatorname{sgn}\Delta''(0) \quad \forall\beta\in[0,h],$$

provided that $\Delta''(0)\neq 0$. To evaluate $\Delta''(0)$ we represent by changing the order of integration

$$G(\varphi;0)=\int_0^h \varphi(a)\hat{f}(a)a(h-a/2)a.$$

Then from the expression for $\Delta''(\beta)$ we obtain that

$$\Delta''(0)=\int_0^h \varphi(a)\hat{f}(a)\left[a(h-a/2)\int_0^h \hat{f}(\sigma)\sigma-\int_0^h \hat{f}(\sigma)\sigma(h-\sigma/2)\sigma\right]a.$$

Notice that the above expression depends linearly on φ, and equals zero if φ is constant. Then the value does not change if we replace $\varphi(a)$ with $\int_0^a \varphi'(\beta)\beta$. Changing once again the order of integration we obtain that

$$\Delta''(0)=\int_0^h \varphi'(\beta)\int_\beta^h\int_0^h \hat{f}(a)\hat{f}(\sigma)[a(h-a/2)-\sigma(h-\sigma/2))]\sigma a\beta=\int_0^h \varphi'(\beta)\Gamma(\beta)\beta.$$

Obviously $\Gamma(0) = \Gamma(h) = 0$. If $\Gamma(\beta)$ changes its sign in $[0, h]$, then $\Gamma'(\beta)$ must have at least two zeros in $(0, h)$. We have that

$$\Gamma'(\beta) = -\hat{f}(\beta) \int_0^h \hat{f}(\sigma)[\beta(h - \beta/2) - \sigma(h - \sigma/2))]\sigma.$$

Since p is strictly positive, the integral must vanish at least two times in $(0, h)$. This, however, cannot happen. Indeed, the integral is a quadratic function of β which takes the same negative value

$$\int_0^h \hat{f}(\sigma)\sigma(h - \sigma/2)\sigma$$

for $\beta = 0$ and $\beta = 2h$. Hence $\Gamma'(\cdot)$ has at most one zero in $[0, h]$. Since $\Gamma(0) = 0$, we have $\operatorname{sgn}\Gamma(\beta) = \operatorname{sgn}\Gamma'(0)$, if the last value is nonzero. We already showed that $\Gamma'(0)$ is strictly positive. Thus we obtain: if $\varphi'(\beta) \prec 0$ on $[0, t]$, then $\Delta(\beta) \prec 0$. Now we conclude from (22) that:

$$\text{if } \operatorname{sgn}(v')\operatorname{sgn}(\varphi') \prec 0 \quad \text{then} \quad V(\varphi)Q(1) \prec S(1)Q(\varphi). \tag{23}$$

Here and below $\operatorname{sgn}(x) = 1$ if x is nonnegative and nonidentically equal to zero function on $[0, h]$, $\operatorname{sgn}(x) = -1$ in the symmetric case, and $\operatorname{sgn}(x) = 0$ if x is identically zero. If both v and φ are non-constant, then the inequality in the right-hand side of (23) holds in strict sense.

By the same argument we obtain also:

$$\text{if } 0 \prec \operatorname{sgn}(s')\operatorname{sgn}(\varphi') \quad \text{then} \quad S(1)R(\varphi) \prec S(\varphi)R(1). \tag{24}$$

2. The third relation in (20)–(21) can be rewritten as

$$\int_0^h \varphi(a)\hat{f}(a)a(h - a/2)\,\mathrm{d}a \int_0^h a(h - a/2)\,\mathrm{d}a - \int_0^h \varphi(a)a(h - a/2)\,\mathrm{d}a \int_0^h \hat{f}(a)a(h - a/2)\,\mathrm{d}a \prec 0.$$

Since the above expression is linear in φ and equals zero for a constant function φ, one can replace $\varphi(a)$ with $\int_0^a \varphi'(\beta)\,\mathrm{d}\beta$. After a change of the order of integration the above inequality can be equivalently written as

$$\int_0^h \varphi'(\beta)\Delta(\beta)\,\mathrm{d}\beta \prec 0,$$

where

$$\Delta(\beta) = \int_0^h a\left(h - \frac{a}{2}\right)\mathrm{d}a \int_\beta^h a\left(h - \frac{a}{2}\right)\hat{f}(a)\,\mathrm{d}a - \int_0^h \alpha\left(h - \frac{\alpha}{2}\right)\hat{f}(a)\,\mathrm{d}a \int_\beta^h a\left(h - \frac{a}{2}\right)\mathrm{d}a$$

Thanks to the non-increase of $\hat{f}$ one can prove (again, we skip this technical proof) that $\Delta(\beta) \leq 0$ on $[0, h]$ Hence we can conclude that

$$\text{if } \ 0 \prec \operatorname{sgn}(\varphi') \quad \text{then} \quad Q(\varphi)R(1) \prec Q(1)R(\varphi). \tag{25}$$

If f is strictly increasing (thus $\hat{f}$ is strictly decreasing) and φ is not constant, then the last inequality holds in strict sense.

3. Finally, consider the second relation in (20)–(21), which can be written as

$$B(1)\int_0^h \varphi(a)\hat{f}(a)a(h-a/2)\,\mathrm{d}a - Q(1)\int_0^h\int_0^a \varphi(a)b(\sigma)\,\mathrm{d}\sigma\,\mathrm{d}a \prec 0. \tag{26}$$

Since the above expression is linear in φ and equals zero for a constant φ, we can replace $\varphi(a)$ by $\int_0^a \varphi'(\beta)\,\mathrm{d}\beta$. Changing the order of integration we obtain the equivalent condition

$$\int_0^h \varphi'(\beta)\int_\beta^h \left[B(1)\hat{f}(a)a(h-a/2) - Q(1)\int_0^a b(\sigma)\,\mathrm{d}\sigma\right]\,\mathrm{d}a\,\mathrm{d}\beta \prec 0, \tag{27}$$

where

$$B(1) = \int_0^h (h-\sigma)b(\sigma)\,\mathrm{d}\sigma, \quad Q(1) = \int_0^h \hat{f}(\sigma)\sigma(h-\sigma/2)\,\mathrm{d}\sigma.$$

Thus we obtained that:

$$\text{if (27) holds then } \ B(1)Q(\varphi) \prec B(\varphi)Q(1). \tag{28}$$

Combining the above considerations we obtain the following auxiliary conclusion.

Lemma 1 *If φ, v, s, f and b satisfy all conditions listed in (23), (24), (25), (28), and if c satisfies (24) (with s replaced with c), then $\mathcal{P}_\varphi(\tau) \prec \mathcal{P}_\varphi(0)$.*

4.2 Proofs of Propositions 3 and 4

First we shall consider the claims of Propositions 3 and 4 in the case without technological progress: $f(t-a) = 1$.

We apply Lemma 1 with $\varphi(a) = a$ for the average age, and with $\varphi(a) = v(a)$ for the average productivity. One only needs to elaborate the inequality (27) in these particular cases. Since the multiplier $\varphi'(\beta)$ has a constant sign, we consider the function

$$\Delta(\beta) = \int_\beta^h \left[B(1)a(h-a/2) - Q(1)\int_0^a b(\sigma)\,\mathrm{d}\sigma\right]\,\mathrm{d}a,$$

where $Q(1) = h^3/3$. Obviously $\Delta(h) = 0$, and taking into account the definition of $B(1)$ we get that also $b(0) = 0$. If $\Delta(\beta)$ changes its sign in $[0,h]$ then $\Delta'(\beta)$ must have at least two zeros in $(0,h)$. But since

$$\Delta'(\beta) = -B(1)\beta(h-\beta/2) + R(1,1)\int_0^\beta b(\sigma)\,\mathrm{d}\sigma,$$

which has one more zero at $\beta = 0$, we conclude that $\Delta''(\beta)$ must have at least two zeros in $(0,h)$. But

$$\Delta''(\beta) = -B(1)(h-\beta) + R(1,1)b(\beta),$$

which has an additional zero at $\beta = h$. Hence, also $\Delta'''(\beta)$ must have two zeros in $(0,t)$. This means that the equation

$$B(1) + R(1,1)b'(\beta) = 0$$

has at least two zeros. Since the function $b'(\cdot)$ is assumed monotone, this is possible only if it is a constant, which means that $b(a) = b'(0)(b-h)$. One can directly check that in this case $\Delta(\beta)$ is identically zero, therefore it does not change its sign as well. Let us represent $b(\sigma) = \frac{b(0)}{h}(h-\sigma) + \delta(\sigma)$, where obviously $\delta(0) = \delta(h) = 0$. Then one may calculate

$$\Delta''(0) = -h\int_0^h (h-\sigma)\delta(\sigma)\,\mathrm{d}\sigma.$$

We see that if $\delta(\sigma)$ has a constant sign, then $\Delta(\beta)$ has the opposite sign. It remains to notice that $\delta(\sigma)$ is positive for a concave function $b(\cdot)$ and negative for a convex one. We conclude that

$$\text{if} \quad \mathrm{sgn}(\varphi')\mathrm{sgn}(b'') \prec 0 \quad \text{then} \quad B(1)Q(\varphi) \prec B(\varphi)Q(1).$$

Then Lemma 1 implies the claim (i) and (ii) of propositions 3 and 4.

Now we consider the case of technological progress. We apply Lemma 1 with

$$\hat{f}(a) = f_0(t-a) = f^0 + \kappa.(t-a), \quad b(a) = b.(h-a),$$

As before, we only need to elaborate condition (27), which can be written as

$$\int_0^h \varphi'(\beta)\Delta(\beta)\,\mathrm{d}\beta \le 0, \tag{29}$$

where

$$\Delta(\beta) = \int_\beta^h [B(1)\hat{f}(a)a(h-a/2) - Q(1)\int_0^a b(\sigma)\,\mathrm{d}\sigma]\,\mathrm{d}a.$$

One can calculate that $\Delta(\beta) = -b\kappa\frac{h^3}{48}\beta^2(h-\beta)(5h-2\beta)$, which is negative in $(0,h)$. Then the claims (iii) of propositions 3 and 4 follow from Lemma 1.

4.3 Proof of Proposition 5

The validity of the considerations below is established for values of the parameter κ in some interval $\kappa_* < 1 < \kappa^*$ and values $\tau \in (0, \tau^*]$, $\tau^* > 0$. For such values of κ and τ the capital stock at the optimal solution is supposed positive, so that the formulas from Section 2 are valid. The value $\kappa^* > 1$ will be a subject to some more constraints specified below.

First consider the emissions result. It will be convenient to introduce the following notation:

$$\begin{aligned} d_0(\kappa) &= 1 + \frac{1}{3}[1 + \frac{1}{15}(1-\kappa^2)]y^2h^3, \\ d_1(\kappa) &= 1 + \frac{1}{3}[1 + \frac{2}{15}(1-\kappa^2)]y^2h^3, \\ d_2(\kappa) &= 1 + \frac{2}{3}[1 + \frac{1}{15}(1-\kappa)^2]y^2h^3, \end{aligned}$$

and to suppose that κ^* is chosen in such a way that $d_i(\kappa) > 0$ for $\kappa \in [1, \kappa^*]$, which is always possible. In Xepapadeas and De Zeeuw [5] it is obtained that

$$\frac{\mathrm{d}S^b(\tau)}{\mathrm{d}\tau} = -\frac{s^2h^3}{3}\frac{d_1(1)}{d_2(1)}, \qquad \frac{\mathrm{d}S^v(\tau)}{\mathrm{d}\tau} = -\frac{s^2h^3}{3}\frac{d_1(\kappa)}{d_2(\kappa)}.$$

We suppose that $\kappa^* > 1$ is chosen in such a way that for $0 \le \kappa < \kappa^*$ the numerator in the last ratio is positive. Then the first claim of Proposition 5 is equivalent to the following inequalities obtained successively:

$$\frac{s^2h^3}{3}\frac{d_1(\kappa)}{d_2(\kappa)} < \frac{s^2h^3}{3}\frac{d_1(1)}{d_2(1)} \iff 5(1-\kappa^2) < 4(1-\kappa)^2 \iff -9\kappa^2 + 8\kappa + 1 < 0, \tag{30}$$

and the last inequality is true for $\kappa > 1$. This proves the first claim of the proposition.

Next we investigate the effect of emission tax on profits. For the benchmark case Xepapadeas and De Zeeuw [5] obtained that

$$\frac{\mathrm{d}\Pi^b(\tau)}{\mathrm{d}\tau} = \frac{1}{3}sh^3\left[\left(\frac{d_1(1)}{d_2(1)}\right)^2 s\tau - \frac{d_1(1)}{d_2(1)}(p_0y-c-d)\right],$$

while for the case of varying productivity they derived that

$$\frac{\mathrm{d}\Pi^v(\tau)}{\mathrm{d}\tau} = \hat{\pi}\tau + \tilde{\pi},$$

where (after some rearrangement)

$$\hat{\pi} = \frac{1}{45}\left[1-\kappa\right]^2 (p_1^v)^2 y^2h^3 + \frac{s^2h^3}{3}\left[\frac{d_1(\kappa)}{d_2(\kappa)}\right]^2,$$

$$\tilde{\pi} = -\frac{sh^3}{3d_2(\kappa)}\left[d_1(\kappa)(p_0y-c-d) - \frac{1}{45}(1-\kappa^2)y^2h^3p_0y - \frac{1}{15}d_1(\kappa)(1-\kappa)^2p_0y\right].$$

Then (16) implies that $\tilde{\pi} < 0$ if $\kappa^* > 1$ is chosen sufficiently close to one.

From the above formulas we get that

$$0 < \frac{\mathrm{d}\Pi^v}{\mathrm{d}S^v}_{|\tau=0} = \frac{\frac{\mathrm{d}\Pi^v}{\mathrm{d}\tau}(0)}{\frac{\mathrm{d}S^v}{\mathrm{d}\tau}(0)} = \frac{\frac{sh^3}{3d_2(\kappa)}\left[d_1(\kappa)(p_0y-c-d) - \frac{1}{45}(1-\kappa^2)y^2h^3p_0y - \frac{1}{15}d_1(\kappa)(1-\kappa)^2p_0y\right]}{\frac{s^2h^3}{3}\frac{d_1(\kappa)}{d_2(\kappa)}}$$

$$= \frac{p_0y-c-b}{s} - \frac{3d_2(\kappa)}{s^2h^3d_1(\kappa)}\frac{1}{15}(1-\kappa)p_0y\left[\frac{1}{3}(1+\kappa)y^2h^3 + d_1(\kappa)(1-\kappa)\right]$$

$$= \frac{\mathrm{d}\Pi^b}{\mathrm{d}S^b}_{|\tau=0} + (\kappa-1)r(\kappa).$$

Obviously the remainder $r(\kappa)$ is positive if $\kappa^* > 1$ is chosen sufficiently close to one. Hence, for $\kappa \in (1, \kappa^*]$ we obtain the last claim of the proposition.

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