A Theory of Random-Lifetime-Rational Junk-Food Consumption

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Abstract

Random-lifetime-rational (RLR) junk-food consumption balances the marginal satisfaction with the marginal deterioration of health. An RLR person discounts the instantaneous marginal satisfaction from junk-food consumption by its implications for his survival probability. His change rate of health evaluation is increased (decreased) by junk-food consumption when health is better (worse) than a critical level. The moderating direct effects of age and relative price on junk-food consumption may be amplified, or dimmed, by the change in his health. The stationary health of a person ignoring his age declines with his time-preference rate and rises with the marginal effect of junk food on his relative health-improvement rate.

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1. Introduction

Food can be classified as junk or healthy in accordance with the concentration of ingredients such as sugar, fat and salt. Due to a high concentration of these ingredients, junk food is often tastier than its low in calories, fat and salt substitute. Due to cheaper ingredients and/or preparation process, junk food is often less expensive than health food. These possible short-term taste and cost advantages of junk food might be offset by the long-term adverse effects of junk-food consumption on health and life expectancy. This paper presents a theory of RLR consumption of junk food whereby people are aware of the possible short-term advantages and the long-term disadvantages associated with junk-food consumption. In addition to the taste and price differentials, they take into account the risk differential in deciding upon the composition of junk-food and health-food products in their diet.

Taste, price and risk differences are not exclusive to junk-food products and their healthier substitutes. They may also provide an explanation to decisions on the consumption of commodities such as coffee, tea, beer and self-rolled cigarettes. The comparison of the taste, price and health impeding effects of coffee, tea, beer and self-rolled cigarettes to those of their healthier substitutes (decaffeinated coffee, herbal tea, light beer and filtered cigarettes, respectively) within a lifetime utility maximization framework with uncertain life expectancy constitutes a complementary approach to the rational addiction model proposed by Gary Becker and Kevin Murphy (1988) and applied by Frank Chaloupka (1991), Gary Becker, Michael Grossman and Kevin Murphy (1994), Nilss Olekalns and Peter Bardsley (1996), Michael Grossman, Frank

Chaloupka and Ismail Sirtalan (1998) and many others to the consumption of cigarettes, alcohol and coffee.

RLR food consumers are defined as maximizers of their expected lifetime utility from consumption of junk food and health food subject to the evolution of their health and the effect of age and health on their random life expectancy. Their path of junk-food consumption balances the marginal satisfaction with the marginal deterioration of health. RLR consumers of junk food discount the instantaneous marginal satisfaction from junk-food consumption by its implications for their survival probability. Junk-food consumption increases (reduces) the change rate of their evaluation of health when their health is better (worse) than a critical level. The moderating direct effects of age and relative price on RLR junk-food consumption may be amplified, or dimmed, by the change in health. The stationary health of junk-food consumers ignoring their age, but otherwise rational, declines with their time-preference rate and rises with the marginal effect of junk food on their relative health-improvement rate. However, off steady state their joint trajectory of junk-food consumption and health neither converges to, nor orbits, steady state.

The conceptual framework leading to the aforementioned results is structured as follows. The building blocks of the analysis generating an RLR choice of a diet of junk food and health food are presented in section 2. Similar to Levy (2000, 2002a and 2002b), life expectancy is taken to be random and the probability of dying is related to health and age. The expected lifetime-utility maximization problem is presented in section 3 and the properties of the RLR diet of junk-food and value of health are discussed in section 4. The long-run (stationary) consumption of junk food and health are presented in section 5 for the case where people ignore their age. A brief summary of the conclusions is given in section 6.

2. Building Blocks

The analysis of the RLR junk-food consumption employs the following notations:

t = a continuous time index, $t \in (0, T)$ where T is a positive scalar indicating the upper bound on human longevity;

$c_j(t)$ = the individual's consumption of junk food at instance t ;

$c_h(t)$ = the individual's consumption of health food at instance t ;

$x(t)$ = the individual's age-adjusted health condition at instance t, a unit interval index $0 \le x(t) \le 1$ with $x = 0$ representing a terminally ill person and $x = 1$ a perfectly healthy person;

$p(t)$ = the junk food-health food price ratio;

α = the junk food-health food taste ratio;

$y(t)$ = the individual's income at instance t ;

$\hat{y}$ = a positive scalar indicating the full capacity income;

$\phi(t)$ = the probability density of dying at instance t ;

$u(c_j(t), c_h(t))$ = the individual's satisfaction from food at instance t; and

$\rho(t)$ = the individual's rate of time preference at instance t .

The subscripts *j* and *h* can be interpreted as (the only) two types of meals: the *j*-th meal consists of junk food and the *h*-th one of health food. In which case, $c_j(t)$ and $c_h(t)$ indicate the numbers of these meals consumed at *t*.

The individual's health condition, x, is adjusted to the adverse effects of normal aging. That is, x indicates the individual's health relative to his age. This definition of x is used for distinguishing between the effect of age (i.e., youth vis-àvis old age) and the effect of health on the individual's probability of survival. (See assumption 7.) This definition also explains why age (and thereby aging) is not included in the motion equation of the individual's health. (See assumption 6.)

The building blocks of the RLR junk-food consumption model are summarized by the following assumptions.

Assumption 1 (instantaneous satisfaction): The individual's instantaneous satisfaction from eating is represented by a utility function $u(c_j(t), c_h(t))$ having the following properties. Food is essential -- $u(0,0) = 0$. However, neither junk food nor health food is essential -- $u(0, c_h) > 0 < u(c_j, 0)$. The marginal satisfaction with respect to each type of food is positive and diminishing -- $u_j, u_h > 0$, $u_{jj}, u_{hh} < 0$ -- and health food and junk food are substitutes -- $u_{jh} < 0$.[1]

Consistent with this assumption the following explicit utility function is considered

$$u_t = [\alpha c_j(t) + c_h(t)]^{\beta} \tag{1}$$

[1] It is possible that junk food and/or health food are addictive for some people. John Cawley's (1999) empirical findings on the consumption of calories lend support to the hypothesis that some types of junk food are addictive. However, addiction and, in particular, the controversial concept of *rational* addiction are not the scope of the present analysis. Consistently with Karen Dynan's (2000) empirical findings with panel household data, the present analysis assumes that food consumption is neither addictive nor a formed habit. That is, the *stocks* of junk-food consumption and health-food consumption are not considered as moderating the individual's level of satisfaction from the *flows* of these commodities and hence are not introduced into the individual's utility function. Instead, the analysis focuses on the roles of price, taste and risk differences in explaining the individual's choice of junk-food and health-food consumption flows.

where $\alpha > 0$ is the relative taste coefficient, $0 < \beta < 1$ is the elasticity of the individual's satisfaction from the composite diet, and the ratio of the satisfaction-elasticities with respect to junk food and health food is $\alpha(c_j / c_h)$.

Assumption 2 (instantaneous income): The ratio of the individual's instantaneous income to the full capacity income is equal to the individual's age-adjusted health condition. That is,

$$y(t) = x(t)\hat{y} \qquad (2)$$

revealing that the full capacity income is only attained by a perfectly healthy individual ($x = 1$), and that the income of a terminally ill person ($x = 0$) is nil. In this context, $1 - x$ can be interpreted as the individual's degree of incapacitation. To simplify matters, the full capacity income is assumed to be independent of age.

Assumption 3 (instantaneous budget constraint): For simplicity sake, there is no borrowing or lending and the individual's instantaneous income is fully spent on buying junk food and health food. Taking the price of health food as a numeraire, the budget constraint is given by

$$p(t)c_j(t) + c_h(t) = x(t)\hat{y}. \qquad (3)$$

Assumption 4 (health change): The individual's age-adjusted health is deteriorated by eating junk food and improved by a natural recovery process. Health-food only helps maintaining the individual's health relative to his age at the same level.[2] Correspondingly, the instantaneous change in the individual's age-adjusted health is given by a logistic function displaying a diminishing relative health-improvement rate in junk-food consumption, a diminishing health-improvement rate (r) in the level of health, and a unit upper bound and a zero lower bound on the individual health. That is, it is assumed that

$$\dot{x}(t) = \underbrace{[1-\delta c_j(t)][1-x(t)]}_{r} x(t) \tag{4}$$

where, δ is a positive scalar indicating the marginal adverse effect of junk-food consumption on the relative rate of improvement of the individual's age-adjusted health. Loosely interpreted, δ is the health sensitivity to junk food.

The underlying rationale of the abovementioned effect of junk food on health is as follows. By rearranging Eq. (4), $1-\delta c_j(t) = [\dot{x}(t)/x(t)]/[1-x(t)]$. That is, $1-\delta c_j$ is the individual's health-improvement rate relative to his degree of incapacitation ($1-x$). This relative health-improvement rate is hindered by junk-food consumption and is negative for sufficiently large values of δ and c_j. The case of a negative relative health-improvement rate ($1-\delta c_j < 0$) does not violate the assumption that x lies within the (positive) unit interval as long as the initial value of x is smaller than 1. Furthermore, when x is close to zero and the consumption of junk food is sufficiently low (i.e., $c_j < 1/\delta$), $1-\delta c_j$ can be interpreted as the recovery rate from a near death situation.

[2] Health-food fans may argue that, *ceteris paribus*, health food not only helps maintain personal health but also improves personal health. The incorporation of the latter assertion complicates the analysis and renders the model unsolvable.

Assumption 5 (survival probability): Let T be the limit to human longevity, $F(t)$ the cumulative distribution function associated with the probability density of dying $\phi(t)$ where $0 \le t \le T$. Then $\Phi(t) = 1 - F(t)$ indicates the probability of living beyond t. It is assumed that $\Phi(t) = \Phi(x(t), T - t)$ with $\Phi_{T-t} > 0$ (the youth effect), $\Phi_x > 0$ (the age-adjusted health effect) and $\Phi(0, T - t) = 0 = \Phi(x(t), 0)$. That is, the probability of living beyond *t* declines with the individual's age and rises with the individual's age-adjusted health. It converges to zero as the individual's age approaches T and when his health is completely deteriorated ($x = 0$).

Assumption 6 (time-consistent preferences): The individual's rate of time preference is positive and time invariant. That is, $\rho(t) = \rho$ for every $t \in (0, T)$.

3. RLR Choice

It is postulated that RLR individuals chose their junk and health food diet path so as to maximize their expected lifetime satisfaction from food subject to their health motion equation. Since the duration of life is random, expected-lifetime-satisfaction-maximizing food consumers multiply their accumulated satisfaction from food between the starting point of their planning horizon, 0, to their possible time of death t (i.e., multiply $\int_0^t e^{-\rho\tau} u_\tau d\tau$) by the probability density of dying at time t (i.e., $\phi(t)$). The products of $\phi(t)$ and $\int_0^t e^{-\rho\tau} u_\tau d\tau$ associated with any possible life expectancy $0 \le t \le T$ are considered by such consumers. The sum of all these products is these consumers' expected lifetime-satisfaction from food. It is given by the following double-integral expression

$$V = \int_0^T \phi(t) \int_0^t e^{-\rho\tau} u_\tau d\tau dt \,. \tag{5}$$

Integrating by parts, this expected lifetime-satisfaction is equivalently rendered by a mathematically more manageable single-integral expression:

$$V = \int_0^T \Phi(x(t), T-t) e^{-\rho t} u_t dt \,. \tag{6}$$

That is, the expected lifetime utility is the sum of the discounted instantaneous utility from food consumption accruing during the maximum lifespan and weighted by the probability of prevailing. (A detailed mathematical explanation is given in Appendix A.)

The analysis of the RLR diet trajectory is further simplified by expressing c_h as a function of c_j. Recalling the instantaneous budget constraint,

$$c_h(t) = x(t)\hat{y} - p(t)c_j(t) \,. \tag{7}$$

The substitution of Eq. (7) into Eq. (1) renders the instantaneous satisfaction function as

$$u_t = \left[[\alpha - p(t)]c_j(t) + x(t)\hat{y}\right]^\beta \,. \tag{8}$$

Note that as long as the difference between the relative taste and the relative price of junk food is positive (i.e., $\alpha - p(t) > 0$) the marginal instantaneous satisfaction from junk food, in this concentrated form, is positive and diminishing. In turn, V is concave in the control variable c_j. Of course, an RLR person follows a strictly health-food diet when $\alpha - p(t) < 0$.

By substituting Eq. (8) into Eq. (7) for u_t the RLR junk-food consumption path can now be found by

$$\max_{\{c_j\}} \int_0^T \Phi(x(t), T-t))e^{-\rho t}\left[[\alpha - p(t)]c_j(t) + x(t)\hat{y}\right]^{\beta} dt$$

subject to the health motion equation 4.

4. RLR Junk-Food Consumption and Shadow Value of Health

The present-value Hamiltonian corresponding to the aforementioned constrained maximization problem is

$$H(t) = \Phi(x(t), T-t)e^{-\rho t}\underbrace{[(\alpha - p)c_j(t) + x(t)\hat{y}]}_{Z}^{\beta} + \lambda(t)\underbrace{[1-\delta c_j(t)][1-x(t)]x(t)}_{\dot{x}} \quad (9)$$

where the co-state variable $\lambda(t)$ indicates the RLR shadow present value of the individual's age-adjusted health at t. Since $0 < \beta < 1$, H is concave in the state variable (x). If $\alpha - p(t) > 0$, H is also concave in the control variable (c_j). It is assumed, henceforth, that the relative taste-price differential $(\alpha - p)$ is positive; in which case, there exists an interior solution and, in addition to the state equation (Eq. (4)), the following conditions are necessary and sufficient for maximum expected lifetime satisfaction from junk-food consumption[3]:

$$\dot{\lambda} = -\frac{\partial H}{\partial x} = -[\Phi_x \underbrace{Z^{\beta}}_{u} - \Phi \underbrace{\beta Z^{\beta-1}\hat{y}}_{u_x}]e^{-\rho t} - \lambda(1-2x)(1-\delta c_j) \quad (10.1)$$

and

$$\frac{\partial H}{\partial c_j} = \Phi e^{-\rho t}\underbrace{\beta Z^{\beta-1}[\alpha - p(t)]}_{u_j} - \lambda\delta x(1-x) = 0\,. \quad (10.2)$$

[3] The time-index t is omitted for tractability.

The optimality condition, Eq. (10.2), indicates that along the RLR junk-food consumption path there should be a balance between the marginal satisfaction from junk-food consumption, discounted by both the individual's time preference and prospects of survival, and the value of the marginal damage to the individual health caused by consuming junk-food.

The adjoint equation, Eq. (10.1), implies, in conjunction with the optimality condition, that along the RLR junk-food consumption path the rate of change of the shadow value of health is given by

$$\frac{\dot{\lambda}(t)}{\lambda(t)} = -\left[(\eta/\beta)c_j + [1 + (\eta/\beta)\frac{x\hat{y}}{\alpha - p}\right]\delta(1-x) - (1-2x)(1-\delta c_j) \qquad (11)$$

where η denotes the *survival elasticity* ($\Phi_x \frac{x}{\Phi}$) which, for simplicity, is henceforth assumed to be constant. As $\frac{\partial(\dot{\lambda}/\lambda)}{\partial c_j} \gtreqless 0$ for $x \gtreqless (1+\eta/\beta)/(2+\eta/\beta)$, the value of health for an RLR person is increased (reduced) by junk-food consumption when his health is better (worse) than a critical level, which rises with the ratio of the elasticity of survival to the elasticity of satisfaction from eating (η/β).

The change in the RLR junk-food consumption over time is given by the following no-arbitrage rule[4]:

[4] Eq. (12) is obtained by differentiating Eq. (10.2) with respect to time, substituting the right-hand sides of Eq. (10.1) and Eq. (10.2) for $\dot{\lambda}$ and λ, multiplying both sides of the resultant equation by $e^{\delta t}/\Phi Z^{\beta-2}\beta(1-\beta)(\alpha-p)$ and collecting terms.

$$\dot{c}_j = \frac{\overbrace{[(1-2x)(1-c_j)-\rho]\beta Z(\alpha-p)+\delta(1-x)[\eta Z^2+\beta Zx\hat{y}]}^{A}}{\beta(1-\beta)(\alpha-p)}$$

$$+\left[\frac{\overbrace{[\eta/x+(1-2x)/(1-x)x](\alpha-p)Z-(1-\beta)\hat{y}}^{B}}{(1-\beta)(\alpha-p)}\right]\dot{x}-\left[\frac{Z}{(1-\beta)(\alpha-p)}\right]\dot{p}+\left[\frac{Z}{(1-\beta)}\right]\frac{\dot{\Phi}}{\Phi} \quad (12)$$

This equation and Eq. (4) portray the joint evolution of an RLR person's junk-food consumption and health. They lead to the following conclusions.

Recalling our assumptions, $(1-\beta)(\alpha-p)>0$. Hence, the direction of the effect of an improvement in the RLR person's health on junk food consumption depends on the sign of *B*, which is positive, equal to zero, or negative when the survival elasticity is greater than, equal to, or smaller than a critical size. That is, $\frac{d\dot{c}_j}{d\dot{x}} = \frac{B}{(1-\beta)(\alpha-p)} \gtreqless 0$ as $\eta \gtreqless \frac{(1-\beta)x\hat{y}}{(\alpha-p)Z} - \frac{1-2x}{1-x}$.

The direct effects of changes in the prospects of survival and the relative price of junk food on the RLR junk-food consumption are given by differentiating Eq. (12) with respect to $\dot{\Phi}/\Phi$ and $\dot{p}$, respectively. Recalling Eq. (4), these direct effects on junk food consumption affect the individual's age-adjusted health at a rate of $-\delta$, which, by virtue of Eq. (12), also affects junk food consumption. The full effects of changes in the prospects of survival and the relative price of junk food on the RLR junk-food consumption are equal to the sum of these direct and indirect effects.

As can be seen from Eq. (12) and assumptions 1 and 3, the adverse effect of age on survival ($\dot{\Phi}<0$) has a direct moderating effect ($Z/(1-\beta)\Phi$) on the RLR junk-food consumption over time. However, this decline in consumption of junk food improves the individual's age-adjusted health by $\delta Z/(1-\beta)\Phi$ and hence indirectly changes the RLR junk-food consumption by $\delta ZB/(1-\beta)^2\Phi(\alpha-p)$. Recalling that $\frac{d\dot{c}_j}{d\dot{x}}=\frac{B}{(1-\beta)(\alpha-p)}$, the indirect effect of aging on the RLR junk-food consumption is positive (negative) if η is greater (smaller) than $\frac{(1-\beta)x\hat{y}}{(\alpha-p)Z}-\frac{1-2x}{1-x}$ and hence dimming (amplifying) the direct moderating effect of age on RLR junk-food consumption.

As can be expected, a rise in the relative price of junk food over time has a moderating direct effect ($-Z/(1-\beta)(\alpha-p)$) on the RLR junk-food consumption. This decline in junk-food consumption leads to an improvement in the individual's age-adjusted health by $\delta Z/(1-\beta)(\alpha-p)$ and hence indirectly changes the RLR junk-food consumption by $\delta ZB/[(1-\beta)(\alpha-p)]^2$. Recalling that $\frac{d\dot{c}_j}{d\dot{x}}=\frac{B}{(1-\beta)(\alpha-p)}$, this indirect effect of a rise in the relative price of junk food on junk-food consumption is positive (negative) if η is greater (smaller) than $\frac{(1-\beta)x\hat{y}}{(\alpha-p)Z}-\frac{1-2x}{1-x}$ and hence dimming (amplifying) the direct moderating effect of a relative price rise on the RLR junk-food consumption.

5. Stationary RLR Junk-Food Consumption and Health Index

The notion of steady state (SS) is used in this section to indicate possible long-run levels. Of course, the derivation of stationary junk-food consumption and stationary health index is inconsistent with the assumption that $\Phi_{T-t}>0$. This assumption is now relaxed. That is, the following analysis is conducted under the assumption that some people ignore aging ($\Phi_{T-t}=0$) and believe that their survival in the future depends only on their health. In other words, these people believe that there is no upper bound on life expectancy ($T \to \infty$). For these *forever-young-feeling*, but in all other aspects rational, people the evolution of junk-food consumption is given by

$$\dot{c}_j = \frac{\overbrace{[(1-2x)(1-c_j)-\rho]\beta Z(\alpha-p)+\delta(1-x)[\eta Z^2+\beta Zx\hat{y}]}^{A}}{\beta(1-\beta)(\alpha-p)}$$

$$+\left[\frac{\overbrace{[\eta/x+(1-2x)/(1-x)x](\alpha-p)Z-(1-\beta)\hat{y}}^{B}}{(1-\beta)(\alpha-p)}\right]\dot{x}-\left[\frac{Z}{(1-\beta)(\alpha-p)}\right]\dot{p}$$

.(13)

The substitution of $\dot{p}=\dot{c}_j=\dot{x}=0$ and the definition of Z into Eq. (13) implies that in steady state

$$[(1-2x_{ss})(1-c_{j_{ss}})-\rho]\beta(\alpha-p)+\delta(1-x_{ss})[\eta(\alpha-p)c_{j_{ss}}+(\beta+\eta)x_{ss}\hat{y}]=0 \quad (14)$$

and, as the substitution of $\dot{x}=0$ into Eq. (4) implies that $c_{j_{ss}}=1/\delta$, it is obtained that

$$x_{ss}^2-\left[1-\frac{(\alpha-p)[2(1-1/\delta)+\eta/\beta]}{\delta(1+\eta/\beta)\hat{y}}\right]x_{ss}+\frac{(\alpha-p)[1/\rho+\rho-1-\eta/\beta]}{\delta(1+\eta/\beta)\hat{y}}=0\,. \quad (15)$$

The solution of this quadratic equation yields double stationary health conditions:

$$x_{ss}^{I}=0.5\left[1-\frac{(\alpha-p)[2(1-1/\delta)+\eta/\beta]}{\delta(1+\eta/\beta)\hat{y}}\right]+0.5\left\{\left[1-\frac{(\alpha-p)[2(1-1/\delta)+\eta/\beta]}{\delta(1+\eta/\beta)\hat{y}}\right]^2-4\frac{(\alpha-p)[1/\rho+\rho-1-\eta/\beta]}{\delta(1+\eta/\beta)\hat{y}}\right\}^{0.5}$$

(16)

and

$$x_{ss}^{II}=0.5\left[1-\frac{(\alpha-p)[2(1-1/\delta)+\eta/\beta]}{\delta(1+\eta/\beta)\hat{y}}\right]-0.5\left\{\left[1-\frac{(\alpha-p)[2(1-1/\delta)+\eta/\beta]}{\delta(1+\eta/\beta)\hat{y}}\right]^2-4\frac{(\alpha-p)[1/\rho+\rho-1-\eta/\beta]}{\delta(1+\eta/\beta)\hat{y}}\right\}^{0.5}$$

.(17)

Numerical simulations are used for assessing the effects of the model's parameters on these RLR stationary levels of health. The simulations reveal that for various choices of parameter-values only $x_{SS}{}^{II}$ is, as required by construction, within the unit interval (0,1). Hence, the reported simulation results are generated by using Eq. (17). The reported simulations refer to a forever-young feeling person for whom:

junk-food is fifty percent tastier than health food, $\alpha = 1.5$;

junk-food is fifty percent cheaper than health food, $p = 0.5$;

the elasticity of satisfaction from eating is $\beta = 0.5$;

the survival elasticity is $\eta = 1$ (i.e., $\Phi = x$);

the marginal (adverse) effect of junk-food consumption on the relative rate of improvement of the individual health is $\delta = 0.0003$;

the daily rate of time preference is $\rho = 0.00026$ (which is equivalent to about 10 percent per annum); and

the daily full-capacity income is $\hat{y} = \$100$.

For this forever-young-feeling person, the stationary health index is 0.578: namely, 57.8 percent of a perfectly healthy individual in his cohort.

The numerical simulations reveal that this stationary health index is not sensitive to changes in the relative taste of junk food, in the relative price of junk food, in the elasticity of satisfaction from eating, in the elasticity of living beyond *t*, and in the full-capacity income.

In contrast, and as can be expected, the numerical simulations indicate that the stationary health index is considerably lowered by the rate of time preference. For instance, a one-percent rise in ρ from the aforementioned benchmark level, all other things remain the same, reduces x_{SS} by 0.998 percent.

It is also found the stationary health index rises considerably with the marginal effect of junk-food consumption on the relative rate of improvement of the individual health. The rise of the stationary health index is due to the moderating effect of an increase in δ on the stationary consumption of junk food ($c_{j_{ss}} = 1/\delta$). For instance, a one-percent rise in δ from the aforementioned benchmark level, all other things remain the same, increases x_{ss} by 1.006 percent.

However, the trajectories of health index and junk-food consumption of the "forever-young feeling" (otherwise rational) people neither converge to, nor orbit, the stationary combination. (See Appendix B.)

6. Conclusion

We analyzed RLR junk-food consumption by incorporating the taste, price and risk differences between junk food and its healthier substitute into an expected-lifetime-utility-maximizing framework. Our analysis proposed that the RLR combination of junk food and health food maintains a balance between the marginal satisfaction from junk-food consumption and the value of the marginal damage to the individual health caused by consuming junk-food, where the marginal satisfaction from junk-food consumption is discounted by both the individual's time preference and prospects of survival.

We argued that junk-food consumption increases (reduces) the rate of change of RLR people's evaluation of their health when their health is better (worse) than a critical level, which rises with the ratio of the elasticity of survival to the elasticity of satisfaction from eating.

We also argued that the adverse effect of age on survival and a rise in the relative price of junk food have direct moderating effects on the RLR junk-food consumption over time. However, the decline in consumption of junk food improves the individual's age-adjusted health and hence indirectly changes the RLR junk-food consumption. The indirect effect of aging on junk-food consumption can be positive, or negative, and hence dimming, or amplifying, the direct moderating effects of age and relative price on junk-food consumption if the elasticity of survival is larger, or smaller, than a critical value.

We derived the steady-state health index for the case where people ignore their age or believe that there is no upper bound on life expectancy. The numerical simulations revealed that the steady-state health index declines considerably with the individual's rate of time preference and rises considerably with the marginal effect of junk-food consumption on the relative rate of improvement of the individual's health. The trajectories of the health index and junk-food consumption neither converge to, nor orbit, the computed steady state.

Finally, we note that it is possible that, for any given combination of food consumption, a healthier person is happier, and that a healthier and happier person may have a higher propensity to save and invest. The model can be extended to incorporate this possible effect of health state on people's instantaneous utility and capital formation. The capital formation equation may further reflect a tradeoff between the positive effect of the consumption of healthy food on people's propensity to invest and the adverse effect of the higher cost of healthy food on instantaneous saving and, in turn, on investment. Correspondingly, the earning equation of the extended model should take into account the effect of capital stock.

Appendix A: An explanation of the transition from Eq. (6) to Eq. (7)

$F(t)$ is the cumulative density function associated with the probability density of dying at t (i.e., the probability of living up to t). Hence,

$$\phi(t) = F'(t) \tag{A1}$$

and Eq. (6) can be rendered as

$$J = \int_0^T F'(t)\left\{\int_0^t e^{-\rho\tau} u_\tau d\tau\right\}dt = \int_0^T vdU \tag{A2}$$

where,

$$v = \int_0^t e^{-\rho\tau} u_\tau d\tau \tag{A3}$$

and

$$U = -(1 - F(t))\,. \tag{A4}$$

The integration by parts rule suggests that

$$J = \int_0^T vdU = Uv - \int_0^T Udv\,.$$

(A5)

Note, however, that

$$Uv = -\left[(1-F(t))\int_0^t e^{-\rho\tau}u_\tau d\tau\right]_0^T = 0 \tag{A6}$$

because when evaluated at the lower limit

$$Uv = -\left[(1-F(0))\int_0^0 e^{-\rho\tau}u_\tau d\tau\right] = 0 \tag{A7}$$

and when evaluated at the upper limit

$$Uv = -\left[(1-F(T))\int_0^T e^{-\rho\tau}u_\tau d\tau\right] = 0 \tag{A8}$$

as

$$F(T) = 1. \tag{A9}$$

Hence,

$$J = -\int_0^T Udv\,. \tag{A10}$$

By virtue of equation (A3)

$$dv = e^{-\rho\tau}d\tau \tag{A11}$$

and the substitution of equations (A4) and (A11) into (A10) implies

$$J = \int_0^T e^{-\rho t}u_t\Omega(t)dt \tag{A12}$$

where

$$\Omega(t) \equiv -U_t = 1 - F(t) \tag{A.13}$$

and indicating the probability of living at least until t .

Appendix B: The nature of the steady-state

In order to find whether the individual's health and consumption of junk food convergence to the aforementioned stationary levels of 0.578 and 3333.333, respectively, the system of equations (13) and (4) is linearized at the vicinity of this stationary point. The eigenvalues of the state-transition matrix are given by

$$\lambda_{1,2} = 0.5\{[M_{c_j}(ss) + N_x(ss)] \pm \sqrt{[M_{c_j}(ss) + N_x(ss)]^2 - 4[M_{c_j}(ss)N_x(ss) - N_{c_j}(ss)M_x(ss)}\} \tag{B.1}$$

with $M_{c_j}(ss) = 2068.761$ and $M_x(ss) = 45{,}288{,}789$ indicating the stationary values of the derivatives the right-hand-side of Eq. (13) with respect to c_j and x, and $N_{c_j}(ss) = -7.32298E - 05$ and $N_x(ss) = 0$ (as it is proportional to $1 - \delta c_j^{ss} = 0$) the stationary values of the derivatives the right-hand-side of Eq. (4) with respect to c_j and x. As λ_1 and λ_2 are both positive (2067.156 and 1.604, respectively) the individual's health and junk-food consumption trajectories neither converge to, nor orbit, the stationary combination.

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