# On the Effect of Drug Enforcement on Property Crime[1]

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**Abstract**

Although a part of the property crime rate can be explained by predatory crime committed by drug users, research often recommends not to enforce against drug crime, because law enforcement resource allocations accompanying strong drug law enforcement lead to more property crime. We develop a dynamic model including a drug court program to analyse the impact of drug enforcement on property crime. In this model it is almost always optimal to enforce, but the amount of drug enforcement expenditures and its time path vary depending on the parameter settings. Key parameters are the elasticity of demand of the drug and the estimation of the social costs of drug use. Whether enforcement is done, depends also on the initial value of the number of users. In this model it is optimal not to enforce only for an initial stock of users greater than a certain threshold and very low social costs of drug use or a low absolute value for the short term elasticity of demand.

# 1 Introduction

The war against drugs is one of the most discussed issues nowadays. Not only how increasing drug markets should be combated but also the effects of drug enforcement on other crimes especially on property crime are disputed. According to Benson et al. (1992) drug enforcement may affect other crimes in at least two ways. Firstly, a part of the property crime rate can be explained by predatory crime perpetrated by drug users. The Arrestee Drug Abuse Monitoring Program of the National Institute of Justice found out that more than 60 percent of adult male arrestees tested positive for drugs in twenty out of twenty-three cities in the U.S.A. in 1996 (Office of National Drug Control Policy, 1998).

Secondly, law enforcement resources are scarce and they must be allocated among competing uses (i.e. control of drug markets, investigation of robberies, burglaries, rapes, assaults, and murders, etc.). Consequently, if shifting law enforcement resources to drug enforcement is accompanied by a reduced law enforcement effort against property crime, the property crimes become a less risky – i.e., more attractive – source of income for drug users and non-drug users alike.

Benson and Rasmussen (1991) state another explanation for the failure of increased drug control efforts to reduce property crime. If drug users finance their addiction through property crime but their demand for drugs is inelastic and drug control efforts focus on the supply side of the market, then prices will rise, total expenditures on drugs will increase, and existing users may commit even more crimes.

Up to now most of the research in this context considered only a static setting. In this paper we develop a dynamic model, including a drug court program. Drug courts channel drug law offenders into court-supervised treatment programs instead of prisons or jails (Office of National Drug Control Policy, 1998). In our model we do not want to ask whether drug enforcement causes property crime, but we want to find an optimal path for drug enforcement, which minimizes the social costs caused by drug crime and property crime committed by drug users.

# 2 The Model

We study a continous time optimization problem where the decision maker minimizes the sum of social costs caused by drug use and drug related property crime, enforcement costs and drug court costs by an appropiate choice of the level of drug enforcement, $v(t)$. In this model we focus on enforcement against the supply side, e.g. the control of drug markets.

The decision maker is considered as a representation of the law enforcement agency or government. The state of the model is represented by the number of users, $A(t)$.

Assume that the propensity for users to commit a property crime depends on the money needed for consumption of the illicit drug. If $p$ denotes the price of the illict drug and $\eta$ the absolute value of the short term elasticity of demand, then consumption per user and spending on the drug per user is given by $p^{-\eta}$ and $p^{1-\eta}$, respectively. Assume furthermore that the propensity to commit a crime, $\varphi$, is modeled by a linear function of $p^{1-\eta}$, i. e.

$$\varphi(p^{1-\eta}) = q + sp^{1-\eta}. \tag{2.1}$$

Denote the probability of getting arrested given the commission of a crime by $\alpha_1$. Further, getting arrested leads to admission to a drug court or other "treatment" programs administered by the criminal justice system, and with some probability, $\alpha_2$, treatment is successful and removes the criminal from the status of being a drug user.

Summing up, the social costs per user are given by

$$\rho\, p^{1-\eta} + \beta\, \varphi(p^{1-\eta}) + \alpha_1\, d\, \varphi(p^{1-\eta}), \tag{2.2}$$

where $\rho$ stands for the social costs of drug consumption per gram, $\beta$ denotes the social costs per property crime and $d$ represents the costs of an arrest and an ensuing drug court program.

The law enforcement expenditures are measured linearly, i. e.

$$h\, v(t). \tag{2.3}$$

Enforcement affects the drug price, the inflow and outflow of the stock of drug users as pointed out by Rydell and Everingham (1994): "[...] the money spent on supply control causes increases in the cost to producers of supplying the cocaine. That increased cost of supply gets passed along to the consumer as price increases, which in turn causes the number of users to decline as inflows to cocaine use decrease and outflows increase."

Furthermore, according to Kleiman (1993) the efficacy of a given level of enforcement on a drug market depends on the size of that market. Considering the number of users as a measure of the drug market's size, the drug price, $p$, depends on both $A(t)$ and $v(t)$, i. e.

$$p = p(A(t), v(t)) = p\left(\frac{v(t)}{A(t) + \epsilon}\right), \tag{2.4}$$

where $\epsilon$ is a small positive constant avoiding zero denominators if $A(t)$ approaches zero.

According to the statements above, it is reasonable to assume

$$\frac{\partial p}{\partial A} < 0, \quad \frac{\partial p}{\partial v} > 0. \tag{2.5}$$

Similar to Tragler et al. (1997) we assume that the price is constant, if there is no drug enforcement.

Denote the inflow into the stock of users by $I$, and the quit rate by $Q$, then according to Rydell & Everingham (1994) the following mathematical properties can be derived:

$$I = I(p), \quad \frac{\partial I}{\partial p} < 0; \tag{2.6}$$

$$Q = Q(p), \quad \frac{\partial Q}{\partial p} > 0. \tag{2.7}$$

Since arrested drug users take part in a drug court program, there is an additional outflow of the stock of users, namely the successfully treated arrestees, $\alpha_1 \alpha_2 \varphi(p^{1-\eta}) A(t)$.

Combining the equations (2.2)–(2.4) the utility functional is given by

$$J = \int_0^\infty e^{-rt} \Big\{ A(t) \Big[ \rho\, p(A(t), v(t))^{-\eta} + \beta\, \varphi\Big(p(A(t), v(t))^{1-\eta}\Big) + \alpha_1 d\, \varphi\Big(p(A(t), v(t))^{1-\eta}\Big) \Big] + h\, v(t) \Big\} dt, \tag{2.8}$$

where $r$ denotes the discount rate. Consequently the objective of the decision-maker is

$$\min_{v(t) \geq 0} J$$

subject to the system dynamic

$$\dot{A}(t) = I(p(A(t), v(t)) - Q(p(A(t), v(t))A(t) - \alpha_1\, \alpha_2\, \varphi\Big(p(A(t), v(t))^{1-\eta}\Big)\, A(t). \tag{2.9}$$

The decision-making problem is that on the one side enforcement reduces the social costs of drug consumption and the growth of the number of users, but on the other side enforcement raises the drug price and thus enforcement raises the number of drug-related property crimes, which increases the social costs of property crime and the drug court program costs.

In what follows, we distinguish between two types of controlling:

1. The control is proportional to the number of users, that means

$$v(t) = GA(t) \qquad \forall t, \tag{2.10}$$

with $G$ being the factor of proportionality.[1] By inserting (2.10) in (2.9), the system dynamics reduces to a first-order ordinary differential equation:

$$\dot{A} = I(p(A,G)) - Q(p(A,G))\,A - \alpha_1\,\alpha_2\,\varphi(p(A,G)^{1-\eta})A. \tag{2.11}$$

The utility functional (2.8) becomes

$$J = \int_0^\infty e^{-rt}\Big\{A\Big[\rho\,p(A,G)^{-\eta} + \beta\,\varphi(p(A,G)^{1-\eta}) + \alpha_1\,d\,\varphi(p(A,G)^{1-\eta}) + h\,G\Big]\Big\}\,dt. \tag{2.12}$$

The law enforcement agency may wish to choose $G$ optimally, i.e.

$$G = \arg\min_G J.$$

In the case that the law enforcement agency is not aware of the drug problem or is not willing to spend money for drug enforcement, which means that

$$v(t) = 0 \qquad \forall t, \tag{2.13}$$

$G$ is set to zero and the optimization problem becomes a descriptive model.

2. The control is chosen without restriction (except for the non-negativity assumption).

# 3 Analysis

In this section we provide the analysis of the models presented above. The numerical computations have been performed with the program package MATHEMATICA (Wolfram, 1997).

[1] If there is no danger of confusion time arguments $t$ of $A$ and $v$ will be omitted in what follows.

## 3.1 Specification of the Price, Initiation and Quit Rate Functions

We model price as linear in the ratio of enforcement to market size, i. e.

$$p(A, v) = b + c\frac{v}{A + \epsilon}, \tag{3.1}$$

which is the simpliest function satisfying the mathematical properties assumed in the section before. Consequently, the drug price in the absence of enforcement is given by $b$, and $c$ measures the efficiency of enforcement.

Furthermore, we assume the following functions for the initiation and quit rate:

$$I(p) = kp^{\omega}, \quad \omega < 0; \tag{3.2}$$

$$Q(p) = \mu p^{\gamma}, \quad \gamma > 0, \tag{3.3}$$

where $\omega$ and $\gamma$ are elasticities specifying how price supresses initiation and stimulates desistance, respectively. The parameters $k$ and $\mu$ represent proportionality constants.

## 3.2 Derivation of the Base Parameter Values

We assume that monetary units of the parameter values are in \$ $1,000$. From Tragler et al. (1997) we get that in the base year

- the number of users, $A$, is $6,500,000$;
- the enforcement spending amounts to \$ 9,500,000,000, hence $v$ equals $9,500,000$;
- the cocaine price per gram is \$ 106.73;
- the total consumption is 291 metric tons;
- the social costs per gram consumed sum up to \$ 100;
- the inflow into use is $1,000,000$;
- the outflow including treatment which is not based on the drug court program is $785,800$;
- the absolute value of the short term elasticity of demand, $\eta$, is 0.5;
- the elasticity of initiation, $\omega$, is $-0.25$;

- the elasticity of exit, $\gamma$, is 0.25;
- the discount rate, $r$, equals 0.04.

As in Caulkins et al. (1997), we assume that a 1% increase in enforcement spending causes a $0.\overline{36}$% increase in price with base price value \$ 106.73 which yields $b = 0.06792$ and $c = 0.02655$, respectively, which implies that the per gram cocaine price without any enforcement is around \$ 68.

Furthermore, the term $\rho A\, p(A, v)^{-\eta}$ in the utility functional represents the social costs caused by consumption. With a total consumption of 291 metric tons and social costs of \$ 100 per gram consumed, the total social costs due to drug consumption amount to \$ 29,100,000,000. From

$$\rho\, A\Big(b + c\frac{v}{A+\epsilon}\Big)^{-\eta} = 29,100,000,$$

we finally get $\rho = 1.46259$.

For the initiation and the outflow of drug use, the following equalities must hold:

$$k\Big(b + c\frac{v}{A+\epsilon}\Big)^{\omega} = 1,000,000,$$
$$\mu\Big(b + c\frac{v}{A+\epsilon}\Big)^{\gamma} A = 785,800,$$

which yield $k$=571,572.9 and $\mu$=0.21151, respectively.

Rydell et al. (1996) estimate that the probability that treating a heavy user is successful equals 13.2%. In Rydell et al.'s model, a heavy user consumes 118.93 grams per year. But we consider a mixture of heavy users and light users with a total consumption of 291 metric tons, which yields an average consumption of 44.77 grams in the base year. So succesful treatment gets rid of the equivalent of 118.93/44.77 of this type of users. Consequently, $\alpha_2$ equals 0.132 times this ratio.

The National Criminal Victimization Survey estimates that there were 1.3 million robberies, 5.482 million household burglaries, 1.763 milllion motor vehicle thefts, and 23.7 million other thefts in 1994 (Bureau of Justice Statistics, 1996). Because theft often does not lead to an arrest (and a drug court program), thefts are not included in our considerations. Assuming furthermore that 22.5% of all property crimes are perpetrated by drug users, 0.295788 property crimes per capita are commited by drug users. This number should equal $sp^{1-\eta}$, and therefore we get $s$=0.90539. Furthermore, we assume that one third of the drug-related property crimes are committed in order to finance other goods than drugs, hence $q$ equals 0.14789.

From Miller et al. (1996) we get that the average social loss of one robbery equals \$ 8,000, that of one burglary \$ 1,400, and that of one motor vehicle theft \$ 3,700. The weighted social damage of property crime $\beta$ is therefore 2.87863.

In 1994, there were the following numbers of arrest (Bureau of Justice Statistics, 1996): 172,290 for robbery, 396,100 for burglary and 200,200 for motor vehicle theft. If one assumes that there are 1.2 offenders per offense on average, then the probability of arrest given commision of a robbery, MVT and burglary appears to be 0.11, 0.095 and 0.06, respectively. The probability of getting arrested, $\alpha_1$, is therefore 0.07483.

The proportionality constant of the law enforcement expenditures in the objective function, $h$, is set to one.

The constant $\epsilon$ guaranteeing the non-negativity of the denominator of the fraction of the price function is assumed to be 0.001.

Furthermore, the costs of an arrest and an ensuing drug court program amount to \$ 5,000, thus $d$ is 5.

Finally the base parameters values are summed up in table 3.1.

Table 3.1: Base parameter values

| Parameter | Base Value |
|---|---|
| $\alpha_1$ | 0.07483 |
| $\alpha_2$ | 0.35065 |
| $b$ | 0.06792 |
| $\beta$ | 2.87863 |
| $c$ | 0.02655 |
| $d$ | 5 |
| $\epsilon$ | 0.001 |
| $\eta$ | 0.5 |
| $\gamma$ | 0.25 |
| $h$ | 1 |
| $k$ | $571,572.9$ |
| $\mu$ | 0.21151 |
| $\omega$ | $-0.25$ |
| $q$ | 0.14789 |
| $r$ | 0.04 |
| $\rho$ | 1.46259 |
| $s$ | 0.90539 |

## 3.3 The Model with a Control Proportional to the Number of Users

Neglecting the small constant $\epsilon$ and substituting $v = GA$, (3.1) reduces to

$$p = b + cG, \tag{3.4}$$

and the system dynamics becomes a first-order linear ordinary differential equation, which can easily be solved analytically:

$$A_{pro}(t) = \frac{kp^{\omega}}{\Omega} + \left(A_0 - \frac{kp^{\omega}}{\Omega}\right) e^{-\Omega(t-t_0)}, \qquad A_0 = A(t_0) \tag{3.5}$$

where

$$\Omega = \mu p^{\gamma} + \alpha_1 \alpha_2 \varphi(p^{1-\eta}).$$

Using the fact that $\Omega \geq 0$,

$$\hat{A}_{pro} := \lim_{t\to\infty} A_{pro}(t) = \frac{kp^{\omega}}{\Omega} \tag{3.6}$$

is the steady state to be approached. Solving the utility functional (2.12) by substituting (3.5) and (3.4) yields

$$J_{pro} := \frac{\Psi}{\Omega + r}\left(\frac{kp^{\omega}}{r} + A_0\right), \tag{3.7}$$

where

$$\Psi = \rho p^{-\eta} + \beta\varphi(p^{1-\eta}) + \alpha_1 d\varphi(p^{1-\eta}) + hG.$$

The minimum of $J_{pro}$ is reached at $G = 2.52581$ and its value there is \$ 1,056,575,250,000. In figure 3.1 it can be seen that to spend nothing on drug control is more expensive than to spend an amount up to \$ 8,503 per addict. The steady state number of users as a function of $G$ can be seen in figure 3.2. It is plausible that the steady state number of users is a decreasing function of $G$.

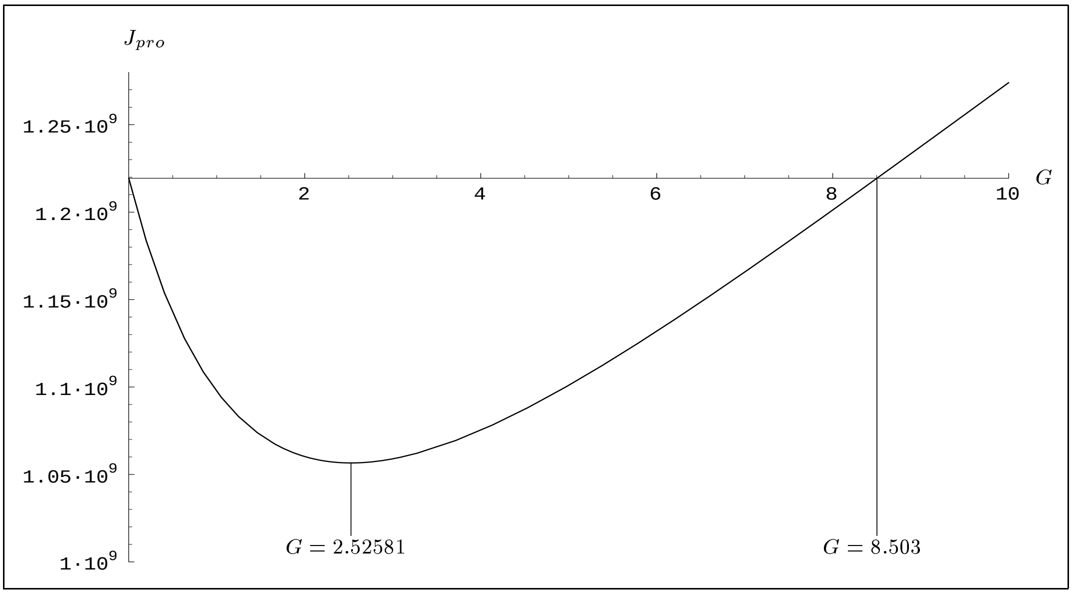


Figure 3.1: $J_{pro}$ with $A_0 = 100,000$ as a function of $G$

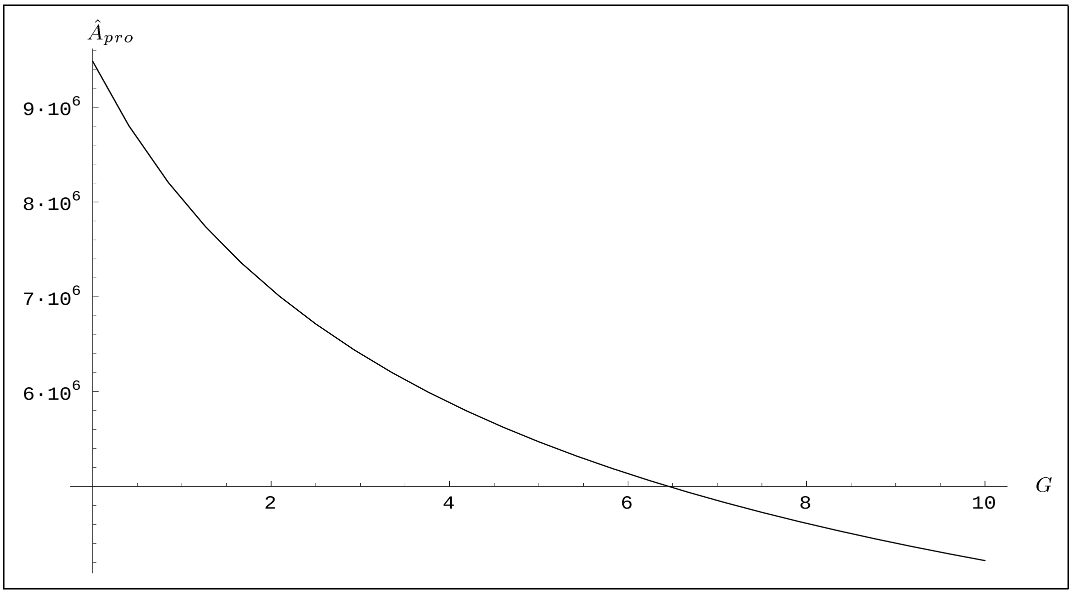


Figure 3.2: $\hat{A}_{pro}$ as a function of $G$

## 3.4 The Model with non-restricted control

### 3.4.1 Results of the Base Case

In order to apply the maximum principle (see, e.g., Feichtinger & Hartl, 1986, or Leonard & Long, 1992) the minimization problem must be changed into a maximization problem. Since minimizing a function is equivalent to maximizing the negative function, the objective of the decision maker is

$$\max_{v \geq 0} -J.$$

Let us define

$$F(A,v) := A\left[\rho p^{-\eta} + \beta\varphi(p^{1-\eta}) + \alpha_1 d\varphi(p^{1-\eta})\right] + hv \tag{3.8}$$

and

$$f(A,v) := \dot{A} = kp^{\omega} - \mu p^{\gamma} A - \alpha_1\alpha_2\left(q + sp^{1-\eta}\right)A. \tag{3.9}$$

The current value Hamiltonian $H$ for the optimization problem is then simply given by

$$H = -F(A,v) + \lambda f(A,v), \tag{3.10}$$

where $\lambda$ denotes the current value costate variable. Applying Pontryagin's maximum principle to (3.10) yields the necessary optimality condition

$$v = \arg\max_{v} H. \tag{3.11}$$

In this section we only provide the analysis where the non-negativity constraint for $v$ is satisfied. The whole analysis is given in Appendix A.

Assuming $v > 0$, we derive

$$\lambda = \frac{F_v}{f_v} \tag{3.12}$$

from the condition $H_v = 0$. The numerical analysis shows that the concavity of $H$ with respect to $v$ holds at least in the steady state. The costate equation runs

$$\dot{\lambda} = r\lambda - H_A, \tag{3.13}$$

from which together with (3.12) a differential equation for the control $v$ can be derived (for details, see Appendix B)

$$\dot{v} = \frac{f\left(F_v f_{vA} - F_{vA} f_v\right) + F_v f_v\left(r - f_A\right) + f_v^2 F_A}{F_{vv} f_v - F_v f_{vv}}. \tag{3.14}$$

For the numerical computations we use the base parameter values from table 3.1. The steady state values for this probelm can be computed by setting $\dot{A}$ and $\dot{v}$ equal to zero (grey lines in figure 3.3). In figure 3.3 it can be seen that the intersection of the isoclines is a saddle point equilibrium $(\hat{A}, \hat{v})$. The values of $\hat{A}$ and $\hat{v}$ are given in table 3.2. The two stable manifolds (thick black curves) yield the optimal trajectories.

| $\hat{A}$ | $\hat{v}$ | $J^*$ |
|---|---|---|
| $7,028,458$ | $14,509,184$ | $1,038,262,056$ |

Table 3.2: Steady state values

The optimal drug enforcement time path according to our parameter setting is increasing (but less than proportionally to the number of users) for an initial value of users smaller than $\hat{A}$. For initial values higher than $\hat{A}$, the optimal enforcement path is decreasing.

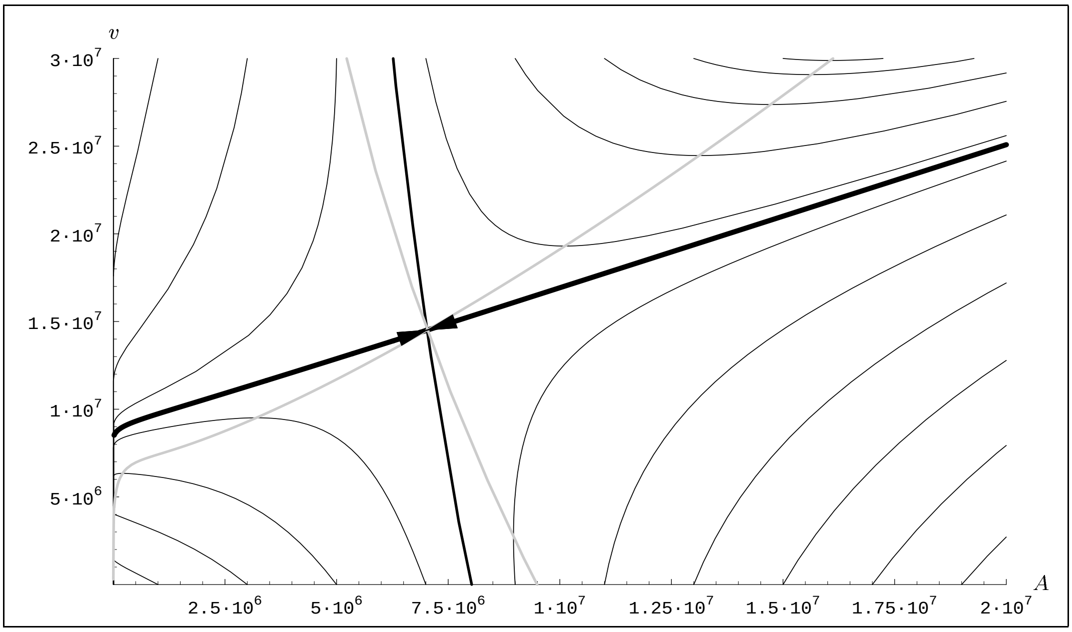


Figure 3.3: Phase portrait in the $A$-$v$-plane

### 3.4.2 Sensitivity Analysis

Since most of the base assumptions are only estimations, it is important to analyse how the results change due to deviations of the base parameter values. Table 3.3 gives the percent changes of the steady state values $\hat{A}$ and $\hat{v}$ caused by a one percent change of the base parameter values.

| Parameter | Value $\cdot$ 1.01 | $e_{\hat{A}}$ | $e_{\hat{v}}$ |
|---|---|---|---|
| $\alpha_1$ | 0.07558 | -0.104 | -0.045 |
| $\alpha_2$ | 0.35416 | -0.101 | -0.048 |
| $b$ | 0.06860 | 0.113 | -1.626 |
| $\beta$ | 2.90742 | -0.008 | 0.027 |
| $c$ | 0.026821 | -0.426 | 0.449 |
| $d$ | 5.05 | -0.001 | 0.004 |
| $\epsilon$ | 0.00101 | 0 | 0 |
| $\eta$ | 0.505 | -0.772 | 2.361 |
| $\gamma$ | 0.2525 | 0.358 | 0.887 |
| $h$ | 1.01 | 0.428 | -1.454 |
| $k$ | 577,288.629 | 1 | 1 |
| $\mu$ | 0.21362 | -0.968 | -0.683 |
| $\omega$ | -0.2525 | 0.355 | 1.104 |
| $q$ | 0.14937 | -0.047 | 0.035 |
| $r$ | 0.0404 | 0.077 | -0.263 |
| $\rho$ | 1.47722 | -0.417 | 1.424 |
| $s$ | 0.91445 | -0.063 | -0.053 |

Table 3.3: Percent changes $e_{\hat{A}}$ and $e_{\hat{v}}$ of $\hat{A}$ and $\hat{v}$, respectively, with respect to a one percent change of the base parameter values

A graphical representation of the changes of the steady state values due to a one percent change of the parameters $\eta$, $\rho$, $r$, $h$ and $s$ is given in figure 3.4. As a parameter change also affects the slope of the stable manifolds, with the thickness of the arrows in figure 3.4 we indicate how the slope of the manifolds changes relative to the base case. The thicker the arrow is, the greater is the slope. The reference case is given in the legend of the figure.

Especially, note the strong influence of $\eta$ on $\hat{v}$. Figure 3.5, in which the parameter $\eta$ is reduced from 0.5 to 0.4, shows how sensitive the model behaves with respect to changes in $\eta$.

In the following paragraphs we want to motivate this kind of tilting of the stable manifolds. The following considerations cannot explain exactly why the stable manifolds slope upward and downward for $\eta = 0.5$ and $\eta = 0.4$,

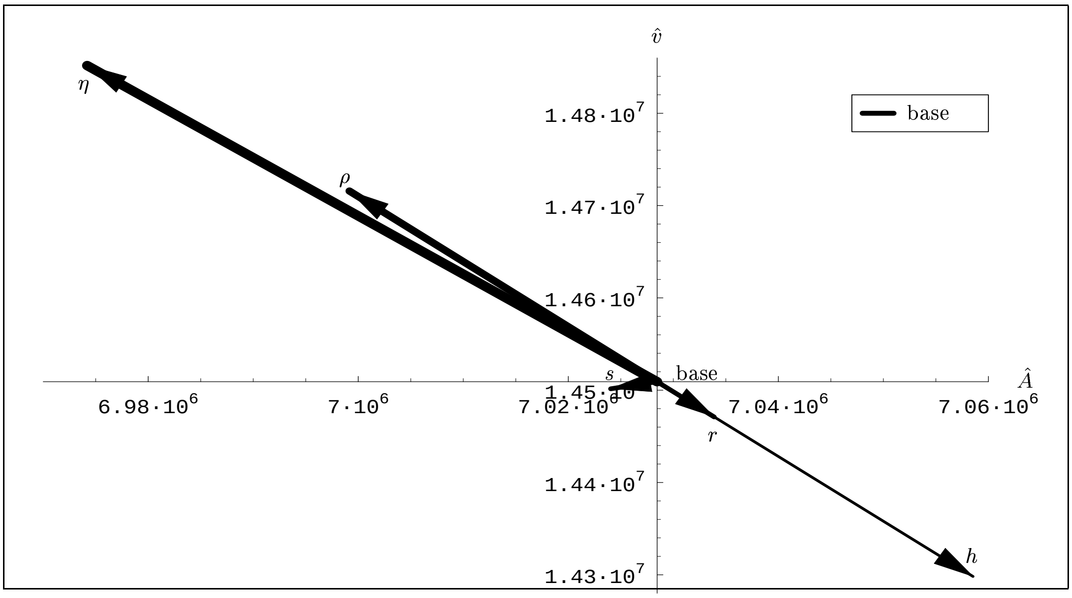


Figure 3.4: A graphical representation of the changes of the steady state values due to a one percent change of the parameters $\eta$, $\rho$, $r$, $h$ and $s$.

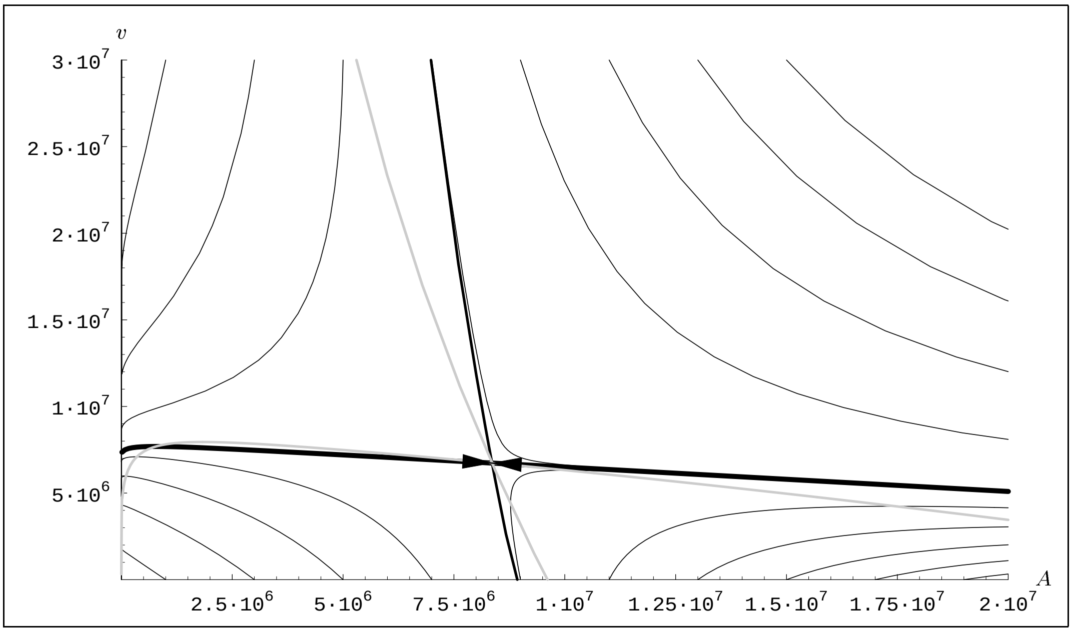


Figure 3.5: Phase-portrait in the $A$-$v$-plane with $\eta = 0.4$

respectively, but we try to explain why it is possible, that the manifolds tilt due to a change of $\eta$ from 0.5 to 0.4.

First of all, we look how enforcement $v$ affects the objective function. A rise of $v$ alters the objective function threefold. Firstly, an increase of $v$ raises directly the objective function by increasing the enforcement costs $hv$.

Secondly, the objective function is affected by $v$ through the drug price, $p$. For a fixed $A$, a rise of $v$ increases the price, $p$, which reduces the consumption costs, $A\,\rho\,p^{-\eta}$, on the one hand. On the other hand, an increase of $v$ leads to an increase of the costs of property crime, $A\,\beta\,\varphi(p^{1-\eta})$, the cost of the drug court program, $A\,\alpha_1\,d\,\varphi(p^{1-\eta})$, since $\varphi$ is an increasing function of $p^{1-\eta}$. The functions $p^{-\eta}$ and $p^{1-\eta}$ are shown in figures 3.6 and 3.7, respectively, both for $\eta = 0.4$ and $\eta = 0.5$.

Thirdly, the objective function also depends indirectly on $v$ through $A$, because a rise of enforcement reduces $\dot{A}$. Since the drug price, $p$, also depends on $A$, $v$ affects $p$ twice.

With the help of these considerations, we are able to explain, why the tilting can happen because of a reduction of $\eta$ from 0.5 to 0.4. If the initial value of $A$ is small, $\dot{A}$ is positive. In the case of $\dot{A} > 0$, $p$ decreases due to the rise of $A$. Consequently, $p^{-\eta}$ increases and $p^{1-\eta}$ decreases. For $\eta = 0.5$, an increase of $v$ can compensate for the additional costs of the increase of $A$, because on the one hand an increase of $v$ reduces directly the fall of $p$ and on the other hand a rise of $v$ decreases indirectly the fall of $p$ by a reduced rise of $A$.

But for $\eta = 0.4$ the consumption costs per capita are substantially lower than for $\eta = 0.5$ and the slope of $p^{-\eta}$ is less for $\eta = 0.4$ than for $\eta = 0.5$ for $b \leq p \leq 1$ (compare figure 3.6). So the additional costs of a rise of enforcement (an increase of $hv$, a reduced decrease of $p^{1-\eta}$) can overweigh the profit of a reduced increase of the consumption costs per addict. Thus it is possible that it is even optimal to reduce enforcement, because $(\beta + \alpha_1 d)\,\varphi(p^{1-\eta})$ and $h\,v$ decrease.

In figure 3.4 it can be seen that $\rho$ has also a great influence on the steady state values and on the slope of the stable manifolds. The higher the value of $\rho$, the higher is the weight of consumption in the objective function and consequently the more $v$ rises after a small increase of $A$.

If the costs of consumption are left out (cf. Benson et al., 1992) which means $\rho = 0$, it seems to be optimal not to enforce, because a positive $v$ only seems to rise the remaining objective function terms. But the strategy "no enforcement" is only optimal in a static optimization. In our model, which is a dynamic one, it is optimal to enforce for a low stock of users even in this case and to continuously decrease the level of enforcement while the number of users is increasing until a threshold is reached. For a number of users

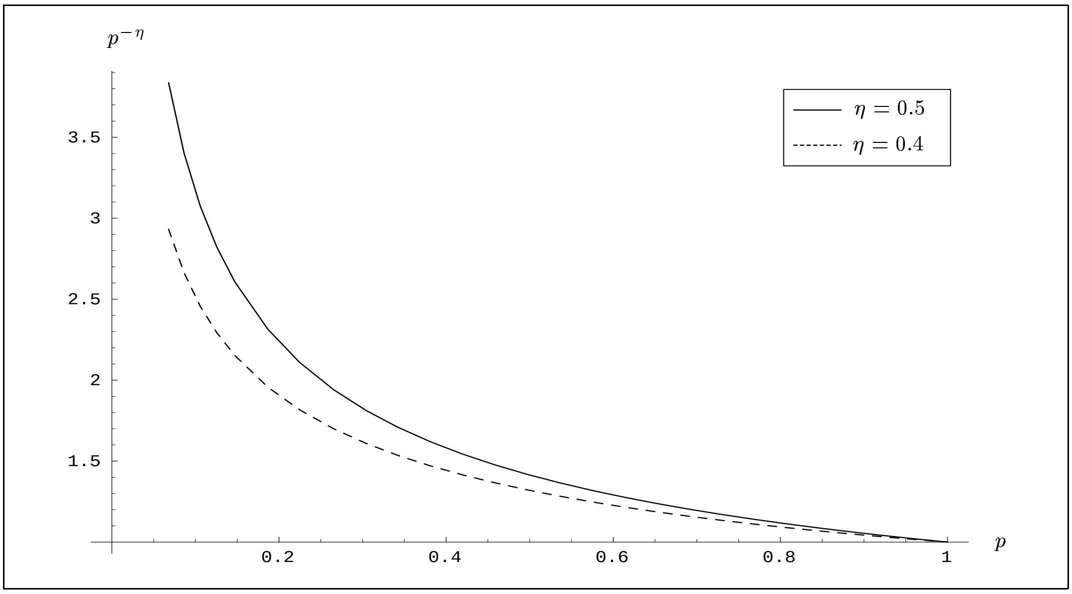


Figure 3.6: The function $p^{-\eta}$ for $\eta = 0.5$ and $\eta = 0.4$ for $b \leq p \leq 1$.

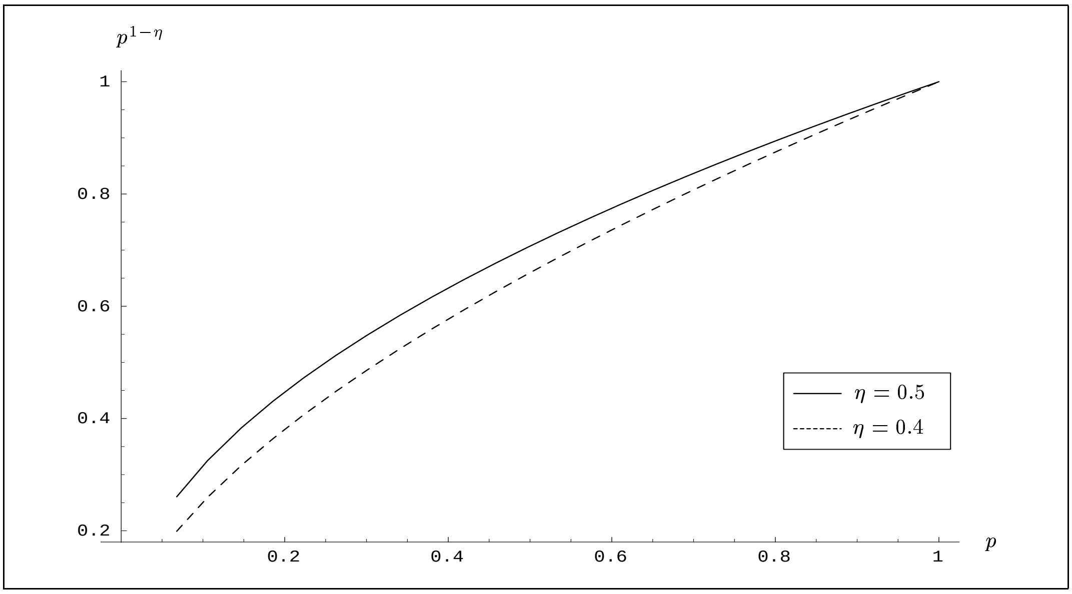


Figure 3.7: The function $p^{1-\eta}$ for $\eta = 0.5$ and $\eta = 0.4$ for $b \leq p \leq 1$.

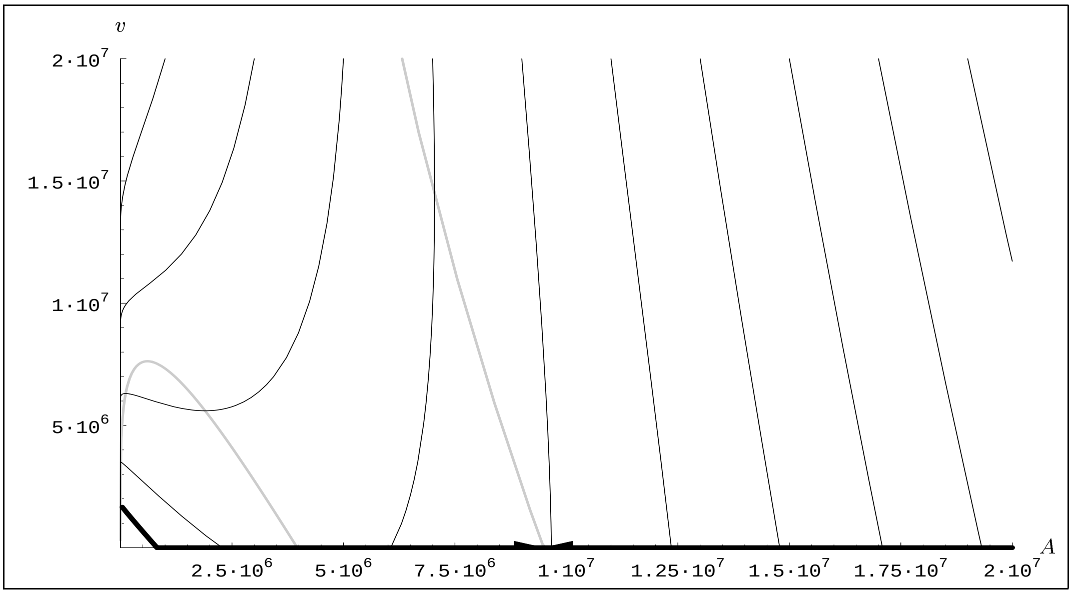


Figure 3.8: Phase portrait for $\rho = 0$ in the $A$-$v$-plane.

greater than this threshold it is optimal to stop enforcement (see figure 3.8)

The following considerations motivate this result. As mentioned above, a rise of enforcement $v$ lowers $\dot{A}$, the growth of the number of users. In the contrary to a rise of $v$, a rise of $A$ reduces $p^{1-\eta}$, since an increase of $A$ reduces $p$.

In the case of $\dot{A} > 0$, a rise of $v$ lowers the rise of $A$. But $p^{1-\eta}$ increases directly due to the rise of $v$, since a rise of $v$ increases $p$, and indirectly due to the reduced rise of $A$. Consequently, the effects of a rise of $v$ on the cost of property crime, $A\,\beta\,\varphi(p^{1-\eta})$, and on the costs of the drug court program, $A\,\alpha_1\,d\,\varphi(p^{1-\eta})$, are ambiguous, since $\varphi$ is an increasing function of $p^{1-\eta}$. If $A$ is very small, the additional costs of an increase of $v$ (the rise of $p^{1-\eta}$) can be offset by the reduced costs of the rise of $A$. Otherwise, an increase of $v$ in order to lower the rise of $A$ is too costly, because a rise of $v$ would increase $p^{1-\eta}$ too much. So the additional costs of a rise of $A$ can be compensated by setting $v$ to zero, because for $v = 0$, $p^{1-\eta}$ reaches a minimum. Summing up, it can be said, that whether enforcement is done or not depends on whether the gain of lowering $\dot{A}$ overweighs the additional costs of enforcement. The derivation of this threshold can be seen in Appendix A.

In the case of $\dot{A} < 0$, the increase of $p^{1-\eta}$ due to the fall of $A$ can be annihilated by setting $v = 0$. This effect overweighs the costs of the lower fall of $A$.

The steady state values of $A$ and $v$ and the values of the utility functional, splitted up in costs of consumption, property crime, drug court program and enforcement, as functions of $\rho$ can be seen in figure 3.9 and 3.10. The higher $\rho$ is, the more enforcement will be done and the less is the steady state number of users, because of a decreasing initiation function and increasing quit rate function of $v$. Furthermore, the higher the weight of consumption, $\rho$, is, the higher are the costs of consumption. The drug court costs and property crime costs are decreasing for an increasing $\rho$ due to the decreasing number of users.

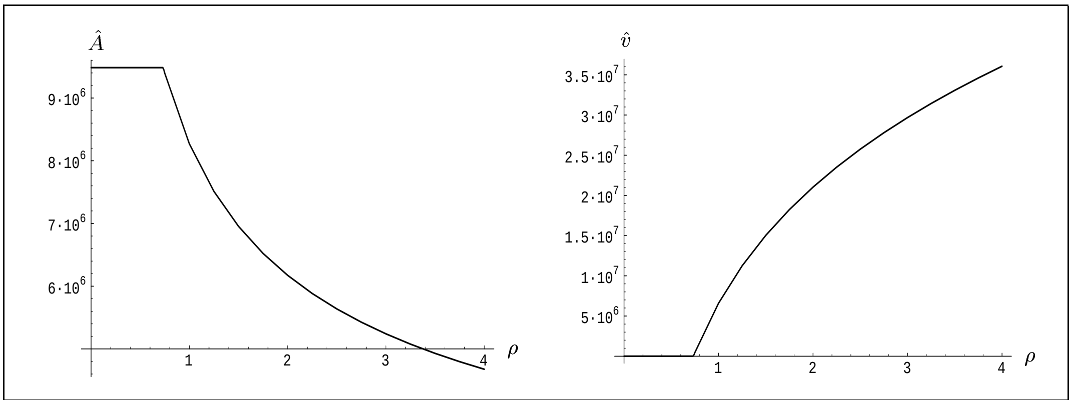


Figure 3.9: The steady state values of $A$ and $v$ as function of $\rho$

The change of the slope of the stable manifolds due to a rise of $h$ (figure 3.4) is simply explained. The more costly enforcement is, the less enforcement will be done.

As stated above, parameter changes affect also the slope of the stable manifolds and thus the path of the optimal solution. The intensity of the changes can also be illustrated by categorizing the optimal solution in five strategies:

$$\begin{array}{llll} a) & v^*(0) > 0 & \hat{v} > 0 & \hat{v} > v^*(0) \\ b) & v^*(0) > 0 & \hat{v} > 0 & \hat{v} < v^*(0) \\ c) & v^*(0) > 0 & \hat{v} = 0 & \\ d) & v^*(0) = 0 & \hat{v} = 0 & \\ e) & v^*(0) = 0 & \hat{v} > 0, & \end{array}$$

where $v^*(0)$ is the initial optimal enforcement spending and $\hat{v}$ the value for $t \to \infty$. The possible changes of the strategies due to changes of $\rho$ and the elasticities can be seen in table 3.4.

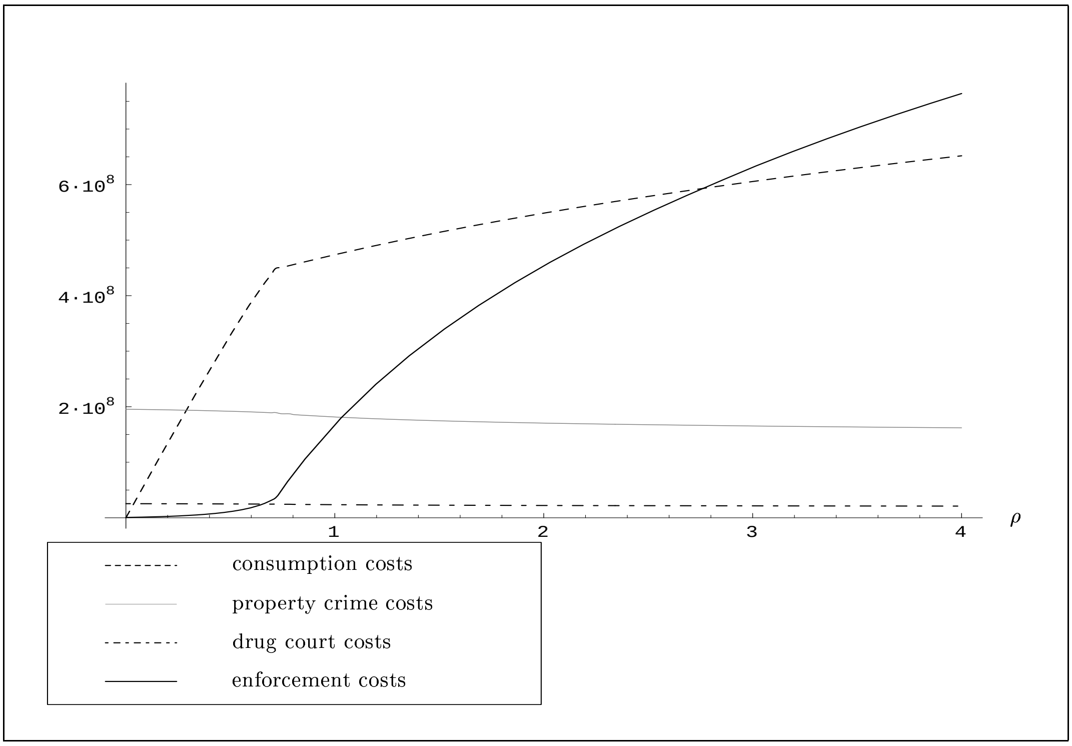


Figure 3.10: The values of the utility functional, split up in costs of consumption, property crime, drug court program and enforcement as functions of $\rho$.

| Paramter value | Strategy | | $J^*$ | | $\hat{A}$ | $\hat{v}$ |
|---|---|---|---|---|---|---|
| | $A_0 = 10^5$ | $A_0 = 2 \cdot 10^7$ | $A_0 = 10^5$ | $A_0 = 2 \cdot 10^7$ | | |
| base | $a)$ | $b)$ | 1,038,236,848 | 1,919,876,496 | 7,028,458 | 14,509,183 |
| $\rho = 0.7312955$ | $c)$ | $d)$ | 705,367,929 | 1,231,183,235 | 9,484,580 | 0 |
| $\rho = 2.925182$ | $a)$ | $b)$ | 1,426,201,728 | 2,791,031,936 | 5,293,551 | 29,133,823 |
| $\eta = 0.25$ | $c)$ | $d)$ | 667,702,210 | 1,157,190,586 | 9,734,490 | 0 |
| $\eta = 1$ | $a)$ | $b)$ | 1,456,703,971 | 6,077,135,067 | 4,185,789 | 36,769,789 |
| $\omega = -0.125$<br>$\gamma = 0.125$<br>$k = 756,024$<br>$\mu = 0.1599053$ | $a)$ | $b)$ | 1,063,365,877 | 1,894,183,394 | 7,874,665 | 6,838,394 |
| $\omega = -0.5$<br>$\gamma = 0.5$<br>$k = 326,696$<br>$\mu = 0.370045744$ | $a)$ | $b)$ | 727,290,703 | 1,933,944,634 | 2,160,301 | 25,222,538 |

Table 3.4: Optimal strategies

### 3.4.3 Impact of Delays of the Start of Controlling

Reality shows that in politics there is a difference between the moment the decision maker is aware of the problem and the start of controlling. If control starts $\tau$ periods after the problem arises the drug model remains uncontrolled in the first $\tau$ periods. Since the strategy "no enforcement" is not optimal in the base setting, the social costs are higher than in the case with controlling from the beginning. In figure 3.11 it can be seen how much a delay in starting the control increases the utility functional $J$.

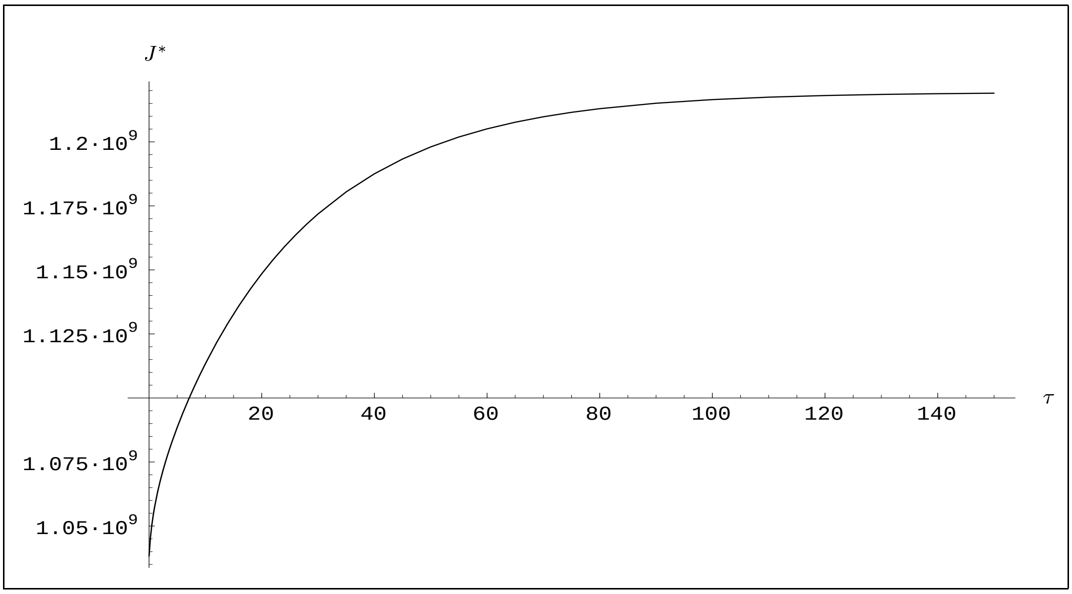


Figure 3.11: Optimal utility functional value $J^*$ as a function of $\tau$, where $\tau$ denotes the time when controlling starts and $A_0 = 100,000$.

# 4 Conclusions and Extensions

Summing up it can be said that whether it is optimal to enforce or not depends on various parameters and on the initial stock of users. Sensitivity analysis shows that the short term elasticity of demand and the estimation of the social costs of drug consumption have a great influence on the time path of the optimal solution, because they determine the gain of drug enforcement. In certain parameter settings it is optimal to stop enforcement, if the stock of users exceeds a threshold, which means, in an advanced drug epidemic it can be optimal not to enforce. If there are only few users, it is optimal to enforce in all analyzed parameter settings. But early in a drug epidemic the decision-maker may be not aware of the problem, and a delay in the start of controlling raises substantially the drug crime costs.

Since the computation of some parameters depends on the values of the elasticities, these parameters should also be changed if the elasticities are changed. Such an analysis will be provided in an ensuing paper.

# A Application of the Maximum Principle

In order to apply the maximum principle to the optimization problem subject to the non-negativity constraint for $v$, the Lagrangean $L$ has to be defined, i.e.

$$L = H + \pi v,$$

where $\pi$ represents the Lagrange multiplier. Then the necessary optimality conditions,

$$L_v = 0, \tag{A.1}$$

$$\dot{\lambda} = r\lambda - L_A, \tag{A.2}$$

$$\pi \geq 0, \quad \pi v = 0 \tag{A.3}$$

have to be satisfied on every continuity point of $v$, whereas $\lambda$ and $\pi$ have to be continuous and piecewisely continuous, respectively.

In the case $v > 0$, $\pi = 0$ can be derived from the complementary slackness condition in (A.3). Consequently, the optimality conditions reduce to

$$H_v = 0$$

$$\dot{\lambda} = r\lambda - H_A,$$

which are the necessary optimal conditions without the non-negativity restriction. The further analysis is already provided in Section 3.4.1.

For $v = 0$, the system dynamics (3.9) and (A.2) reduce to two ordinary differential equations with the following solutions:

$$A(t) = \frac{kp^\omega}{\Omega} + \left(A(t_0) - \frac{kp^\omega}{\Omega}\right) e^{-\Omega(t-t_0)}, \tag{A.4}$$

$$\lambda(t) = -\frac{\Psi}{r+\Omega} + \left(\lambda(t_0) + \frac{\Psi}{r+\Omega}\right) e^{(r+\Omega)(t-t_0)}, \tag{A.5}$$

where $\Psi$ and $\Omega$ are constants as defined in Section 3.3, and $t_0$ denotes the time when enforcement is stopped.

Because of the continuity assumption of $\lambda$, $\lambda(t_0)$ must equal $\frac{F_v(A(t_0),0)}{f_v(A(t_0),0)}$ (see (3.12)). Furthermore, $\pi$ equals $-H_v$ evaluated at $v = 0$. The thick lines in figure 3.8 with $\lambda \equiv -\frac{\Psi}{r+\Omega}$ for $v = 0$ are the optimal trajectories, because on the one hand, for small initial values of $A$, the assumptions of the theorem of Michel (Feichtinger and Hartl, 1986) are satisfied only for $\lambda = -\Psi/(r + \Omega)$ . On the other hand, for high initial values of $A$, $\lambda = -\Psi/(r + \Omega)$ is the only function guaranteeing $\pi \geq 0$. Since $\lambda$ is constant, $A(t_0)$ can be computed from

$$-\frac{\Psi}{r+\Omega} = \lambda(t_0) = \frac{F_v(A(t_0),0)}{f_v(A(t_0),0)}.$$

For initial values of $A$ greater than $A(t_0)$ it is optimal not to enforce. But for an initial value of $A$ lower than $A(t_0)$ it is optimal to enforce in the beginning and to reduce successively enforcement until $A(t_0)$ is reached, where enforcement is stopped.

# B Derivation of Equation (3.14)

Differentiating $H_v = 0$ with respect to time yields

$$\dot{v} = -\frac{\dot{A}\left(-F_{vA} + \lambda f_{vA}\right) + \dot{\lambda} f_v}{\left(-F_{vv} + \lambda f_{vv}\right)} \tag{B.1}$$

Using

$$H_A = -F_A + \lambda f_A$$

and substituting (3.13) and (3.12) in (B.1), we get

$$\dot{v} = -\frac{f\left(-F_{vA} + \frac{F_v}{f_v} f_{vA}\right) + F_v\left(r - f_A\right) + f_v F_A}{-F_{vv} + \frac{F_v}{f_v} f_{vv}}.$$

Multiplicating this fraction with $\frac{f_v}{f_v}$ yields (3.14).

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